

## A Note on Occupation Times of Stationary Processes

Marina Kozlova, *Abo Akademi University, Finland*  
Paavo Salminen, *Abo Akademi University, Finland*

### Abstract

Consider a real valued stationary process  $X = \{X_s, s \in \mathbb{R}\}$ . For a fixed  $t \in \mathbb{R}$  and a set  $D$  in the state space of  $X$ , let  $g_t$  and  $d_t$  denote the starting and the ending time, respectively, of an excursion from and to  $D$  (straddling  $t$ ). Introduce also the occupation times  $I^+_t$  and  $I^-_t$  above and below, respectively, the observed level at time  $t$  during such an excursion. In this note we show that the pairs  $(I^+_t, I^-_t)$  and  $(t - g_t, d_t - t)$  are identically distributed. This somewhat curious property is, in fact, seen to be a fairly simple consequence of the known general uniform sojourn law which implies that conditionally on  $I^+_t + I^-_t = v$  the variable  $I^+_t$  (and also  $I^-_t$ ) is uniformly distributed on  $(0, v)$ . We also particularize to the stationary diffusion case and show, e.g., that the distribution of  $I^-_t + I^+_t$  is a mixture of gamma distributions.

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Published on: June 9, 2005

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