Brownian couplings, convexity, and shy-ness

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Abstract

Benjamini, Burdzy and Chen (2007) introduced the notion of a *shy coupling*: a coupling of a Markov process such that, for suitable starting points, there is a positive chance of the two component processes of the coupling staying at least a given positive distance away from each other for all time. Among other results, they showed that no shy couplings could exist for reflected Brownian motions in C^2 bounded convex planar domains whose boundaries contain no line segments. Here we use potential-theoretic methods to extend this Benjamini *et al.*(2007) result (a) to all bounded convex domains (whether planar and smooth or not) whose boundaries contain no line segments, (b) to all bounded convex planar domains regardless of further conditions on the boundary.

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