On the parabolic generator of a general one-dimensional Lévy process

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Abstract

The purpose of this note is twofold. Firstly to complete a recent accumulation of results concerning extended version of Ito's formula for any one dimensional Lévy processes, X. Secondly, we use the latter to characterise the parabolic generator of X, [{bf A}:= left{ (f,g) : f(X_cdot,cdot) - int_0^cdot g(X_s, s)ds text{ is a local martingale} right}.] We also establish a necessary condition for a pair of functions to be in the domain of the parabolic generator when X has a Gaussian component.

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