

## The needlets bispectrum

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### Abstract

The purpose of this paper is to join two different threads of the recent literature on random fields on the sphere, namely the statistical analysis of higher order angular power spectra on one hand, and the construction of second-generation wavelets on the sphere on the other. To this aim, we introduce the needlets bispectrum and we derive a number of convergence results. Here, the limit theory is developed in the high resolution sense. The leading motivation of these results is the need for statistical procedures for searching non-Gaussianity in Cosmic Microwave Background radiation.

AMS 2000 subject classifications: Primary 62G20; secondary 62M15, 60B15, 60G60.

Keywords: Bispectrum, Needlets, Spherical Random Fields, Cosmic Microwave Background Radiation, High Resolution Asymptotics.



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