A class of unbiased location invariant Hill-type estimators for heavy tailed distributions

Jiaona Li, *School of Mathematics and Statistics, Southwest University, Chongqing, 400715*
Zuoxiang Peng, *School of Mathematics and Statistics, Southwest University, Chongqing, 400715*
Saralees Nadarajah

Abstract

Based on the methods provided in Caeiro and Gomes (2002) and Fraga Alves (2001), a new class of location invariant Hill-type estimators is derived in this paper. Its asymptotic distributional representation and asymptotic normality are presented, and the optimal choice of sample fraction by Mean Squared Error is also discussed for some special cases. Finally comparison studies are provided for some familiar models by Monte Carlo simulations.

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References

Beirlant, J., Dierckx, G., Guillou, A. and Stǎricǎ, C. (2002). On exponential representations of log-spacings of extreme order statistics. Extremes, 5, 157–180. MR1965977

Caeiro, F. and Gomes, M. I. (2002). A class of asymptotically unbiansed semi-parametric estimators of the tail index. Sociedad de Estadistica e Investigacion Operativa, 11, 345–364. MR1947602

de Haan, L. and Ferreira, A. (2006). Extreme Value Theory. Springer, New York. MR2234156

Dekkers, A. L. M., Einmahl, J. H. J. and de Haan, L. (1989). A moment estimator for the index of an extreme-value distribution. Annals of Statistics, 4, 1833–1855.

Embrechets, P., Kluppelberg, C. and Mikosch, T. (1997). Modelling Extremal Events for Insurance and Finance. Springer, New York. MR1458613

Fraga Alves, M. I. (2001). A location invariant Hill-type estimator. Extremes, 4, 199–217. MR1907061

Gomes, M. I., de Haan, L. and Henriques Rodrigues, L. (2008). Tail index estimation through the accomodation of bias in the weighted log-excesses. Journal of the Royal Statistical Society B, 70, 31–53.

Gomes, M. I. and Henriques Rodrigues, L. (2008). Tail index estimation for heavy tails:

accommodation of bias in the excesses over a high threshold. Extremes, doi 10.1007/s10687-008-0059-1.

Gomes, M. I. and Martins, M. J. (2002). "Asymptotically unbiased" estimators of the tail index based on external estimation of the second order parameter. Extremes, 5, 5–31. MR1947785

Gomes, M. I. and Martins, M. J. (2004). Bias reduction and explicit estimation of the tail index. Journal of Statistical Planning Inference, 124, 361–378. MR2080370

Hill, B. M. (1975). A simple general approach to inference about the tail of a distribution. Annals of Statistics, 3, 1163–1174. MR0378204

Ling C. X., Peng, Z. and Nadarajah, S. (2007a). A location invariant moment-type estimator (I). Theory Probability and Mathematical Statistics, 76, 22–30. MR2368736

Ling C. X., Peng, Z. and Nadarajah, S. (2007b). A location invariant moment-type estimator (II). Theory Probability and Mathematical Statistics, 77, 167–178.

Peng, L. (1998). Asymptotically unbiased estimator for the extreme-value index. Statistics and Probability Letters, 2, 107–115. MR1627906

Peng, L. and Qi, Y. (2006a). A new calibration method of constructing empirical likelihood-based confidence intervals for the tail index. Australian and New Zealand Journal of Statistics, 48, 59–66. MR2234779

Peng, L. and Qi, Y. (2006b). Confidence regions for high quantiles of a heavy tailed distribution. Annals of Statistics, 4, 1964–1986. MR2283723

Qi, Y. (2008a). On the tail index of a heavy tailed distribution. Annals of the Institute of Statistical Mathematics. To appear.

Qi, Y. (2008b). Bootstrap and empirical likelihood methods in extremes. Extremes, doi 10.1007/s10687-007-0049-8.