Electronic Journal of Statistics > Vol. 2 (2008) open journal systems

Two simple sufficient conditions for FDR control

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Abstract

We show that the control of the false discovery rate (FDR) for a multiple testing procedure is implied by two coupled simple sufficient conditions. The first one, which we call ``self-consistency condition'', concerns the algorithm itself, and the second, called ``dependency control condition'' is related to the dependency assumptions on the p-value family. Many standard multiple testing procedures are self-consistent (e.g. step-up, step-down or step-up-down procedures), and we prove that the dependency control condition can be fulfilled when choosing correspondingly appropriate rejection functions, in three classical types of dependency: independence, positive dependency (PRDS) and unspecified dependency. As a consequence, we recover earlier results through simple and unifying proofs while extending their scope to several regards: weighted FDR, p-value reweighting, new family of step-up procedures under unspecified p-value dependency and adaptive step-up procedures. We give additional examples of other possible applications. This framework also allows for defining and studying FDR control for multiple testing procedures over a continuous, uncountable space of hypotheses.

AMS 2000 subject classifications: Primary 62J15; secondary 62G10.

Keywords: False Discovery Rate, multiple testing, step-up, step-down, step-up-down, weighted p-values, PRDS condition.

Full Text: PDF

Blanchard, Gilles, Roquain, Etienne, Two simple sufficient conditions for FDR control, Electronic Journal of Statistics, 2, (2008), 963-992 (electronic). DOI: 10.1214/08-EJS180.

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Electronic Journal of Statistics. ISSN: 1935-7524