

On the frequentist coverage of Bayesian credible intervals for lower bounded means

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Abstract

For estimating a lower bounded location or mean parameter for a symmetric and logconcave density, we investigate the frequentist performance of the $100(1-\alpha)\%$ Bayesian HPD credible set associated with priors which are truncations of flat priors onto the restricted parameter space. Various new properties are obtained. Namely, we identify precisely where the minimum coverage is obtained and we show that this minimum coverage is bounded between $1 - \frac{3\alpha}{2}$ and $1 - \frac{3\alpha}{2} + \frac{\alpha^2}{1+\alpha}$; with the lower bound $1 - \frac{3\alpha}{2}$ improving (for $\alpha \leq 1/3$) on the previously established ([9]; [8]) lower bound $\frac{1-\alpha}{1+\alpha}$. Several illustrative examples are given.

AMS 2000 subject classifications: 62F10, 62F30, 62C10, 62C15, 35Q15, 45B05, 42A99.

Keywords: Bayesian credible sets, restricted parameter space, confidence intervals, frequentist coverage probability, logconcavity.



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
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