Lower bounds for posterior rates with Gaussian process priors

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Abstract

Upper bounds for rates of convergence of posterior distributions associated to Gaussian process priors are obtained by van der Vaart and van Zanten in [van der Vaart and van Zanten, Rates of contraction of posterior distributions based on Gaussian process priors, Ann. Statist., 36, 2008] and expressed in terms of a concentration function involving the Reproducing Kernel Hilbert Space of the Gaussian prior. Here lower-bound counterparts are obtained. As a corollary, we obtain the precise rate of convergence of posteriors for Gaussian priors in various settings. Additionally, we extend the upper-bound results of [van der Vaart and van Zanten, ibidem] about Riemann-Liouville priors to a continuous family of parameters.

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