Distributions that are both log-symmetric and R-symmetric

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Abstract

Two concepts of symmetry for the distributions of positive random variables Y are log-symmetry (symmetry of the distribution of $\log Y$) and R-symmetry [7]. In this paper, we characterise the distributions that have both properties, which we call doubly symmetric. It turns out that doubly symmetric distributions constitute a subset of those distributions that are moment-equivalent to the lognormal distribution. They include the lognormal, some members of the Berg/Askey class of distributions, and a number of others for which we give an explicit construction (based on work of A.J. Pakes) and note some properties; Stieltjes classes, however, are not doubly symmetric.

AMS 2000 subject classifications: Primary 62E10; secondary 60E05.

Keywords: double symmetry, lognormal distribution, moment equivalence, weighted distribution.

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