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Asymptotic properties of an estimator of the drift coefficients of multidimensional Ornstein-Uhlenbeck processes that are not necessarily stable

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Abstract

In this paper, we investigate the consistency and asymptotic efficiency of an estimator of the drift matrix, F, of Ornstein-Uhlenbeck processes that are not necessarily stable. We consider all the cases. (1)~The eigenvalues of F are in the right half space (i.e., eigenvalues with positive real parts). In this case the process grows exponentially fast. (2)~The eigenvalues of F are on the left half space (i.e., the eigenvalues with negative or zero real parts). The process where all eigenvalues of F have negative real parts is called a stable process and has a unique invariant (i.e., stationary) distribution. In this case the process does not grow. When the eigenvalues of F have zero real parts (i.e., the case of zero eigenvalues and purely imaginary eigenvalues) the process grows polynomially fast. Considering (1) and (2) separately, we first show that an estimator, $hat{F}$, of F is consistent. We then combine them to present results for the general Ornstein-Uhlenbeck processes. We adopt similar procedure to show the asymptotic efficiency of the estimator.

AMS 2000 subject classifications: Primary 62M05; secondary 60F15.

Keywords: Ornstein-Uhlenbeck processes, stable process, drift coefficient matrix, estimation, consistency, asymptotic efficiency.

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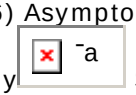
[7] Basawa, I.V., Feigin, P.D. and Heyde, C.C. (1976) Asymptotic properties of maximum likelihood estimators for stochastic processes. Sankhy  Ser. A 38 no. 3, 259–270. MR0652549

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