Electronic Journal of Statistics > Vol. 1 (2007) open journal systems

Needlet algorithms for estimation in inverse problems

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Abstract

We provide a new algorithm for the treatment of inverse problems which combines the traditional SVD inversion with an appropriate thresholding technique in a well chosen new basis. Our goal is to devise an inversion procedure which has the advantages of localization and multiscale analysis of wavelet representations without losing the stability and computability of the SVD decompositions. To this end we utilize the construction of localized frames (termed ``needlets") built upon the SVD bases. We consider two different situations : the ``wavelet" scenario, where the needlets are assumed to behave similarly to true wavelets, and the ``Jacobi-type" scenario, where we assume that the properties of the frame truly depend on the SVD basis at hand (hence on the operator). To illustrate each situation, we apply the estimation algorithm respectively to the deconvolution problem and to the Wicksell problem. In the latter case, where the SVD basis is a Jacobi polynomial basis, we show that our scheme is capable of achieving rates of convergence which are optimal in the L_2 case, we obtain interesting rates of convergence for other L_p norms which are new (to the best of our knowledge) in the literature, and we also give a simulation study showing that the NEED-VD estimator outperforms other standard algorithms in almost all situations.

AMS 2000 subject classifications: Primary 62G05, 62G20; secondary 65J20.

Keywords: statistical inverse problems, minimax estimation, second-generation wavelets.

Full Text: PDF

Kerkyacharian, Gérard, Petrushev, Pencho, Picard, Dominique B., Willer, Thomas, Needlet algorithms for estimation in inverse problems, Electronic Journal of Statistics, 1, (2007), 30-76 (electronic). DOI: 10.1214/07-EJS014.

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