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Central limit theorems in linear structural error-in-variables models with explanatory variables in the domain of attraction of the normal law

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Abstract

Linear structural error-in-variables models with univariate observations are revisited for studying modified least squares estimators of the slope and intercept. New marginal central limit theorems (CLT's) are established for these estimators, assuming the existence of four moments for the measurement errors and that the explanatory variables are in the domain of attraction of the normal law. The latter condition for the explanatory variables is used the first time, and is so far the most general in this context. It is also optimal, or nearly optimal, for our CLT's. Moreover, due to their Studentized and self-normalized forms, the obtained CLT's are automatically free of unknown parameters of the joint distribution of the error and explanatory variables. Consequently, they lead to a variety of readily available, or easily derivable, large-sample approximate confidence intervals (CI's) for the slope and intercept. In contrast, in related CLT's in the literature so far, the variances of the limiting normal distributions, in general, are complicated and depend on various, typically unknown, moments of the error and explanatory variables. Thus, the corresponding CI's for the slope and intercept in the literature, unlike those of the present paper, are available only under some additional model assumptions.

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Keywords: central limit theorem, domain of attraction of the normal law, explanatory variables, large-sample approximate confidence interval, linear structural error-in-variables model, measurement errors, modified least squares estimators, self-normalization, Studentization, reliability ratio.

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