

Sensitivity of principal Hessian direction analysis

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Abstract

We provide sensitivity comparisons for two competing versions of the dimension reduction method principal Hessian directions (pHd). These comparisons consider the effects of small perturbations on the estimation of the dimension reduction subspace via the influence function. We show that the two versions of pHd can behave completely differently in the presence of certain observational types. Our results also provide evidence that outliers in the traditional sense may or may not be highly influential in practice. Since influential observations may lurk within otherwise typical data, we consider the influence function in the empirical setting for the efficient detection of influential observations in practice.

AMS 2000 subject classifications: Primary 62F35; secondary 62H12.

Keywords: dimension reduction, influence function, influential observations, principal hessian directions.



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