# New multivariate central limit theorems in linear structural and functional error-in-variables models

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## Abstract

This paper deals simultaneously with linear structural and functional error-in-variables models (SEIVM and FEIVM), revisiting in this context generalized and modified least squares estimators of the slope and intercept, and some methods of moments estimators of unknown variances of the measurement errors. New joint central limit theorems (CLT's) are established for these estimators in the SEIVM and FEIVM under some first time, so far the most general, respective conditions on the explanatory variables, and under the existence of four moments of the measurement errors. Moreover, due to them being in Studentized forms to begin with, the obtained CLT's are a priori nearly, or completely, data-based, and free of unknown parameters of the distribution of the errors and any parameters associated with the explanatory variables. In contrast, in related CLT's in the literature so far, the covariance matrices of the limiting normal distributions are, in general, complicated and depend on various, typically unknown parameters that are hard to estimate. In addition, the very forms of the CLT's in the present paper are universal for the SEIVM and FEIVM. This extends a previously known interplay between a SEIVM and a FEIVM. Moreover, though the particular methods and details of the proofs of the CLT's in the SEIVM and FEIVM that are established in this paper are quite different, a unified general scheme of these proofs is constructed for the two models herewith.

AMS 2000 subject classifications: Primary 60F05, 62J99; secondary 60E07.

Keywords: linear structural/functional error-in-variables model, measurement errors, explanatory variables, domain of attraction of the normal law, slowly varying function, identifiability assumptions, generalized/ modified least squares estimator, central limit theorem, multivariate Student statistic, positive definite matrix, Cholesky square root of a matrix, symmetric positive definite square root of a matrix, Lindeberg's condition, generalized domain of attraction of the multivariate normal law, spherically symmetric random vector, full random vector.

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