

Likelihood inference in exponential families and directions of recession

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Abstract

When in a full exponential family the maximum likelihood estimate (MLE) does not exist, the MLE may exist in the Barndorff-Nielsen completion of the family. We propose a practical algorithm for finding the MLE in the completion based on repeated linear programming using the R contributed package `texttt{rcdd}` and illustrate it with three generalized linear model examples. When the MLE for the null hypothesis lies in the completion, likelihood ratio tests of model comparison are almost unchanged from the usual case. Only the degrees of freedom need to be adjusted. When the MLE lies in the completion, confidence intervals are changed much more from the usual case. The MLE of the natural parameter can be thought of as having gone to infinity in a certain direction, which we call a generic direction of recession. We propose a new one-sided confidence interval which says how close to infinity the natural parameter may be. This maps to one-sided confidence intervals for mean values showing how close to the boundary of their support they may be.

AMS 2000 subject classifications: Primary 62F99; secondary 52B55.

Keywords: exponential family, existence of maximum likelihood estimate, Barndorff-Nielsen completion.



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Geyer, Charles J., Likelihood inference in exponential families and directions of recession, *Electronic Journal of Statistics*, 3, (2009), 259-289 (electronic). DOI: 10.1214/08-EJS349.

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