

## Estimation in a class of nonlinear heteroscedastic time series models

Joseph Ngatchou-Wandji, *LMNO and Département de Mathématiques, Université de Caen, Campus 2, BP 5186, Bo*

### Abstract

Parameter estimation in a class of heteroscedastic time series models is investigated. The existence of conditional least-squares and conditional likelihood estimators is proved. Their consistency and their asymptotic normality are established. Kernel estimators of the noise's density and its derivatives are defined and shown to be uniformly consistent. A simulation experiment conducted shows that the estimators perform well for large sample size.

AMS 2000 subject classifications: Primary 62M10; secondary 62F12.

Keywords: Conditional least-squares estimation, Conditional likelihood estimation, Heteroscedastic models, Kernel density estimation, LATEX2 $\epsilon$ .



Full Text: [PDF](#)

Ngatchou-Wandji, Joseph, Estimation in a class of nonlinear heteroscedastic time series models, *Electronic Journal of Statistics*, 2, (2008), 40-62 (electronic). DOI: 10.1214/07-EJS157.

## References

- An, H., Chen, M. and Huang, F. (1997). The geometric ergodicity and existence of moments for a class of non-linear time series model. *Statist. Probab. Lett.*, 31, 213-224. [MR1440637](#)
- Bai, J. and Ng, S. (2001). A consistent test for conditional symmetry in time series models. *J. Econometrics*, 103, 225-258. [MR1838200](#)
- Berkes, I. and Horváth, L. (2004). The efficiency of the estimators of the parameters in GARCH processes. *Ann. Statist.*, 32, 633-655. [MR2060172](#)
- Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. *J. Econometrics*, 31, 307-327. [MR0853051](#)
- Brockwell, P.J. and Davis, R.A. (1991). Time Series: Theory and Methods. Second edition. Springer-Verlag. [MR1093459](#)
- Brockwell, P.J. and Davis, R.A. (1996). Introduction to Time Series and Forecasting. Springer-Verlag. [MR1416563](#)
- Chatterjee, S. and Das, S. (2003). Parameter estimation in conditional heteroscedastic models. *Comm Statist.*, 32, 1135-1153. [MR1983236](#)
- Chen, M. and An, H.Z. (1998). A note on the stationarity and the existence of moments of the GARCH model. *Statist. Sin.*, 8, 505-510. [MR1624371](#)
- Engle, R.F. (1982). Autoregressive conditional heteroscedasticity with estimates of the

Fan, J., Yao, Q. and Tong, H. (1996). Estimation of conditional densities and sensitivity measures in nonlinear dynamical systems. *Biometrika*, 83, 189-206. [MR1399164](#)

Fan, J. and Yim, T.H. (2004). A cross validation method for estimating conditional densities. *Biometrika*, 90, . [MR2126035](#)

Francq, C. and Zakoïan, J.M. (2004). Maximum likelihood estimation for pure GARCH and ARMA-GARCH process. *Bernoulli*, , 605-637. [MR2076065](#)

Francq, C. and Zakoïan, J.M. (2007). Quasi-likelihood in GARCH processes when some coefficients are equal to zero. *Stoch. Proc. Appl.*, 9, 1265-1284.

Giraitis, L. and Robinson, P.M. (2001). Wittle estimation of ARCH models. *Econometr. Theory*, 17, 608-631. [MR1841822](#)

Guégan, D. and Diebolt, J. (1994). Probabilistic properties of the  $\beta$ -ARCH model. *Statist. Sin.*, 4, 71-87. [MR1282866](#)

Hall, P. and Yao, Q. (2003). Inference in ARCH and GARCH models with heavy-tailed errors. *Econometrica*, 71, 285-317. [MR1956860](#)

Heyde, C.C. (1997). *Quasi-Likelihood And Its Application*. Springer. [MR1461808](#)

Horová, I., Vieu, P. and Zelinka, J. (2002). Optimal choice of the nonparametric estimates of a density and its derivatives. *Statist. Decisions*, 20, 355-378. [MR1960709](#)

Hyndman, R. J., Bashtannyk, D. M. and Grunwald, R.J. (1996). Estimating and visualizing conditional densities. *J. Comput. Graph. Statist.*, 54, 315-336. [MR1422114](#)

Hyndman, R. J. and Yao, Q. (2002). Nonparametric estimation and symmetry tests for conditional density functions. *J. Nonparametr. Stat.*, 14, 259-278. [MR1905751](#)

Klimko, L.A. and Nelson, P.I. (1978). On conditional least-squares estimation for stochastic processes. *Ann. Statist.*, 6, 629-642. [MR0494770](#)

Lumsdaine, R. (1996). Consistency and asymptotic normality of the quasi-maximum likelihood estimator for IGARCH(1,1) and covariance stationary GARCH(1,1) models. *Econometrica*, 16, 575-596. [MR1385558](#)

McKeague, I.W. and Zhang, M.J. (1994). Identification of nonlinear time series from first order cumulative characteristics. *Ann. Statist.*, 22, 495-514. [MR1272096](#)

Ngatchou-Wandji, J. (2002). Weak convergence of some marked empirical processes. Application to testing heteroscedasticity. *J. Nonparametr. Statist.* 14, 325-339. [MR1905755](#)

Ngatchou-Wandji, J. (2005). Checking nonlinear heteroscedastic time series models. *J. Statist. Plann. Infer.*, 133, 33-68. [MR2162567](#)

Robinson, P. M. and Zaffroni, P. (2006). Pseudo-maximum likelihood estimation of ARCH ( $\infty$ ) models. *Ann. Statist.*, 34, 1049-1074. [MR2278351](#)

Peng, L. and Yao, Q. (2003). Least absolute deviations estimation for ARCH and GARCH models. *Biometrika*, 90, 967-975. [MR2024770](#)

Shumway, R.H. and Stoffer, D.S. (2001). *Times Series Analysis and Its Applications*. Springer. [MR2228626](#)

Silverman, B.W. (1978). Weak and strong uniform consistency of the kernel estimate of a density and its derivatives. *Ann. Statist.*, 6, 177-184. [MR0471166](#)

Singh, R.S. (1979). On necessary and sufficient conditions for uniform strong consistency of estimators of a density and its derivatives. *J. Mult. Anal.*, 9, 157-164.

Straumann, D. and Mikosch, T. (2006). Quasi-maximum likelihood estimation in heteroscedastic time series: A stochastic recurrence equations approach. *Ann. Statist.*, 34, 2449-2495. [MR2291507](#)

Taniguchi, M. and Kakizawa, Y. (2000). *Asymptotic Theory of Statistical Inference for Time Series*. Springer. [MR1785484](#)

Tjøstheim, D. (1986). Estimation in nonlinear time series models. *Stoch. Proc. Appl.* 21, 251-273. [MR0833954](#)

Tjøstheim, D. (1990). Non-linear time series and Markov chains. *Adv. Appl. Prob.* 22, 587-611. [MR1066965](#)

Tong, H. (1990). *Non-linear Time Series. A Dynamic System Approach*. Oxford: Oxford University Press. [MR1079320](#)

Weiss, A. (1986). Asymptotic theory for ARCH models: estimation and testing. *Econometr. Theory*, 2, 107-31.