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Deconvolution for an atomic distribution

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Abstract

Let $X_1,ldots, X_n$ be i.i.d. observations, where $X_i=Y_i+sigma Z_i $ and Y_i and Z_i are independent. Assume that unobservable Y's are distributed as a random variable UV, where U and V are independent, U has a Bernoulli distribution with probability of zero equal to p and V has a distribution function F with density f. Furthermore, let the random variables Z_i have the standard normal distribution and let $sigma>0$. Based on a sample $X_1,ldots,X_n$, we consider the problem of estimation of the density f and the probability p. We propose a kernel type deconvolution estimator for f and derive its asymptotic normality at a fixed point. A consistent estimator for p is given as well. Our results demonstrate that our estimator behaves very much like the kernel type deconvolution estimator in the classical deconvolution problem.

AMS 2000 subject classifications: Primary 62G07; secondary 62G20.

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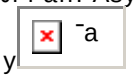
[19] J. Fan. Asymptotic normality for deconvolution kernel density estimators. Sankhy  Ser., 53:97–110, 1991. MR1177770

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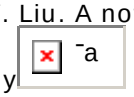
[23] J. Fan and Y. Liu. A note on asymptotic normality for deconvolution kernel density estimators. Sankhy  Ser., 59:138–141, 1997. MR1665140

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