International Conference on Estuaries and Coasts
November 9-11, 2003, Hangzhou, China

SOLUTIONS TO WAVE TRANSMISSION AND REFLECTION BY BOTTOM MOUNTED WAVE-PERMEABLE STRUCTURE IN SHALLOW WATER

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Abstract: We consider the problem of wave partial/full reflection and transmission by wave-permeable structure as solving the pseudo wave-absorbing shape-related function with the focus on the understanding of wave attenuation. For determination of absorbing-reflection effects of wave-permeable breakwaters, one-dimensional case is studied to give the analytical expression of the absorbing term, and the analytical predictions are compared to available physical laboratory data in flume for wave propagation through bottom-mounted permeable breakwaters. Then the expression is also applied in depth averaged Boussinesq type wave equations in simulating the non-linear wave transmission through wave-permeable breakwater, we obtain accurate predictions of reflected and transmitted results combined with diffraction-refraction effects around the wave-permeable breakwater for various incident-wave conditions in the two-dimensional case. The results show that wave permeable breakwaters with proper absorbing effects can be used as an effective alternative to massive gravity breakwaters in reduction of wave transmission in shallow water.

Key words: Wave mixing length, Wave-absorbing Coefficient, Wave permeable breakwaters

1. INTRODUCTIONS

The development of breakwater design methodology often required as a condition of permitting for protection of ecosystem. For permeable structures such as bottom-mounted breakwaters, the incident waves interact with the porous structure causing transmitted and reflected waves to propagate towards the boundaries. The solution of wave permeable breakwaters is expressed by introducing the linear or nonlinear damping item to the governing equations in accordance with energy dissipation. In numerical study the similar 'open boundary' or 'fully absorbing boundary' condition is required, which requires the transmitted and reflected waves small (Larsen and Dancy, 1983). The numerical problem can be approximated by the solutions to weak reflected and fully absorbing of real wave permeable breakwater. Therefore in principle, the same analytical solutions could be applied in numerical simulation of the wave permeable structures. The analytical solutions diversified in different expressions of the energy damping term due to turbulent flow in the permeable structures. The porosity solution (Madsen, 1983) is based on a trial method, and a number of authors solve the problem by different numerical approach (Giles etc., 1998, Kirby, 1999, Hall, 2001), which are also not a direct solution. In this paper an expression is given to solve the problem in one-dimensional condition, and applied in two-dimensional numerical simulation.

2. MATHEMATICAL FORMULATION OF ABSORBING COEFFICIENT

2.1 GOVERNING EQUATIONS FOR ONE-DIMENSIONAL CASE

The one-dimensional wave equation inside the wave permeable breakwaters read:

Continuity equation

$$n\zeta_t + DU_x = \boldsymbol{0} \tag{1}$$

Momentum equation

$$U_t + gn\zeta_x + sU = \boldsymbol{0} \tag{2}$$

where h is the constant water depth, ζ is the free surface elevation, U is the velocity, n is the shape-related coefficient for the absorber, s is the non-linear damping term. We assume time harmonic motion of ω, introducing $U=n\upsilon(x)e^{i\omega t}$ and $\zeta=\eta(x)e^{i\omega t}$, from eq.1 and eq.2 get the relation:

$$\eta_{xx} = \frac{\omega i}{D}\frac{s+\omega i}{g}\eta = r^2\eta \tag{3}$$

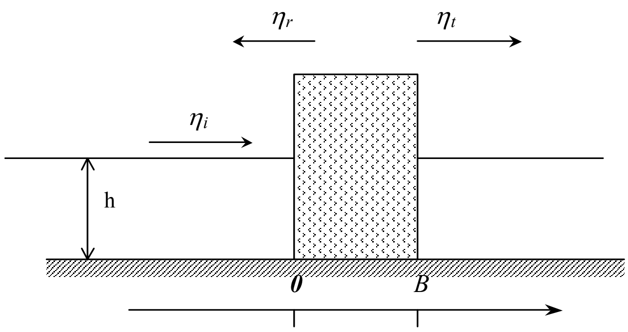

Fig. 1 Definition sketch of the problem

with boundary conditions by assuming continuity of pressure (velocity) and mass (amplitude) for incoming (x<0) and transmitted linear wave (x>B), B is the width of the absorber. The linear wave solutions outside the absorber read (Fig. 1):

$$\begin{cases} \zeta = \mathrm{Re}\left[a_i e^{i(\omega t - k_0 x)} + a_r e^{i(\omega t + k_0 x)}\right]; & x<0 \\ U = \mathrm{Re}\left[\sqrt{g/h}\left(a_i e^{i(\omega t - k_0 x)} + a_r e^{i(\omega t + k_0 x)}\right)\right]; & x<0 \end{cases}$$

The boundary expressions for interfaces are:

$$\begin{cases} \eta_0 = (1+R)\eta_i\,,\ \upsilon_0 = \sqrt{g/D}(1-R)\eta_i;\ x=0 \\ \eta_B = G\eta_i, \qquad \upsilon_B = \sqrt{g/D}G\eta_i; \qquad x=B \end{cases}$$

where R and G are defined as reflection coefficient and transmission coefficient separately in terms of maximum incoming wave amplitude η_{0i}. To fix the problem, introduce phasor constant $r=\alpha+i\beta$. The α is defined as the damping coefficient; the β is defined as $\beta=2\pi/L_s$, where L_s is the mixing wave length in the permeable breakwater and independent of x. The obvious solution is $\eta=\mathrm{Re}[Ce^{\pm rx}]$ inside the absorber, the minus solution is chosen for the permeable structure. Thus the elevation and velocity inside the absorber ($0 < x < B$) read:

$$\begin{cases} \zeta = \mathrm{Re}[Ce^{-\alpha x + i(\omega t - \beta x)}] \\ U = \mathrm{Re}[\dfrac{\omega n}{D|r|}Ce^{-\alpha x + i(\omega t - \beta x + \phi)}];\ -\dfrac{\alpha}{\beta} = \tan\phi \end{cases} \tag{4}$$

with the boundary solutions for coupled R and G:

$$\frac{\boldsymbol{1}-R}{\boldsymbol{1}+R} = \frac{k_{\boldsymbol{0}}}{\beta}\cos^2\phi \text{ and } G = (\boldsymbol{1}+R)e^{-\alpha B} \tag{5}$$

In general, in order to find *R, G, s*, we would have to find α and β. To close the problem, we come up with a reasonable approximation: the average energy dissipation rate by α will be approximated by energy dissipation by linearized damping term *s* in one wavelength. Thus we have:

$$\int_0^T dt \int_{x_0}^{x_0+L_s} sU^2 dx = \int_0^T \int_{x_0}^{x_0+L_s} (E_{x_0} - E_x) dx dt \tag{6}$$

From eq. 3, we have:

$$-k_0^2 + \frac{\omega s}{gh} i = \alpha^2 - \beta^2 + 2\alpha\beta i \tag{7}$$

Solving eq. 6 and eq. 7 with respects to L_x, we have the following relation:

$$s = \frac{2gD}{\omega}\alpha\beta = \frac{1}{2}(1 + \frac{(\alpha^2 + \beta^2)D}{k_0^2 n^2})(\frac{2\alpha L_s}{1 - e^{-2\alpha L_s}} - 1); \ 0 < x < B \tag{8a}$$

Especially when the width of the absorber reduces to a screen (i.e. *B* approximates 0), we have:

$$s = \frac{1}{2}(1 + \frac{D(\alpha^2 + \beta^2)}{k_0^2 n^2})(e^{2\alpha L_s} - 1) \tag{8b}$$

The solution of *s* is based on the determination of L_x, and is set to a constant for specific type of wave permeable breakwater (i.e. shape coefficient *n*) and specific incoming wave (i.e. wave number k_0, and wave frequency ω).

2.2 EXPERIMENT SET-UP AND DETERMINATION OF MIXING WAVE LENGTH

The coupled reflection coefficient *R* and transmission coefficient *G* are related to the factors as:

The shape coefficient for permeable breakwaters, *n*;

The length of the absorber *B;*

The water depth *D;*

The incoming wave period, *T* and the incoming wave length, i.e. the wave number k_0;

The mixing wave length, L_x.

To verify and calibrate data we have carried out a physical experimental in flume (Fig. 2), the measured data is shown in table 1 for different incoming wave and water depth. The incoming wave varied in the wave amplitude a_i,, water depth (*D*) and period (*T*). determination of L_x of eq. 8 are shown in Fig. 2, the reflection and transmission coefficient by theoretical solution of eq. 5 are compared with experimental data of wave permeable breakwater (Fig. 3) in flume, the results shows a satisfactory matching accuracy.

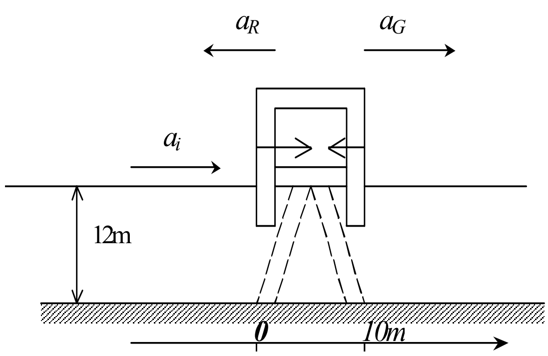

Fig. 2 The cross section configuration of experimental set-up in flume

Table 1 The experimental data in flume

Tide, Period / Wave	2.68m, 5.69s(measured)		2.68m, 5.69s (Calculated)	
	R	G	R	G
$\overline{H}$ (m) =1.27	0.72	0.42	0.70	0.49
$H_{13\%}$(m)= 1.98	0.79	0.36	0.78	0.42
$H_{5\%}$(m) =2.35	0.72	0.35	0.75	0.36
$H_{1\%}$(m) =2.78	0.83	0.36	0.81	0.28

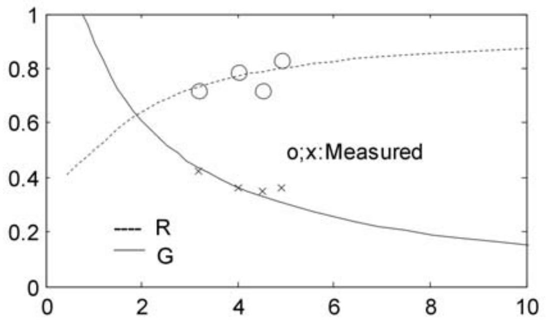

Fig. 4 The R, G as a function of s

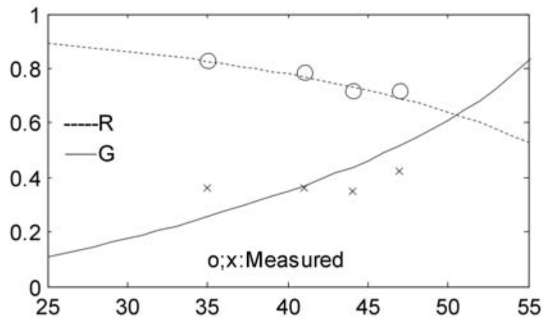

Fig. 5 The R, G as a function of L_x

3. MODIFIED BOUSSINESQ TYPE WAVE EQUATIONS FOR PERMEABLE BREAKWATERS

We introduce the absorbing terms to the modified Boussinesq type equations as following (Madsen P. A,1992; Li Xi, 2002):

Continuity equation

$$\eta_t + P_x + Q_y = 0 \tag{9}$$

X-direction momentum equation

$$n^2 P_t + (\frac{P^2}{h})_x + (\frac{PQ}{h})_y + gh\eta_x + \psi_1 + F_{bx} + S_x = 0 \tag{10a}$$

Y-direction momentum equation

$$n^2 Q_t + (\frac{Q^2}{h})_y + (\frac{PQ}{h})_x + gh\eta_y + \psi_2 + F_{by} + S_x = 0 \tag{10b}$$

In which, x, y and z form a rectangular coordinate system, with the plane coordinates of x ,y on the still water level, z measured vertical upwards, $h \equiv D + \eta$, $P = uh$,$Q = vh$, D is the varying water depth or still water depth, $\eta$is the free surface elevation, u and v is the depth average velocity.

Where dispersion terms ψ_1 ψ_2 in equations 6-(a,b) equals to

$$\begin{aligned}\psi_1 = &-(B+\frac{1}{3})D^2(P_{xxt}+Q_{xyt}) - BgD^3(\eta_{xxx}+\eta_{xyy}) \\ &- DD_x(\frac{1}{3}P_{xt}+\frac{1}{6}Q_{yt}+2DgB\eta_{xx}+BgD\eta_{yy}) - DD_y(\frac{1}{6}Q_{xt}+BgD\eta_{xy})\end{aligned} \tag{11a}$$

$$\begin{aligned}\psi_2 = &-(B+\frac{1}{3})D^2(Q_{yyt}+P_{xyt}) - BgD^3(\eta_{yyy}+\eta_{xxy}) \\ &- DD_y(\frac{1}{3}P_{yt}+\frac{1}{6}Q_{xt}+2DgB\eta_{yy}+BgD\eta_{xx}) - DD_x(\frac{1}{6}Q_{yt}+BgD\eta_{xy})\end{aligned} \tag{11b}$$

F_{bx}, F_{by} are the bottom friction term, they can be written as

$$F_{bx} = \frac{f_b}{2D^2}P\sqrt{P^2+Q^2} \tag{12a}$$

$$F_{by} = \frac{f_b}{2D^2}Q\sqrt{P^2+Q^2} \tag{12b}$$

where friction coefficient f_b is introduced for the wave-current coexistent system by assuming a constant depending on water depth (Yan Yixin, 2002).

S_x, S_y are the non-linear absorbing term

$$S_x = s_x P \tag{13a}$$

$$S_y = s_y Q \tag{13b}$$

where s_x and s_y are absorbing items, defined in one dimensional case as above, here $s_x = s_y$,we apply eq.8 directly in the two dimensional case (the same cross-section of permeable breakwaters as in Fig. 2, only with a changeable water depth). According to eq. 8 the value of s depends on the determination of L_x and varies with water depth D. For this type of breakwater, the value of s is depending on the calibration of shape coefficient n in different of water depth D, hence one-dimensional physical data in flume is necessary for the calibration. The scale of the damping term s of two-dimensional numerical model, is varied in 3~5 in the following application.

4. WAVE TRANSFORMATION AROUND WAVE PERMEABLE BREAKWATER

In order to show the usefulness of the model, we applied the model in the feasibility study of permeable breakwater constructed in Zhejiang, East Coast of China. The breakwater consists of two parts, 110 meters of impermeable rubble mounds breakwater with a slope of 1:20 and 310 × 10.5 meters of bottom mounted wave permeable breakwater. Another impermeable breakwater has been constructed between the two islands; calculation domain of the numerical model is illustrated in Fig. 2a. To verify the model, we also carried out a physical model (Fig. 2b) with scale 1:80 in a 20m×25m wave basin and wave heights of 32 sample points were collected for verification. The input significant wave height and peak spectral period near the site has been extrapolated from the field wind data collections (Table 1).

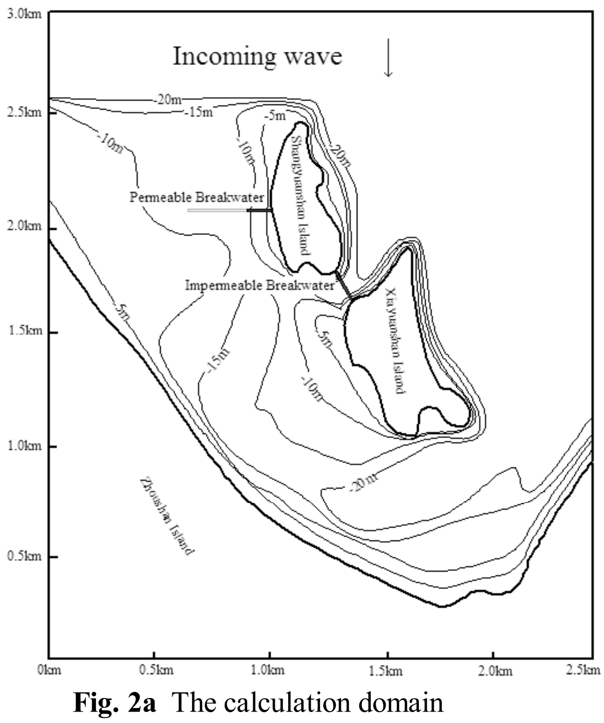

Fig. 2a The calculation domain

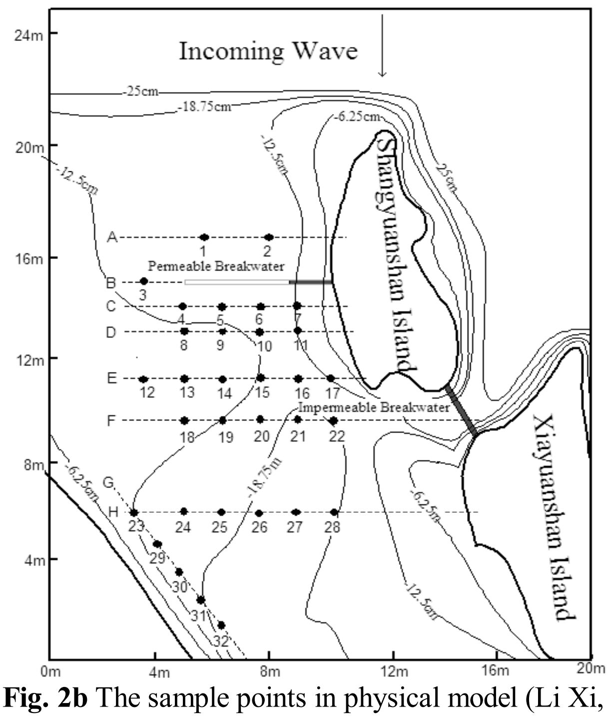

Fig. 2b The sample points in physical model (Li Xi, Yang Yue, etc., 2003)

Table 1 Incoming wave climate (-20m still water depth, 50 years return period)

Tide / Wave	2.68m	1.49m	-1.62m	-2.43m
$\overline{H}$ (m)	1.27	1.26	0.93	0.91
$H_{1\%}$(m)	2.78	2.73	2.00	1.91
$H_{13\%}$(m)	1.98	1.96	1.43	1.40
T_S(s)	5.69	5.59	5.26	5.16

As shown in physical scale model (Fig. 2b), 32 data points in 8 cross-sections were collected for different wave conditions. The calculation results are shown in Figure 3 for significant wave under tide level 2.68m with a regular prototype wave input as 1.98m wave height and 5.69s wave period. In accordance with the physical model situations, A~H sections of wave heights in numerical model are compared with physical model data, as shown in Fig. 4(a−h). It is concluded that that the calculated results agrees well with the measurements.

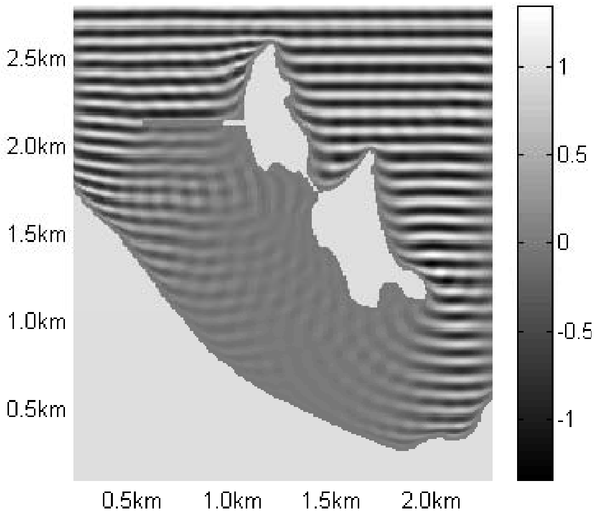

Fig. 3a The calculated wave elevation distribution (Tide=2.68m, H_0=1.98m)

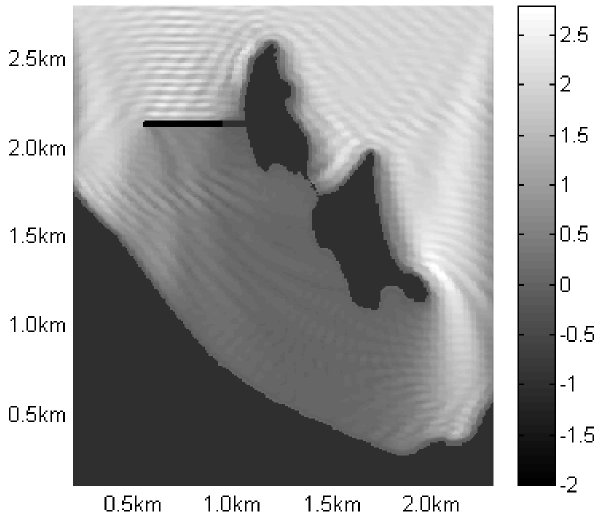

Fig. 3b The calculated wave height distribution (Tide=2.68m, H_0=1.98m)

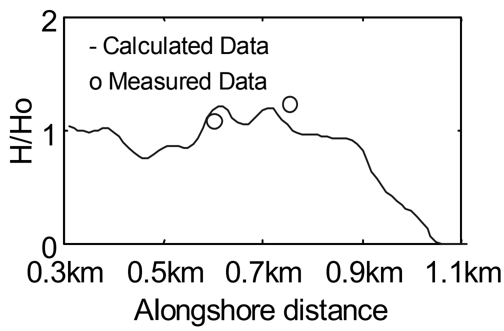

Fig. 4a Section A

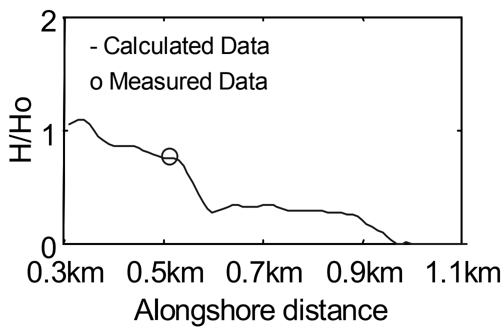

Fig. 4b Section B

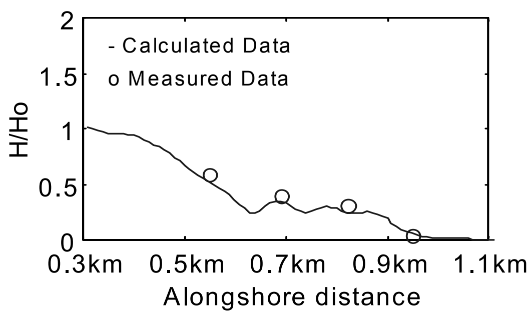

Fig. 4c Section C

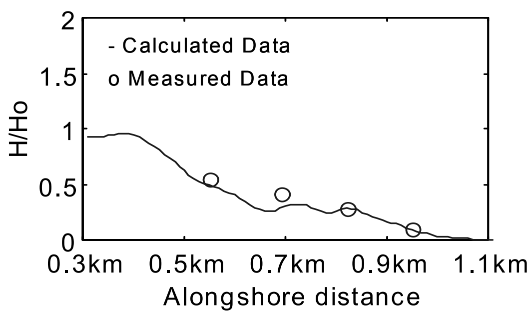

Fig. 4d Section D

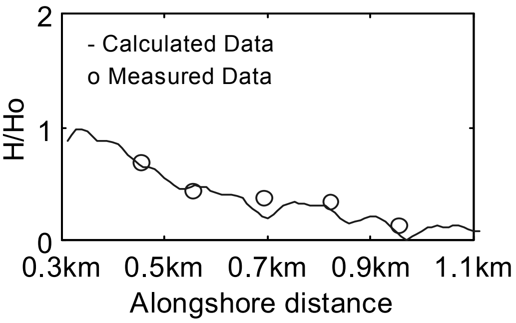

Fig. 4e Section E

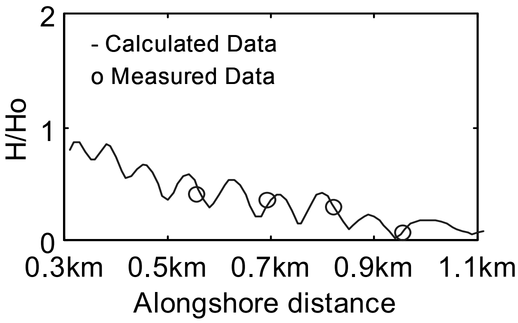

Fig. 4f Section F

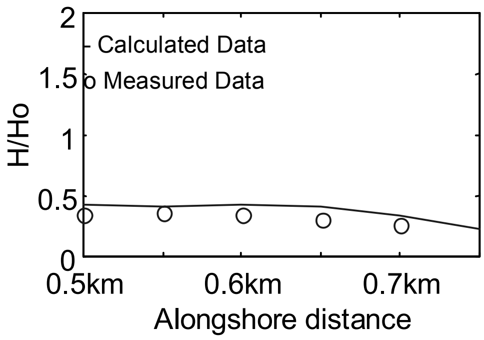

Fig. 4g Section G

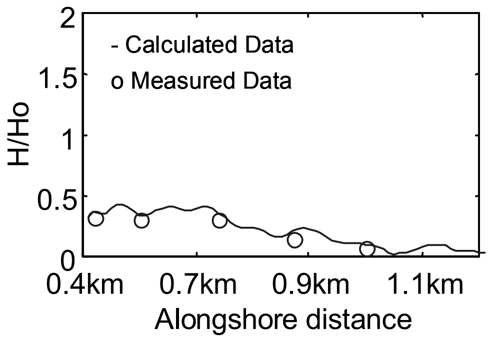

Fig. 4h Section H

5. CONCLUSION AND DISCUSSION

Based on the analysis, it is concluded that Boussinesq type equations are detailed and accurate computation of waves, the solution proved valuable to engineering practice, and more verification and calibration is expected in the future.

ACKNOWLEDGEMENT

This work was carried out with the support of National Open Fund in State Key Laboratory of Coastal and Estuary Engineering.

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