Improved Dynamic Programming for Reservoir Operation Optimization with a Concave Objective Function

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Abstract: Diminishing marginal utility is an important characteristic of water resources systems. With the assumption of diminishing marginal utility (i.e., concavity) of reservoir utility functions, this paper derives a monotonic relationship between reservoir storage and optimal release decision under both deterministic and stochastic conditions, and proposes an algorithm to improve the computational efficiency of both deterministic dynamic programming (DP) and stochastic dynamic programming (SDP) for reservoir operation with concave objective functions. The results from a real-world case study show that the improved DP and SDP exhibit higher computational efficiency than conventional DP and SDP. The computation complexity of the improved DP and SDP is $O(n)$ (order of n, the number of state discretization) compared to $O(n^2)$ with conventional DP and SDP. **DOI: 10.1061/(ASCE)WR.1943-5452.0000205.** *© 2012 American Society of Civil Engineers.*

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Introduction

Diminishing marginal utility with a concave utility function (i.e., concavity) has been recognized as one of the basic characteristics of water resources systems (Loucks et al. 1981; Draper and Lund 2004). For example, willingness to pay for one more unit of water is high in drought periods, and it decreases as water scarcity abates (Harou et al. 2010). For reservoir operation, concavity of the utility function has been applied to both operation rule and solution algorithm design. For example, given a concave utility function, hedging rules for reservoir operation are designed to reduce the current water supply by a certain amount to avoid profit loss in case of severe future water shortage (Draper and Lund 2004; You and Cai 2008a, b). An example of using concavity for algorithm design involves stochastic dual dynamic programming, for which the cumulative utility function is assumed to be concave and is then approximated by Benders cuts (Tilmant and Kelman 2007; Goor et al. 2011). The monotonicity property has even earlier and broader applications in operations research. Veinott (1964) derived monotonic relationships for a multiple-stage dynamic optimization problem. In more recent years, monotonicity has been applied to detecting the optimal forecast period given a certain length of the decision period in supply chain management (Huang and Ahmed 2010). In this study, an application of concavity to improving dynamic programming (DP) and stochastic dynamic programming (SDP) is presented, which illustrates another way to use problem characteristics to improve solution algorithms, particularly for reservoir operation optimization.

The DP and SDP models are among the most popular reservoir operation models (Loucks et al. 1981; Yeh 1985; Labadie 2004) because of their ability to handle nonlinear, noncontinuous objective functions and constraints and temporally sequential reservoir decision making. However, applications of DP, particularly SDP to complex optimization problems, are hindered by the so-called curse of dimensionality, i.e., computation time and storage requirements are proportional to n^m, where n is the number of storage discretization and m is the number of reservoirs (Labadie 2004). There are various approaches to alleviate the curse of dimensionality. One approach is to decompose the multireservoir problem into multiple single-reservoir problems, e.g., dynamic programming with successive approximation (DPSA) (Larson and Korsak 1970; Tilmant and Kelman 2007; Opan 2010). Another approach is on the basis of learning and exploring the relationship between storage and release decision (Lee and Labadie 2007; Castelletti et al. 2010). In operations research, Galil and Park (1992) provided a detailed review of the various implementations of DP that take advantage of model properties, e.g., concavity, convexity, and sparsity. Adding to this category of studies, this paper first derives a monotonic dependence relationship between reservoir storage and release, then uses this relationship to improve DP and SDP for reservoir operation optimization.

In the remainder of this paper, the "Problem Formulation" section introduces the formulation of a dynamic multiple-stage optimization model for reservoir operation. The "Monotonicity and DP Improvement" section derives the monotonic relationship between optimal release/storage carryover decisions and reservoir storage, and then applies this structural relationship between optimal

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decision and model parameters to improve conventional dynamic programming. The "Monotonicity with Stochastic Parameters and Improved SDP" section extends the monotonic relationship to stochastic cases and improves SDP. The "Case Study" section presents an application of improved DP and SDP to the Danjiangkou Reservoir water supply operation. The last section contains discussions and conclusions.

Problem Formulation

The optimization model for a T-period reservoir operation problem can be formulated as a series of two-stage optimization models linked by the recursive function in a backward form (Loucks et al. 1981; Yeh 1985; Labadie 2004). The DP model can be expressed as

$$F_t(s_t) = \max_{s_{t+1}} \quad f_t(r_t) + F_{t+1}(s_{t+1}) \\ s.t. \;\; r_t = s_t + q_t - s_{t+1} \tag{1}$$

In Eq. (1), t = time index; s_t and r_t = reservoir storage and release at period t, respectively; q_t = either the period t inflow or inflow forecast, which is a given input of the DP model; $f_t(r_t)$ = single-period utility function at period t, which is assumed concave in this study; and $F_{t+1}(s_{t+1})$ = maximum cumulative utility function during the periods from $t+1$ to T. Storage loss and utility discount are ignored in Eq. (1), but will be included in an extended analysis in the appendix of this paper.

If $s_t + q_t$ is substituted by $\tilde{s}_t$ and s_{t+1} is selected as the decision variable, then Eq. (1) can be rewritten as

$$F_t(s_t) = \max_{s_{t+1}} \quad f_t(\tilde{s}_t - s_{t+1}) + F_{t+1}(s_{t+1}) \tag{2}$$

Eq. (2) represents a typical two-stage resource allocation problem, where $\tilde{s}_t$ = total available resources; and s_{t+1} and $\tilde{s}_t - s_{t+1}$ (i.e., r_t) = resources allocated to the two stages characterized by $F_{t+1}()$ and $f_t()$, respectively.

Monotonicity and DP Improvement

Monotonicity in Reservoir Operation Analysis

For a general analysis, Eq. (2) can be written into the following form:

$$G(y) = \max_x \quad g_1(x) + g_2(y-x) \tag{3}$$

in which y [analogous to $\tilde{s}_t$ in Eq. (2)] = total available resource to allocate; x and y-x = resource allocated to $g_1()$ and $g_2()$, respectively; $g_1()$ and $g_2()$ = sectional utility functions, which are analogous to $F_{t+1}()$ and $f_t()$ in Eq. (2), respectively; and $G()$ = total utility function. For the resource allocation problem presented by Eq. (3), it has been proved that if the sectional utility functions $g_1()$ and $g_2()$ are concave, then the total utility function, $G()$, is also a concave function (Balinski and Baumol 1968). Referring to the equivalent reservoir operation problem represented by Eq. (2), if the utility function at all single periods, i.e., $f_1(), f_2(), \ldots, f_T()$ are concave functions, then through the sequential backward recursive procedures from period T to period 1, $F_{T-1}(), \ldots, F_1()$ are concave functions. Thus, when the single period utility function $f_t()$ is concave for $t = 1, 2, \ldots T$, the maximum cumulative utility function $F_t()$ is also concave.

The concavity of both the single-period utility function (assumed in this study) and maximum cumulative utility function [derived from the two-stage optimization in Eq. (3)] can be further applied to obtaining structural relationships between reservoir storage and optimal release decisions for the reservoir operation models in Eq. (2). For Eq. (3), denoting the optimal x corresponding to y as x^* and applying the first-order optimality condition,

$$g_1'|_{x=x^*} = g_2'|_{x=y-x^*} \tag{4}$$

Eq. (4) means that at the optimal solution, the marginal utility of both sections $g_1()$ and $g_2()$ should be the same. Subsequently, the monotonic relationship between y and x^* can be derived by proof by contradiction. For the two y values y_1 and y_2 $(y_1 < y_2)$, denoting the corresponding optimal x values as x_1^* and x_2^*, respectively, and assuming $x_1^* \geq x_2^*$ (the null hypothesis),

$$y_1 - x_1^* < y_2 - x_2^* \tag{5}$$

Combining the optimality condition in Eqs. (4) and (5) and using the diminishing marginal utility property of $g_1()$ and $g_2()$,

$$g_1'|_{x=x_1^*} \leq g_1'|_{x=x_2^*} = g_2'|_{x_2=y_2-x_2^*} < g_2'|_{x_1=y_1-x_1^*} \tag{6}$$

In Eq. (6), $g_1'|_{x=x_1^*} < g_2'|_{x_1=y_1-x_1^*}$ is contradictory to Eq. (4), which indicates that the null hypothesis cannot be true, i.e., if $y_1 < y_2$, then $x_1^* < x_2^*$. Thus, it is proven that when $g_1()$ and $g_2()$ are strictly concave, there is a strict monotonic relationship between x^* and y.

Considering an upper bound $\bar{x}$ for x, which usually represents a resource consumption limit constraint, for example, the maximum allowed release from a reservoir in a certain period,

$$G(y) = \max_x \quad g_1(x) + g_2(y-x) \qquad s.t. \;\; x \leq \bar{x} \tag{7}$$

The increase of y will make x^* gradually reach the upper bound; beyond that, x^* remains constant and y-x^* increases with y. Thus, considering resource consumption limitation constraints, the monotonic relationship still holds (i.e., x^* will not decrease if y increases), although not strictly (i.e., x^* will increase if y increases). In other words, if the total available resource increases, the optimal resource allocated to each section will increase or at least remain the same for each section.

Structural relationships between optimal solutions and model parameters can provide insightful information regarding the optimization model, as shown in operations research studies (Veinott 1964; Geoffrion 1976). For example, a monotonic relationship between the optimal production of the current period and the demand in future periods indicates the existence of a forecast horizon, i.e., demands beyond a certain critical horizon have no effect on the current production decision (Huang and Ahmed 2010). In water resources studies, the monotonic relationship contributes to the existence of effective forecast horizon regarding forecast uncertainty (Zhao et al. 2012).

The monotonic relationship between y and x^* in Eq. (3) indicates an equivalent relationship between $s_t + q_t$ (the total available water resources in period t) and s_{t+1}^* (i.e., $s_t + q_t - r_t^*$, the optimal storage carried over to period $t+1$), and also r_t^* (i.e., $s_t + q_t - s_{t+1}^*$, the optimal reservoir release in period t) in Eq. (2). Thus, the increase of total available water resources in period t will induce the increase of both the storage carried over to period $t+1$ and the reservoir release in period t. Considering the storage capacity and release capacity constraints [i.e., an upper bound of current release or storage carryover, which is analogous to the maximum resource consumption constraint in Eq. (7)], this monotonic relationship still holds. Extending the monotonicity property from period t to its subsequent periods, it can be concluded that if the single-period utility function $f_t()$ is concave, then the optimal release decision in periods from t to T, $[r_t^*, r_{t+1}^*, \ldots, r_T^*]$ corresponding to a certain $s_t + q_t$ with any

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realizations of future inflows $[q_{t+1}, q_{t+2}, \ldots, q_T]$ will not decrease with $s_t + q_t$.

This study derives a monotonic relationship between $s_t + q_t$ and s^*_{t+1} (and also r^*_t) for reservoir operation optimization. The monotonic relationship considering storage loss and utility discount is elaborated in the appendix. This monotonic relationship provides the following implications for reservoir operation optimization with a concave utility function

(1) Considering inflow uncertainty, if the inflow upper bound $\bar{Q} = [\bar{q}_1, \bar{q}_2, \ldots, \bar{q}_T]$ (e.g., the maximum of inflow forecast) and lower bound $\underline{Q} = [\underline{q}_1, \underline{q}_2, \ldots, \underline{q}_T]$ (e.g., the minimum of inflow forecast) are provided, then the actual optimal release decision is bounded by $\bar{R} = [\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_T]$ and $\underline{R} = [\underline{r}_1, \underline{r}_2, \ldots, r_T]$, the optimal release sequences corresponding to $\bar{Q}$ and $\underline{Q}$, respectively.

(2) A higher initial storage or a lower ending storage will result in more water available within the operation periods, and correspondingly the same or larger release during the operation periods whereas a lower initial storage or a higher ending storage will result in the same or smaller release.

Although these implications are simple, they are useful for speeding up the search of optimal releases under the various inflow and storage conditions, as described in the following section.

Improving DP Using the Monotonic Relationship

In conventional DP, the optimal storage carryover s^*_{t+1}, corresponding to a current storage s_t, is determined in an exhaustive search approach. To find s^*_{t+1} for each s_t, every s_{t+1} value should be searched (Fig. 1, left). Supposing s_t is discretized into n_t intervals and s_{t+1} into n_{t+1} intervals (equal interval length) (as shown in Fig. 1), to determine the optimal s^*_{t+1} for p, the n_{t+1} discretized s_{t+1} intervals should all be tested. When the single-period utility function is concave, according to the monotonic relationship between s_t and s^*_{t+1} (and also r^*_t), if the optimal state q (at stage $t+1$) corresponding to p (at stage t) is known, then to search for the optimal s^*_{t+1} for $p+1$, only two s_{t+1} values at the states of q and $q+1$ (Fig. 1, right) need tested. Thus, by applying the monotonic relationship, the computation of determining s^*_{t+1} for s_t can be potentially reduced from n_{t+1} to 2.

An improved DP algorithm is proposed (as shown in Fig. 2), including the following steps: (1) start by discretizing s_t and s_{t+1} into intervals of equal length and number them in ascending order, 1 to n_t and 1 to n_{t+1}, respectively; (2) for initialization, set $p = 1$ (the minimum discretized s_t) and search between 1 to n_{t+1} to find its corresponding optimal s^*_{t+1}, and initialize $q = s^*_{t+1}$; (3) for computation, set $p = p + 1$ and search between q and $q + 1$, and use the better of the two to update s^*_{t+1} corresponding to p; (4) repeat step 3 until $p = n_t$. Steps 1 through 4 are designed for DP computation at stage t. By recursive computation from T to 1, the T-stage reservoir optimization operation problem can be solved [Eqs. (1) and (2)]. As illustrated in Fig. 1, when $n = n_t = n_{t+1}$, the computational complexity of the improved algorithm is $n + 2^*(n-1) = 3^*n - 2$, whereas that of the conventional algorithm is n^2, i.e., each of the s_{t+1} states (n) should be tested with each of s_t states (n). Thus, for a large number of state discretization (n), $3^*n - 2 << n^2$, the computational order of the improved algorithm can be considerably reduced.

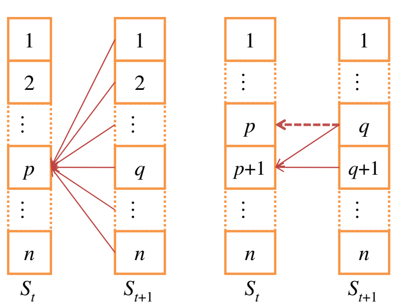

Fig. 1. A schematic diagram of conventional dynamic programming computation (left) and the improved dynamic programming computation (right)

Monotonicity with Stochastic Parameters and Improved SDP

In real-world reservoir operations, hydrological uncertainty is taken into consideration (You and Cai 2008a, b; Zhao et al. 2011), and the cumulative utility function of the two-stage operation optimization model includes an expectation operator to handle the uncertainty

$$F_t(s_t, q_t) = \max_{s_{t+1}} E[f_t(r_t) + F_{t+1}(s_{t+1}, q_{t+1})] \tag{8}$$

In SDP, hydrological uncertainty can be described by state transition probability $P_{q_{t+1}|q_t}$ of reservoir inflow, which represents the conditional probability of q_{t+1} in period $t+1$ on q_t in period t (Kelman et al. 1990; Faber and Stedinger 2001; Zhao et al. 2011). Incorporating $P_{q_{t+1}|q_t}$ into Eq. (8), the recursive function of SDP is obtained as

$$F_t(s_t, q_t) = \max_{s_{t+1}} f_t(r_t) + \sum_{q_{t+1}} P_{q_{t+1}|q_t} F_{t+1}(s_{t+1}, q_{t+1}) \tag{9}$$

Defining

$$FF_t(s_{t+1})|_{q_t} = \sum_{q_{t+1}} P_{q_{t+1}|q_t} F_{t+1}(s_{t+1}, q_{t+1}) \tag{10}$$

then Eq. (9) is rewritten as

$$F_t(s_t, q_t) = \max_{s_{t+1}} f_t(r_t) + FF_{t+1}(s_{t+1})|_{q_t} \tag{11}$$

When assuming a partially-concave dependence relationship between $F_{t+1}(s_{t+1}, q_{t+1})$ and s_{t+1} (i.e., $\frac{\partial^2 F_{t+1}(s_{t+1}, q_{t+1})}{\partial s^2_{t+1}} < 0$), given that the state transition probability $P_{q_{t+1}|q_t}$ is positive, the partial dependence relationship between $FF_{t+1}(s_{t+1})$ [the weighted sum of $F_{t+1}(s_{t+1}, q_{t+1})$, Eq. (10)] and s_{t+1} is concave (i.e., $\frac{\partial^2 FF_{t+1}(s_{t+1})|_{q_t}}{\partial s^2_{t+1}} = \sum_{q_{t+1}} P_{q_{t+1}|q_t} \frac{\partial^2 F_{t+1}(s_{t+1}, q_{t+1})}{\partial s^2_{t+1}} < 0$). Thus, with a fixed q_t [Eq. (11)], $f_t(r_t)$ and $FF_{t+1}(s_{t+1})|_{q_t}$ are concave functions, and an assumption of a partially concave dependence relationship between $F_{t+1}(s_{t+1}, q_{t+1})$ and s_{t+1} leads to the same relationship between $F_t(s_t, q_t)$ and s_t (Balinski and Baumol 1968).

Hence, when assuming that $f_t(r_t)$ is concave, following the procedures described in the "Monotonicity in Reservoir Operation Analysis" section, it can be concluded that (1) $F_{T-1}(s_{T-1}, q_{T-1}), \ldots, F_1(s_1, q_1)$ are partially concave functions of $s_{T-1}, \ldots, s_1$, respectively; and (2) with a fixed q_t, there is a monotonic relationship between s_t and s^*_{t+1} (and also r^*_t). When applying the monotonic relationship to improving SDP, the procedures are shown in Fig. 3 with two steps: step 1, for a fixed value of q_t, run the improved DP (Figs. 1 and 2) to search for s^*_{t+1} corresponding to each of the possible values of s_t; and step 2, updating q_t and repeating step 1 until all possible q_t values are tested and s^*_{t+1} corresponding to all (s_t, q_t) combinations is then identified. By employing the improved SDP in the backward recursive

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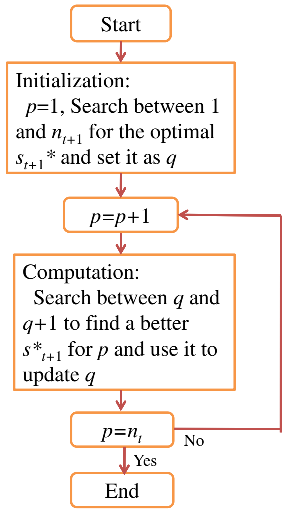

Fig. 2. Flowchart of the improved dynamic programming algorithm

formulation, the multistage stochastic reservoir operation problem can be solved.

Case Study

This study takes the Danjiangkou Reservoir located in central China (Fig. 4) as a case study to test the improved DP and SDP. The Danjiangkou Reservoir has a storage capacity of 17.45 billion m^3 and is used as one of the major storages for water transfer in the central route of the South-North Water Transfer Project (SNWTP) in China (Li et al. 2009). The primary purpose of the reservoir is water supply, although hydropower generation and flooding control are also important functions of this reservoir. This study considers the water supply function of the reservoir only with an objective to minimize water supply shortage.

The shortage index (SI) (Hydrologic Engineering Center 1981; Hsu and Cheng 2002; Tu et al. 2008) is used to define the objective function as follows:

$$SI = \frac{100}{T}\sum_{t=1}^{T}\left(\frac{TS_t}{TD_t}\right)^2 \tag{12}$$

in which T = reservoir operation horizon; and TD_t and TS_t = water demand and water shortage in period t, respectively. The SI is a monotonic convex function of TS, and $dSI/dTS \geq 0$, $d^2SI/dTS^2 \geq 0$; i.e., SI increases with TS, and the marginal SI increases as water shortage becomes more severe. As minimizing a convex function is equivalent to maximizing a concave function, the monotonicity property and the proposed improved DP algorithm are also applicable to reservoir optimization problems with increasing marginal cost, e.g., SI in Eq. (12).

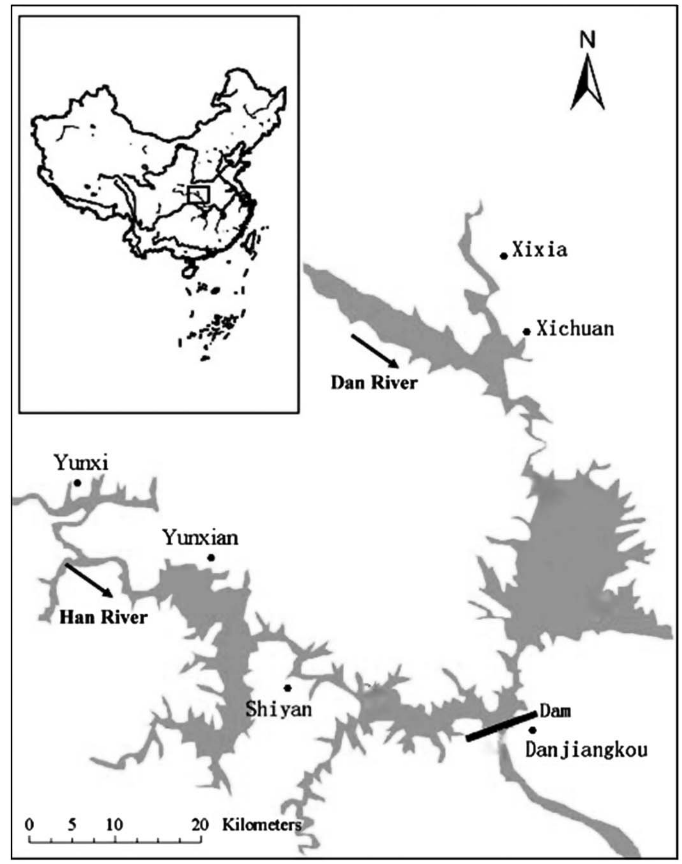

Fig. 4. Location map for the Danjiangkou Reservoir

The constraints include storage capacity constraint, nonnegative release constraint, and water balance constraint, respectively, as follows:

$$\underline{s} \leq s_t \leq \bar{s} \tag{13}$$

$$r_t \geq 0 \tag{14}$$

$$s_t = s_{t-1} + q_t - r_t \tag{15}$$

In Eqs. (13)–(15), s_t, $\underline{s}$, and $\bar{s}$ = reservoir storage in period t, minimum storage, and maximum storage, respectively;

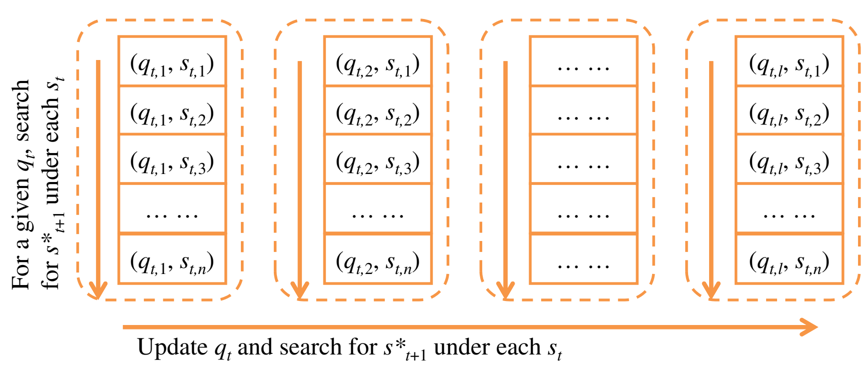

Fig. 3. Flowchart of the improved stochastic dynamic programming algorithm (l and n represent inflow and storage discretization, respectively)

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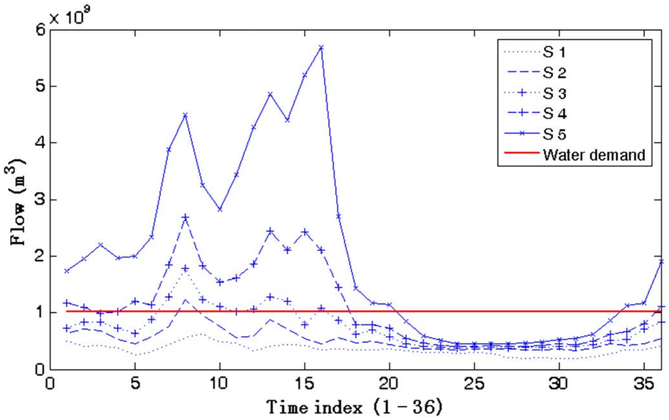

Fig. 5. Inflow scenarios and water demand year round in Danjiangkou Reservoir water supply operation optimization

r_t = reservoir release in period t; and q_t = reservoir inflow in period t. Water shortage, TS_t, in Eq. (12) is calculated with r_t

$$TS_t = \max(0, TD_t - r_t) \tag{16}$$

Eqs. (12)–(16) form a multiple-period reservoir water supply operation optimization model.

This study applies the improved DP to annual operation of the Danjiangkou Reservoir with five deterministic scenarios. The operation period (one year) is split into 36 periods, each with a length of approximately 10 days. A sequence of historical inflows (1956–1995) is employed to calculate the inflow cumulative probability function in each period. Following that, five inflow scenarios are defined on the basis of five inflow sequences corresponding to cumulative probabilities of 10, 30, 50, 70, and 90%, which represent very-dry, dry, normal, wet, and very-wet inflow scenarios and are numbered 1, 2, 3, 4, and 5, as shown in Fig. 5. Both conventional DP and improved DP are applied to optimizing water supply decision making under the five scenarios S 1, S 2, …, and S 5, respectively.

Furthermore, both conventional SDP and improved SDP are applied to stochastic reservoir operation optimization with marginal probabilities and state transition probabilities obtained from historical sequences (Stedinger et al. 1984; Kelman et al. 1990; Faber and Stedinger 2001).

The values of the key parameters of the reservoir operation model are given as follows: $\underline{s}$ is set as 0 and $\bar{s}$ is set as the conservation storage of the reservoir, 17.45 billion m^3; and TD_t ($t = 1, \ldots, 36$) is set as 1.04 billion m^3. In DP formulation, the number of state variable (reservoir storage) discretization ranges from 100 to 1,000 to compare computation times between conventional DP and improved DP. The algorithms are implemented with Matlab (http://www.mathworks.com) on a Lenovo Thinkpad T410 laptop with Intel(R) Core(TM)i5 CPU M560, 4.00 GB of RAM. The computation time is summarized in Tables 1–3.

Table 1. Computation Time (s) of Conventional DP under Different Scenarios and Storage Discretization

	No. of storage discretizations									
Scenario	100	200	300	400	500	600	700	800	900	1,000
S 1	0.5	2.1	4.8	8.5	13.2	19.0	25.9	33.8	42.8	52.8
S 2	0.5	2.1	4.7	8.5	13.2	18.9	25.8	33.7	42.7	52.7
S 3	0.5	2.1	4.7	8.4	13.1	18.9	25.7	33.6	42.5	52.4
S 4	0.5	2.1	4.7	8.4	13.1	18.8	25.6	33.3	42.3	52.1
S 5	0.5	2.1	4.6	8.2	12.9	18.5	25.2	32.9	41.6	51.4

Table 2. Computation Time (s) of Improved DP under Different Scenarios and Storage Discretization

	No. of storage discretizations									
Scenario	100	200	300	400	500	600	700	800	900	1,000
S 1	0.03	0.05	0.07	0.10	0.12	0.14	0.17	0.19	0.21	0.24
S 2	0.03	0.05	0.07	0.09	0.12	0.14	0.17	0.19	0.21	0.24
S 3	0.02	0.05	0.07	0.10	0.12	0.14	0.17	0.19	0.21	0.23
S 4	0.02	0.05	0.07	0.09	0.12	0.14	0.16	0.18	0.20	0.22
S 5	0.02	0.04	0.07	0.09	0.11	0.13	0.15	0.17	0.19	0.21

Table 3. Computation Time (s) Comparison of Conventional SDP and Improved SDP under Different Inflow and Storage Discretization

No. of inflow discretizations	SDP algorithms	No. of storage discretizations				
		100	200	300	400	500
100	Conventional	307	1,275	2,898	5,354	8,118
	Improved	8	16	28	34	40
1,000	Conventional	6,163	25,381	59,410	111,889	170,486
	Improved	122	245	367	486	617

In deterministic cases (one scenario only), conventional DP and improved DP provide the same results, whereas their computation time exhibits differences. As can be seen from Tables 1 and 2, the computation time of DP is approximately proportional to n^2, and that of the improved DP proportional to n. For example, when n increases from 100 to 1,000, conventional DP computation time increases from 0.52 to 53 s (100 times), whereas the improved DP computation time from 0.022 to 0.23 s (10 times). With the stochastic models (Table 3), the improved SDP also exhibits higher computational efficiency than the conventional SDP and for the two flow discretization cases (100 and 1,000). Again, the computation time of the conventional SDP is proportional to n^2, and that of the improved SDP is proportional to n.

Discussion and Conclusions

Diminishing marginal utility is an often realistic and important feature of water resources systems. Given a concave utility function (with diminishing marginal utility), this study proves that a monotonic relationship exists between reservoir storage and release under optimality. Taking advantage of this property, this study provides an approach to improve the computational efficiency of DP and SDP. For a real-world reservoir operation problem with a concave water supply objective, the improved DP and SDP computation complexity is $O(n)$ (order of n), compared to $O(n^2)$ with the conventional DP and SDP.

Although the proposed algorithm is demonstrated for single reservoir operation optimization, it can be extended to multireservoir operation optimization. For example, for cascade reservoir operation, the improved algorithm can be incorporated into dynamic programming with successive approximation (DPSA) (Larson and Korsak 1970), which decomposes a multiple reservoir problem into a number of subproblems with one single reservoir and uses the DP or SDP to solve each of the subproblems (Larson and Korsak 1970; Opan 2010). When all the subproblems exhibit the diminishing marginal utility characteristics and the feasible region is convex, DPSA with the improved DP or SDP can potentially enhance the computational efficiency.

Finally, in real-world reservoir operations, utilities usually depend on complicating factors, e.g., hydropower generation depends on both release and hydraulic head, and may not be concave or

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partially concave (Tilmant and Kelman 2007; Goor et al. 2011). One of the main advantages of conventional DP and SDP is the great flexibility in dealing with any kind of objective functions with no restrictions, which is paid in terms of the curse of dimensionality. As usual, introducing some regularities (i.e., constraints) will mitigate the computational burden. The improved DP and SDP exhibit high computational efficiency, which is based on a concavity assumption of the utility function. Thus, to apply this algorithm to real-world reservoir operation problems, special attention should be paid to the dependence relationships between utility and the various influencing factors.

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Appendix: The Monotonic Relationship Considering Storage Loss and Utility Discount

The derivation in the "Monotonicity and DP Improvement" section does not consider storage loss attributable to evaporation and/or seepage, which is an important factor for reservoir operation in arid or semiarid regions. This section includes this factor into the analysis to further examine the validity of monotonicity property for reservoir operation optimization.

Reservoir storage loss can be calculated through the following two approaches: (1) calculating the storage loss l proportional to storage s, i.e., $l = k * s\,(0 < k < 1)$ (e.g., You and Cai 2008b); and (2) calculating l as a function of s, e.g., $l = eab(\frac{3s}{ab})^{\frac{2}{3}} + d(\frac{3s}{ab})^{\frac{1}{3}}$ (where a and b = constants reflecting the stage-storage relationship; and e and d = coefficients for evaporation and seepage, respectively) (Lund 2006). A storage carryover function $c(s)$ can be defined as

$$c(s) = s - l \tag{17}$$

Under both aforementioned storage loss circumstances, $c(s)$ exhibits the following two properties:

$$c'(s) > 0 \tag{18}$$

which means the more water saved for the future, the more water can be carried over to the future, and

$$c''(s) \le 0 \tag{19}$$

which means that the marginal carryover ratio will decrease as storage increases; i.e., it becomes increasingly difficult to carry over reservoir storage for the future as storage increases, which can be attributed to the increase of surface area (contributing to evaporation) and water head (contributing to seepage) with reservoir storage increase (Lund 2006).

When considering the effect of storage loss, $g_2(y - x)$ in Eq. (3) will be replaced by a composition function, $g_2[c(y - x)]$. The concavity of $g_2[c(y - x)]$ depends on the properties of first- and second-order derivatives of $g_2()$ and $c()$

$$\frac{d^2 g_2(c(x))}{dx^2} = g_2'' c' + g_2' c'' \tag{20}$$

Because $c'() > 0$, $c''() \le 0$, and $g_2'' \le 0$ (i.e., diminishing marginal utility), it can easily be concluded that if $g_2' \ge 0$ (i.e., $g_2()$ is a monotonically increasing function or the utility will monotonically increase with the allocated resource), then

$$\frac{d^2 g_2(c(x))}{dx^2} = g_2'' c' + g_2' c'' \le 0 \tag{21}$$

which means that $g_2(c())$ is a concave function and the monotonicity property holds. In the case of $g_2' < 0$, to validate the concavity of $g_2(c())$, more details about c', c'', g_2', and g_2'' are needed to verify if $\frac{d^2 g_2(c(x))}{dx^2} \le 0$ or not.

Therefore, when the effect of storage loss in reservoir operation is considered, if the predefined $f_t()$ is a monotonically increasing concave function, it can sequentially be deduced that $F_{T-1}(), \ldots, F_1()$ are monotonic concave functions and the monotonicity property will hold. Previous studies (e.g., Draper and Lund 2004; You and Cai 2008b) have already adopted the monotonically increasing assumption (i.e., $f_t'() \ge 0$) as well as the concavity assumption (i.e., $f_t''() \le 0$) in reservoir system analysis, which reflects a simple relationship: more water supply induces more utility, but the utility increase rate declines.

Moreover, for long-term reservoir operation, utility discount may also be an important factor. Because it does not change the concavity of the single-period utility function, both concavity of maximum cumulative utility function and monotonicity property will hold when utility discount is taken into consideration.

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