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ELEMENT-FREE METHOD AND ITS APPLICATION IN TIDAL CURRENT AND SEDIMENT TRANSPORTATION

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Abstract: In this paper, based on the moving least squares method and the finite difference scheme, a new element-free method was proposed and used to simulate the horizontal two dimension tidal current and sediment transportation with complicated boundary. Reasonable agreement is found between the measured date and the model results.

Key words: Moving least squares method, Element-free method, Tidal current, Sediment transportation

1. INTRODUCTION

Presently, there are many numerical simulation methods on tidal current and sediment transportation, as the new science methods, the Finite Element Method and the Finite Difference Method are widely used. But because it's need to use the information of the elements and spots in the calculated process over and over again, the calculated speed is affect in a certain degree. By using the smooth function from the moving least squares method, the element-free method, keeps a few characteristic of the FEM and frees from the elements because it needs the information of spots only. The calculated speed is fast and the information is simple by using the element-free method.

In this paper, we try to introduce the element-free method to the calculation of the tidal current and sediment transportation.

2. MOVING LEAST SQUARES METHOD

2.1 FUNDAMENTAL FORMULA

Given a field function: $u(x)$, where $\boldsymbol{x}=(x,y)^T$ is any point of the field. Then giving the value of $\mathbf{u}(x)$ at n points is following:

$$\boldsymbol{u}(\boldsymbol{x}_i)=u_i,\ i=1,2,...,n \tag{1}$$

Then, we could determine an approximate function of $\mathbf{u}(x)$ by using the moving least squares method as follow:

$$G\boldsymbol{u}(\boldsymbol{x})=\sum_{j=1}^{m} p_j(\boldsymbol{x})a_j(\boldsymbol{x})=\boldsymbol{p}^T(\boldsymbol{x})\boldsymbol{a}(\boldsymbol{x}) \tag{2}$$

Where $\mathbf{a}(\mathbf{x})$ is m dimension coefficient vector, $\mathbf{p}(\mathbf{x})$ is m dimension base vector.

Towards two dimension field, $\mathbf{p}(\mathbf{x})$ could selected as:

$\boldsymbol{p}(\boldsymbol{x})=(1,x,y)^T$ Linear base, m=3

$\boldsymbol{p}(\boldsymbol{x})=(1,x,y,x^2,xy,y^2)^T$, Square base, m=6

$\boldsymbol{p}(\boldsymbol{x})=(1,x,y,x^2,xy,y^2,x^3,x^2y,xy^2,y^3)^T$, Cubic base, m=10

By using the moving least squares method, formula（2）could write as following:

$$Gu(\boldsymbol{x}) = \sum_{i=1}^{n} n_i(\boldsymbol{x})u_i \tag{3}$$

Where , $n_i(\boldsymbol{x}) = \sum_{j=1}^{m} p_j(\boldsymbol{x})[\boldsymbol{A}^{-1}(\boldsymbol{x})\boldsymbol{B}(\boldsymbol{x})]_{ji}$, $\boldsymbol{A}(\boldsymbol{x}) = \sum_{i=1}^{n} w_i(\boldsymbol{x})\boldsymbol{p}(\boldsymbol{x}_i)\boldsymbol{p}^T(\boldsymbol{x}_i)$,

$\boldsymbol{B}(\boldsymbol{x}) = [w_1(\boldsymbol{x})\boldsymbol{p}(\boldsymbol{x}_1),\cdots,w_n(\boldsymbol{x})\boldsymbol{p}(\boldsymbol{x}_n)]$, $w_i(\boldsymbol{x})$ is the value of the influence function of i point at $\mathbf{x} = (x, y)^T$; $n_i(\boldsymbol{x})$ is the value of the form function of i point at $\mathbf{x}$, the partial differentials of the form function are:

$$n_{i,k} = \sum_{j=1}^{m} \{p_{j,k}[\boldsymbol{A}^{-1}\boldsymbol{B}]_{ji} + p_j[\boldsymbol{A}_{,k}^{-1}\boldsymbol{B} + \boldsymbol{A}^{-1}\boldsymbol{B}_{,k}]_{ji}\} \tag{4}$$

$$\begin{aligned} n_{i,kk} = \sum_{j=1}^{m} \{&p_{j,kk}[\boldsymbol{A}^{-1}\boldsymbol{B}]_{ji} + 2p_{j,k}[\boldsymbol{A}_{,k}^{-1}\boldsymbol{B} + \boldsymbol{A}^{-1}\boldsymbol{B}_{,k}]_{ji} \\ &+ p_j[\boldsymbol{A}_{,kk}^{-1}\boldsymbol{B} + 2\boldsymbol{A}_{,k}^{-1}\boldsymbol{B}_{,k} + \boldsymbol{A}^{-1}\boldsymbol{B}_{,kk}]_{ji}\} \end{aligned} \tag{5}$$

Where $\boldsymbol{A}_{,k}^{-1} = -\boldsymbol{A}^{-1}\boldsymbol{A}_{,k}\boldsymbol{A}^{-1}$, $\boldsymbol{A}_{,kk}^{-1} = (-\boldsymbol{A}^{-1}\boldsymbol{A}_{,k}\boldsymbol{A}^{-1})_{,k}$

2.2 INFLUENCE FUNCTION

Suitable choose of the influence function has a direct bearing on the calculated quality of the element-free method. In this paper, we adopt the influence function as following:

$$w_i(r_i) = \begin{cases} \dfrac{r_{mi}^2}{r_i^2 + \varepsilon^2 r_{mi}^2}(1 - \dfrac{r_i^2}{r_{mi}^2})^k , & r_i \le r_{mi} \\ 0 , & r_i > r_{mi} \end{cases} \tag{6}$$

where $r_i = \|\boldsymbol{x} - \boldsymbol{x}_i\|$ is the distance from $\boldsymbol{x}_i$ to $\boldsymbol{x}$, r_{mi} is the influence radius of the i point, ε is a small positive number, k is a positive integer number.

From formula (6), it is easily known that the influence function $w_i(r_i)$ have k-1strata continuous partial differentials on coordinate in the field. Then the form functions in formula (3) also exit the k-1strata continuous partial differentials.

ε and k could choose free in a certain degree and the well choose could improve the calculated quality. In this paper, according to the experience, we give k=4 and ε =1.0, the calculated result is well.

r_{mi} also need to select suitable, it must be satisfied the non-queer characteristic of formula (3) and could reduce the calculated quantity possibly. In this paper, we choice carefully for every points to let there are at least 6 points in the limits of r_{mi}.

3. TWO DIMENSION GOVERNING EQUATIONS

The two dimension governing equation of tidal current and sediment are :

$$\begin{cases} \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} + g\dfrac{\partial z}{\partial x} + g\dfrac{u\sqrt{u^2+v^2}}{C_w{}^2 h} + \varepsilon_H\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}\right) - fv = 0 \\ \dfrac{\partial v}{\partial t} + u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} + g\dfrac{\partial z}{\partial y} + g\dfrac{u\sqrt{u^2+v^2}}{C_w{}^2 h} + \varepsilon_H\left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2}\right) + fu = 0 \\ \dfrac{\partial z}{\partial t} + \dfrac{\partial (uh)}{\partial x} + \dfrac{\partial (vh)}{\partial y} = 0 \end{cases} \tag{7}$$

$$\frac{\partial h\Phi}{\partial t} + u\frac{\partial h\Phi}{\partial x} + v\frac{\partial h\Phi}{\partial y} + \frac{\partial}{\partial x}\varepsilon_x\frac{\partial h\Phi}{\partial x} + \frac{\partial}{\partial y}\varepsilon_y\frac{\partial h\Phi}{\partial y} = S_\Phi \tag{8}$$

Where $(x,y)\in\Omega$, Ω is the plant field, u and v is the depth-averaged horizontal velocity components, z is the water level, h is the depth of water, C_w is Chezy coefficient, ε_H is moving stickiness coefficient, f is Coriolis coefficient, Φ is the content of suspension sand, ε_x and ε_y is the diffuse coefficient of sand, S_Φ is the sand source.

The details of the expression (see Chen H's thesis,1997) is omitted here for simplicity.

4. CALCULATION OF GOVERNING EQUATIONS

Given f_i^n stand for the value of i point at n ocassion. Equation (8) and (9) could spread out to different form:

$$\frac{z_i^{n+1}-z_i^n}{\Delta t}+(\frac{\partial(hu)}{\partial x})_i^n+(\frac{\partial(hv)}{\partial y})_i^n=0 \tag{9}$$

$$\begin{aligned}&\frac{u_i^{n+1}-u_i^n}{\Delta t}+u_i^{n+1}(\frac{\partial u}{\partial x})_i^n+v_i^n(\frac{\partial u}{\partial y})_i^n+g(\frac{\partial z}{\partial x})_i^{n+1}-fv_i^n\\&+\varepsilon_H\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)_i^n+gu_i^{n+1}(\frac{\sqrt{u^2+v^2}}{C_w{}^2h})_i^n=0\end{aligned} \tag{10}$$

$$\begin{aligned}&\frac{v_i^{n+1}-v_i^n}{\Delta t}+u_i^{n+1}(\frac{\partial v}{\partial x})_i^n+v_i^{n+1}(\frac{\partial v}{\partial y})_i^n+g(\frac{\partial z}{\partial y})_i^{n+1}+fu_i^{n+1}\\&+\varepsilon_H\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)_i^n+gv_i^{n+1}(\frac{\sqrt{u^2+v^2}}{C_w{}^2h})_i^n=0\end{aligned} \tag{11}$$

$$\begin{aligned}&\frac{(h\Phi)_i^{n+1}-(h\Phi)_i^n}{\Delta t}+u_i^{n+1}(\frac{\partial h\Phi}{\partial x})_i^n+v_i^{n+1}(\frac{\partial h\Phi}{\partial y})_i^n+(\frac{\partial}{\partial x}\varepsilon_x\frac{\partial h\Phi}{\partial x})_i^n\\&+(\frac{\partial}{\partial y}\varepsilon_y\frac{\partial h\Phi}{\partial y})_i^n=S_{\Phi i}^{n+1}\end{aligned} \tag{12}$$

Where the partial differentials in ( ) of the formula is the same form of formula(4) and (5). Then the calculated formula are:

$$z_i^{n+1}=z_i^n-\Delta t[(\frac{\partial(hu)}{\partial x})_i^n+(\frac{\partial(hv)}{\partial y})_i^n] \tag{13}$$

$$\begin{aligned}u_i^{n+1}=&\{u_i^n-\Delta t[v_i^n(\frac{\partial u}{\partial y})_i^n+g(\frac{\partial z}{\partial x})_i^{n+1}-fv_i^n+\varepsilon_H\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)_i^n]\}\\&\Big/\{1+\Delta t[(\frac{\partial u}{\partial x})_i^n+g(\frac{\sqrt{u^2+v^2}}{C_w{}^2h})_i^n]\}\end{aligned} \tag{14}$$

$$\begin{aligned}v_i^{n+1}=&\{v_i^n-\Delta t[u_i^{n+1}(\frac{\partial v}{\partial x})_i^n+g(\frac{\partial z}{\partial y})_i^{n+1}+\varepsilon_H\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)_i^n+fu_i^{n+1}]\}\\&\Big/\{1+\Delta t[(\frac{\partial v}{\partial y})_i^n+g(\frac{\sqrt{u^2+v^2}}{C_w{}^2h})_i^n]\}\end{aligned} \tag{15)}$$

$$\begin{aligned}(h\Phi)_i^{n+1}=&(h\Phi)_i^n+\Delta t\{(S_\Phi)_i^n-u_i^{n+1}(\frac{\partial h\Phi}{\partial x})_i^n-v_i^{n+1}(\frac{\partial h\Phi}{\partial y})_i^n\\&-(\frac{\partial}{\partial x}\varepsilon_x\frac{\partial h\Phi}{\partial x})_i^n-(\frac{\partial}{\partial y}\varepsilon_y\frac{\partial h\Phi}{\partial y})_i^n\}\end{aligned} \tag{16}$$

5. CACULATION EXAMPLE

As an example, the numerical simulated model of the Bohai Bay Da-Kou-He River coastal sea region is build up, the calculated results shows that the element-free method model is well done with high precision and fast speed and nice stability. Non-equal distance points were arranged in the model and dense ones at offshore and estuary and the area where terrain change complexly.

For the reality situation, the mouth of Da-kou-he River is regarded as an opening boundary. The tide level of the surveying stations on shore is given as water boundary, the level of four continual surveying points (Lian 1 to Lian 4) in the sea are regarded as the adjusting points. The results of tidal current and sediment transportation are well agreeable to the measured data which are showed as the figures.

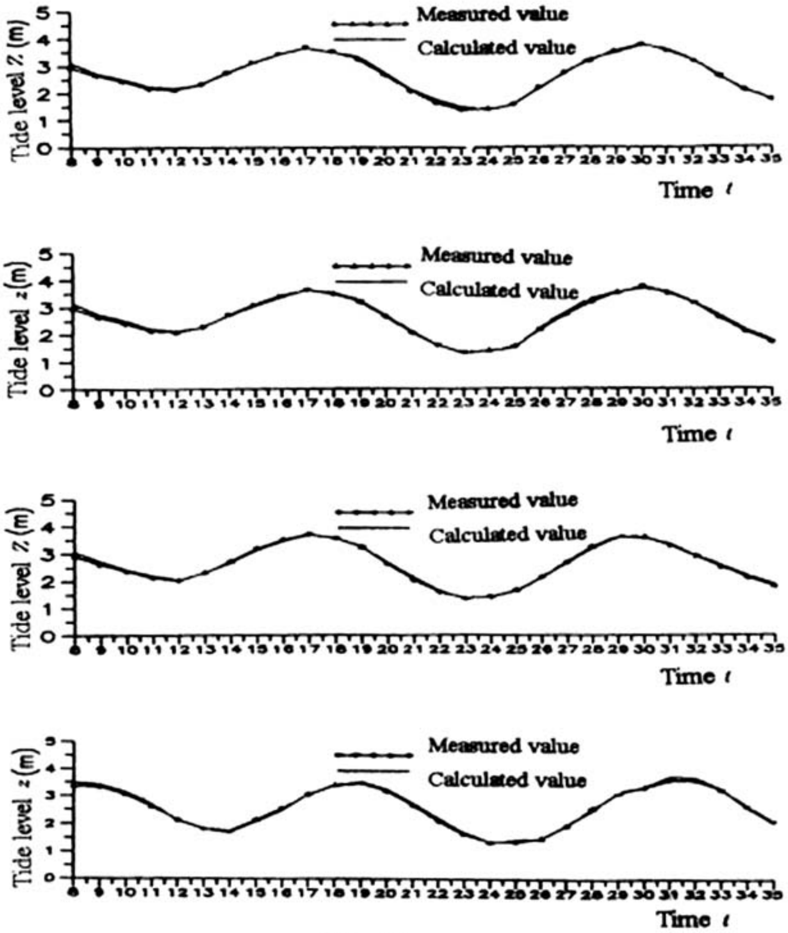

Fig. 1 Process of the tide level (Lian 1-Lian 4)

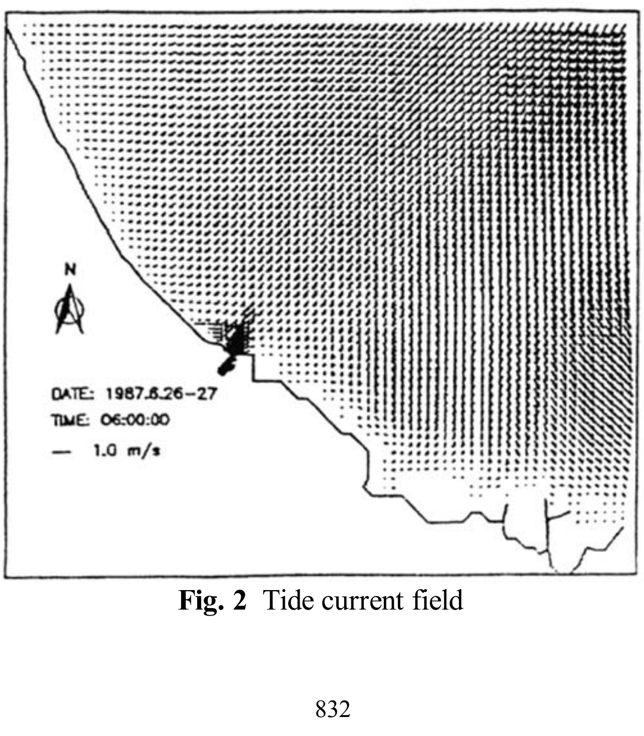

Fig. 2 Tide current field

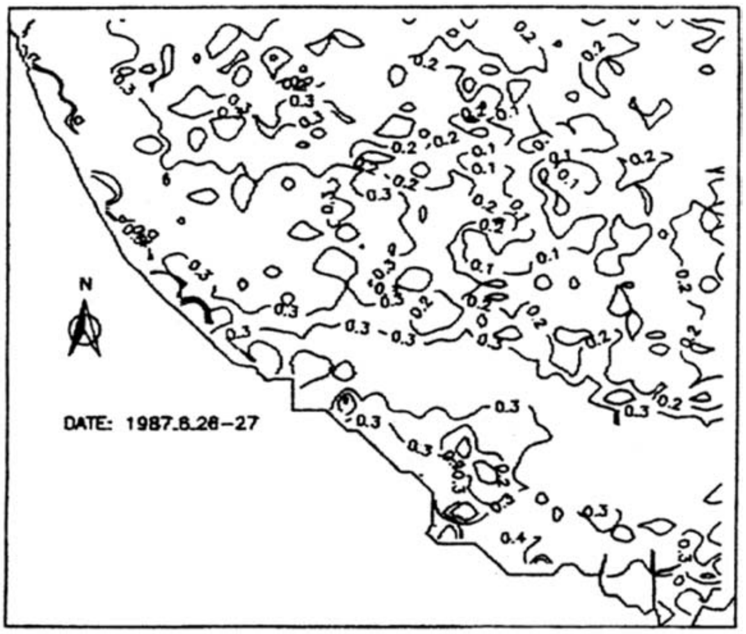

Fig. 3 Contour line of suspension sand

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