

水平圆管中受压扭作用管柱屈曲后的解析解

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摘要 首先建立了考虑自重、受水平圆管约束的管柱在压扭组合作用下的屈曲方程, 此方程为四阶强非线性常微分方程, 难以直接精确求解; 通过管柱的正弦屈曲分析, 找到了方程中的小参数; 利用小参数摄动法求解了强非线性屈曲方程, 得到了管柱螺旋屈曲后的实际屈曲构形解析解, 由此可进一步得到管柱发生螺旋屈曲的临界载荷; 所得解析结果与数值计算结果及本文结果退化后与文献中的相关结果比较都有良好的一致性.

关键词 管柱, 压扭组合, 屈曲, 后屈曲, 螺旋屈曲

引 言

受井眼约束管柱的屈曲是石油工程中的非常重要的问题, 管柱的屈曲行为不但会引起钻柱、油管等的磨损和破坏; 还增加了钻进中的阻力, 甚至引起井眼中管柱锁死而使正常作业无法进行; 尤其是近年来软管的应用及大位移井钻探的发展, 这个问题显得越发突出.

管柱受压后螺旋屈曲的研究, 自从 Lubinski (1962)^[1] 的文章发表以来, 已出现了大量的研究文献. 实际工程中, 管柱除受压外还常承受很大的扭矩作用, 而扭矩对管柱螺旋屈曲影响的研究文献不多. Miska (1995)^[2] 假设屈曲构形为圆柱螺旋线, 用能量法分析了无重管柱在压扭组合作用下螺旋屈曲行为, 得到了螺距与载荷之间的关系; 刘凤梧等 (1997)^[3] 用平衡法, 证明了 Miska 假设的屈曲构形的确为受压扭无重管柱屈曲方程的精确解. 实际工程中, 管柱的自重会对螺旋屈曲行为产生很大影响, 而用通常的能量法难以求得管柱屈曲后的实际构形.

本文首先建立了管柱在压扭组合作用下的屈曲方程, 通过管柱的正弦屈曲分析, 找到了方程中的小参数, 利用摄动法求解了四阶强非线性常微分方程; 最后讨论了管柱所受扭矩和管柱自重对其在水平约束圆管内螺旋屈曲行为的影响.

1 管柱屈曲微分方程的建立

位于水平约束圆管底部的管柱, 受压力 (F_0) 和扭矩 (M_n) 作用屈曲后, 由于自重 (q) 及圆管的约束, 管柱与约束圆管内壁保持连续接触; 设管柱轴线离开直线平衡位置的偏转角为 θ .

建立如图 1 所示, 右手坐标系 $oxyz$, 其中 z 轴沿约束圆管轴线, 管柱轴线上任一点 C 均在半径为 r 的圆柱面上. C 点矢径可表为

$$r = xi + yj + zk \quad (1)$$

其中

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$$x = r \cos \theta, \quad y = r \sin \theta \quad (2)$$

从管柱中截取微元 dz , 其微元平衡方程^[4]为

$$\frac{dF}{dz} + f = 0 \quad (3)$$

$$\frac{dM}{dz} + \frac{dr}{dz} \times F = 0 \quad (4)$$

若不计摩擦则有

$$f = -N \cos \theta i - N \sin \theta j \quad (5)$$

其中 N 为约束圆管与管柱之间的接触分布力, 管柱仍可看作细长梁, 其物理关系为^[5]

$$M = EI \left(\frac{dr}{dz} \times \frac{d^2 r}{dz^2} \right) + GJ \frac{d\gamma}{dz} \frac{dr}{dz} \quad (6)$$

其中 EI 为管柱的抗弯刚度, GJ 为抗扭刚度, γ 为扭转角.

将方程 (1)~(6) 化简可得水平约束圆管内管柱受压扭作用时的屈曲方程

$$\frac{d^4 \theta}{dz^4} + \left[\frac{F_0}{EI} + 3 \frac{M_n}{EI} \frac{d\theta}{dz} - 6 \left(\frac{d\theta}{dz} \right)^2 \right] \frac{d^2 \theta}{dz^2} + \frac{q}{EI r} \sin \theta = 0 \quad (7)$$

令 $\omega_0 = \sqrt{\frac{F_0}{2EI}}$; 引入无量纲参数

$$\xi = \omega_0 z, \quad m_0 = \frac{\sqrt{2} M_n}{\sqrt{F_0} EI} \quad (8)$$

$$\varepsilon = \frac{q}{EI r \omega_0^4}, \quad n = \frac{N}{EI r \omega_0^4} \quad (9)$$

将以上关系代入 (7) 可得无量纲化的管柱屈曲微分方程

$$\theta_\xi'''' - 6\theta_\xi'^2 \theta_\xi'' + 3m_0 \theta_\xi' \theta_\xi'' + 2\theta_\xi'' + \varepsilon \sin \theta = 0 \quad (10)$$

进一步可得屈曲后管柱与约束管壁之间接触力分布关系式

$$n = 4\theta_\xi''' \theta_\xi' + 3\theta_\xi''^2 - \theta_\xi'^4 + 2\theta_\xi'^2 + m_0 (\theta_\xi'^3 - \theta_\xi''') + \varepsilon \cos \theta \quad (11)$$

2 屈曲方程的求解

当管柱所荷载荷增加到某一极限时, 管柱将不再保持其直线平衡状态而发生正弦屈曲, 但这时 θ 仍为小量, 因此可将方程 (10) 线性化, 得到四阶线性常微分方程

$$\theta_\xi'''' + 2\theta_\xi'' + \varepsilon \sin \theta = 0 \quad (12)$$

方程 (12) 与弹性基础上压杆的屈曲方程形式上一致, 其特征方程有完全虚根即管柱发生正弦屈曲的条件为 $\varepsilon \leq 1$; 其临界状态 $\varepsilon = 1$, 对应管柱发生正弦屈曲的临界载荷.

$$F_{crs} = 2\sqrt{\frac{EIq}{r}} \quad (13)$$

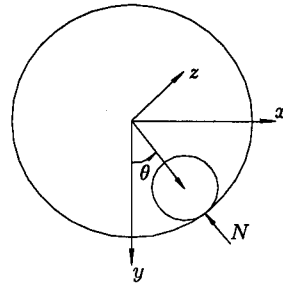


图 1 坐标系

Fig.1 Coordinate system for buckling analysis

此结果与 Dawson (1984)^[6] 用能量法所得结果一致.

当载荷继续增加, 管柱由正弦屈曲过渡到螺旋屈曲, 这时偏转角 θ 不再是小量; 但有 $\varepsilon < 1$, 因此可以考虑将 ε 作为小参数, 利用摄动法求解四阶非线性常微分方程. 令

$$\theta = \theta_0 + \varepsilon\theta_1 \quad (14)$$

将上式代入屈曲方程 (10), 展开后比较 ε 各次幂的系数有

$$\varepsilon^0: \theta_0'''' - 6\theta_0'^2\theta_0'' + 2\theta_0'' + 3m_0\theta_0'\theta_0'' = 0 \quad (15)$$

$$\varepsilon^1: \theta_1'''' - 6\theta_0'^2\theta_1'' + 12\theta_0'\theta_1'\theta_0'' + 2\theta_1'' + 3m_0(\theta_0'\theta_1'' + \theta_1'\theta_0'') + \sin\theta_0 = 0 \quad (16)$$

零阶摄动解为与 ε 零次幂相对应的非线性常微分方程 (15) 的解, 方程 (15) 与无重管柱的非线性屈曲方程相同, 实际中管柱很长, 可暂不考虑管柱端部边界对螺旋屈曲的影响, 对此方程刘凤梧等已求得其精确解. 屈曲方程除平凡解外, 对应管柱螺旋屈曲平衡状态的解记为

$$\alpha = \theta_0' = \frac{3m_0}{8} \pm \sqrt{\frac{9}{64}m_0^2 + 1} \quad (17)$$

其中正、负号表示管柱屈曲构形螺旋的旋向; 以下主要取正号分析.

若令 $\theta_0(0) = 0$, 则有

$$\theta_0 = \alpha\xi \quad (18)$$

将零阶摄动解 (18) 结果代入 (16) 式可得

$$\theta_1'''' - (6\alpha^2 - 3m_0\alpha - 2)\theta_1'' + \sin\theta_0 = 0 \quad (19)$$

管柱可看作无限长, 其边界条件为 $\theta_1(0) = 0$, $\xi \rightarrow \pm\infty$ 时, $\theta_1(\xi)$ 应有界, 可得

$$\theta_1(\xi) = -A \sin \alpha\xi,$$

其中

$$A = \frac{1}{\alpha^3} \frac{1}{7\alpha^2 - 3m_0 - 2} \quad (20)$$

由 (14) 式可得屈曲方程的一阶摄动解为

$$\theta(\xi) = \alpha\xi - A\varepsilon \sin \alpha\xi, \quad \theta'(\xi) = \alpha - A\alpha\varepsilon \cos \alpha\xi \quad (21)$$

将屈曲方程的摄动解 (21) 代入 (11) 式整理, 略去 ε 二次以上的高阶小量有

$$n = 2\alpha^2 - \alpha^4 + [4(2\alpha^2 - 1)A\alpha^2 + 1]\varepsilon \cos \alpha\xi - (10\alpha^2 - 2)A^2\alpha^2\varepsilon^2 \cos^2 \alpha\xi + (3A^2\alpha^4 + A)\varepsilon^2 \sin^2 \alpha\xi + m_0(1 - 4A\varepsilon \cos \alpha\xi + 3A^2\varepsilon^2 \cos^2 \alpha\xi)\alpha^3 \quad (22)$$

由上式可以看出当 $\alpha\xi = 2k\pi$ 时, n 取极大值 n_{\max} , 对应约束圆管底部与管柱的接触力; 当 $\alpha\xi = (2k-1)\pi$ 时 n 取极小值 n_{\min} , 对应约束圆管顶部与管柱的接触力. 当管柱发生螺旋屈曲时, 应与上井壁接触, 即有 $n_{\min} \geq 0$; 根据此条件由 (22) 式可以确定管柱在水平约束圆管中发生螺旋屈曲的临界载荷.

3 结果分析

1) 本文结果与数值计算结果的比较 对于常微分方程 (10), 可利用差分法解边值问题求得非线性常微分方程的数值解, 数值结果见图 2, 由图可见其域内数值结果与本文解析结果有非常良好的一致性。

2) 本文结果退化后与能量法结果的比较 当管柱所受扭矩为零时, $n_{\min} = 1 - \frac{9}{5}\varepsilon - \frac{8}{25}\varepsilon^2$, 由条件 $n_{\min} \geq 0$ 可得管柱在水平约束圆管中发生螺旋屈曲的临界载荷为: $F_{crh} = 2.802\sqrt{\frac{EIq}{r}}$; 与 Chen (1990)^[7] 用能量法所得结果比较相差小于 1%。

3) 扭矩的影响见图 3, 其中 m_0 表示扭矩的相对大小。

4) 自重的影响 (见图 4), 其中 ε 为管柱自重的无量纲量。

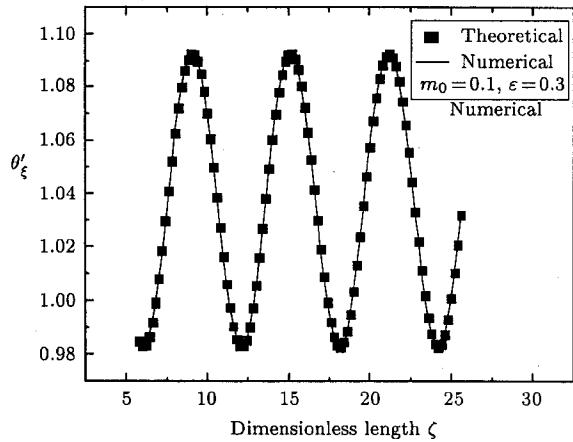


图 2 本文解析解与数值解的比较
Fig.2 Comparison of the analytical and numerical solution

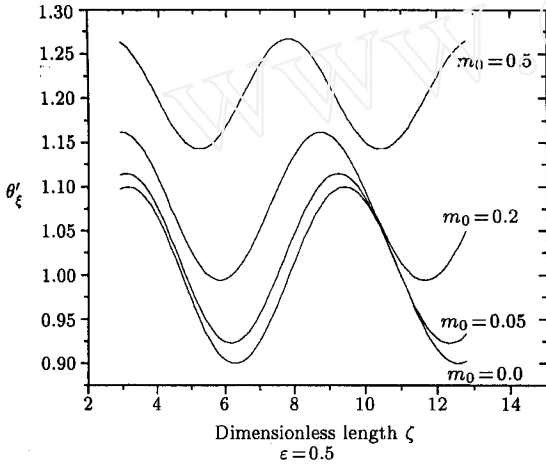


图 3 扭矩对屈曲构形的影响
Fig.3 The effects of torque on real helical shapes

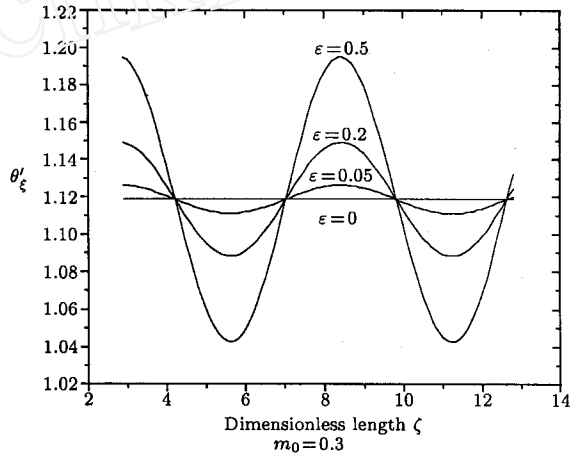


图 4 管柱自重对屈曲构形的影响
Fig.4 The effects of pipe unit weight on real helical shapes

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AN ANALYTIC SOLUTION FOR POST-BUCKLING OF TUBULARS SUBJECTED TO AXIAL AND TORSIONAL LOADING IN HORIZONTAL CIRCULAR CYLINDERS

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Abstract Buckling of radially confined oil tubulars is of significant importance to the oil industry. It will affect the life of the tubulars and cause difficulties in drilling and in oil production. A number of theoretical models have been developed to study the buckling load and the post-buckling configuration of the tubular, but the most of the past studies have not considered the torsional loads. Only recently, some investigators based on energy principle have derived some results for helical buckling of tubulars subjected to axial and torsional loading. The underlying assumption for current energy method is that the helix pitch of the helical buckling shape is constant. However, it is difficult to obtain real post-buckling configuration by the energy methods.

In this paper, the buckling equation for a buckled tubing subjected to axial and torsional loading in horizontal circular cylinder is derived. The equation is a fourth-order strongly nonlinear ordinary differential equation, and it is too difficult to solve it exactly. So a small parameter in the equation is determined by discussing the initiate sinusoidal buckling of the tubular. If the parameter is set to zero, the equation that describe the helical buckling for the case of weightless tubulars is still nonlinear but the exact solution have been derived. Using perturbation method, an approximate analytical solution for the equation that describe the real helical buckling configuration of tubular with weight have been derived. Based on the solution, the critical helical buckling load can be determined. The analytical results obtained by this paper well coincide with the results obtained by numerical method. The degenerative results of the analytical results obtained are also coincident with those given by others using energy method.

Key words tubular, buckling, post-buckling, helical buckling, axial and torsional loading

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