一种高阶非线性钢筋混凝土平板单元

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摘要　在文献[1]高阶钢筋混凝土膜元的基础上,提出了一种具有 14 个节点的高阶钢筋混凝土平板单元.钢筋混凝土材料模型仍采用 Vecchio 的抹平旋转裂缝模型,几何非线性仍采用所谓总体 Lagrange 列式法,非线性方程采用割线刚度位移增量迭代法.数值算例表明本文的方法是可靠的,高阶平板单元虽然列式复杂,但与低阶元相比,其计算量要少且精度要高.

关键词　高阶平板单元,抹平裂缝模型,几何非线性

引　言

钢筋混凝土剪力墙结构通常采用平面应力单元进行分析,由于平面应力单元不能反应剪力墙施工过程中留下的各种缺陷和荷载在墙体平面的偏心作用效应,平面应力单元亦无法模拟剪力墙可能存在的失稳破坏.因此,采用钢筋混凝土平板单元是解决钢筋混凝土剪力墙承载能力的有效方法,最近的工作有 Massicotte, MacGregor 和 Elwi[2], Di 和 Cheung[3] 和 Vecchio[4] 等.然而,由于低阶板元的应力精度差,往往导致过密的有限元网格或过大的预计钢筋混凝土墙板刚度降低,以致计算量过大或计算结果失真.鉴于高阶平面应力膜元的诸多优点[1],本文进一步探讨高阶钢筋混凝土板元.材料模型仍采用 Vecchio 的抹平旋转裂缝模型,几何非线性仍采用总体 Lagrange 列式法.

1　钢筋混凝土抹平旋转裂缝模型

在加载过程中,尽管钢筋砼膜元的两个主应变方向可能是在不断改变或旋转的,仍将钢筋砼膜元视为主轴与主应变轴在全部的加载过程中始终重合的正交各向异性材料,这一模型的数学表达是依据砼单轴 σ-ε 曲线特征和与钢筋砼膜元的双轴实验结果的拟合来最终确定的.

由 Vecchio 模型[6],本文提出的改进模型为

砼受拉时:

$$\sigma_1^c = E_c\varepsilon_1^c, \qquad 0 < \varepsilon^c\varepsilon_1 < \varepsilon_{cr} \tag{1}$$

$$\sigma_1^c = K_a f_t/(1+\sqrt{200\varepsilon_1^c}), \qquad \varepsilon_1^c \ge \varepsilon_{cr} \tag{2}$$

$$\left.\begin{aligned} K_a &= 1 + (11 - 10\varepsilon_1^c/\varepsilon_{cr})\sqrt{200\varepsilon_1^c}, && \varepsilon_{cr} \le \varepsilon_1^c \le 1.1\varepsilon_{cr} \\ K_a &= 1, && \varepsilon_1^c > 1.1\varepsilon_{cr} \end{aligned}\right\}$$

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式中

$$\left.\begin{aligned} &\beta = \frac{1}{0.85 - 0.27\varepsilon_2^c/\varepsilon_1^c}, \quad \alpha = 0 \sim 0.2, \quad E_c = 5\,000\sqrt{f_c'} \\ &f_t = 0.33\sqrt{f_c'}, \quad \varepsilon_0 = 8.322\times 10^{-4}\sqrt{f_c'}, \quad \varepsilon_{cr} = f_t/E_c \end{aligned}\right\}$$

(7)

其中 $n_i(x)$, $m_j(y)$和 $l_j(y)$的表达式可参阅文献[1],因此中面应变为

$$\left.\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 = \sum_{i=1}^{16} u_i N'_{i,x} + \frac{1}{2}\sum_{i=1}^{28}\sum_{j=1}^{28} w_i w_j M'_{i,x} M'_{j,x} \\ \varepsilon_y &= \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 = \sum_{i=1}^{16} v_i N'_{i,y} + \frac{1}{2}\sum_{i=1}^{28}\sum_{j=1}^{28} w_i w_j M'_{i,y} M'_{j,y} \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} = \sum_{i=1}^{16}(u_i N'_{i,y} + v_i N'_{i,x}) + \sum_{i=1}^{28}\sum_{j=1}^{28} w_i w_j M'_{i,x} M'_{j,y} \end{aligned}\right\}$$

板中面的曲率 χ_x, χ_y 和 χ_{xy}分别为

$$\left.\begin{aligned}\chi_x &= -\frac{\partial\psi_x}{\partial x} = -\sum_{i=1}^{16}\psi_{xi}N_{i,x}',\quad \chi_y = -\frac{\partial\psi_y}{\partial y} = -\sum_{i=1}^{16}\psi_{yi}N_{i,y}'\\ \chi_{xy} &= -\left(\frac{\partial\psi_x}{\partial y}+\frac{\partial\psi_y}{\partial x}\right) = -\sum_{i=1}^{16}(\psi_{xi}N_{i,y}' + \psi_{yi}N_{i,x}')\end{aligned}\right\}\tag{12}$$

式中$\left\{\begin{matrix} l_1^T \\ l_2 \end{matrix}\right.$ 为位移向量 u 和荷载参数 λ 构成的 $n+1$ 维空间中平衡路径的 A^{t-1}点指向 A^t 点的单位方向向量[1], Δl^t 为投影增量. 该法具有列式简单,可顺利通过极值点的优点,而且由于约束方程是 u 和 λ 的线性函数,没有 Crisfield 和 Ramm 的弧长法需根据结构特性选根的困扰.

3　算　例

算例 1　在文献[2]中,Massicotte 等分析了 8 类共 26 片钢筋砼板,我们选取其中的 PS1 和 PS2 来作比较计算. 板 PS1 为边长是 1 800 mm 的方板,板 PS2 为 1 800 mm ×3 600 mm 的长方形板,板厚均为 60 mm,沿板底边和顶边各布置有 1 层钢筋,底层和顶层钢筋是相同的,沿 x 方向配筋率 ρ_x 和沿 y 方向配筋率 ρ_y 均为 0.4 %,顶层和底层分布钢筋的中心到板的顶面和底面的距离均为 0.15 h = 9 mm. 砼 f_c' = 35 MPa,初始弹性模量为 26 500 MPa,拉伸强度 f_t = 2.0 MPa,峰值应变 ε_0 = 0.002 4,泊松比 0.2,钢筋为双线性弹性硬化模型,屈服应力为 400 MPa,弹性模量为 200 000 MPa,极限强度为 500 MPa,相应的极限应变选取为 0.66.

利用本文高阶矩形砼板元计算的 PS1 和 PS2 板的结果分别示于图 2 和图 3 中,从图中,我们看到有限元模式对板在达到极限荷载后的平衡路径的影响较之前要大得多. 本文与文献[2]的差别可能来自砼材料模型的不完全相同和单元的应力精度不同,因 Massicotte 采用的是

所谓 NISA-80 程序中的三维退化板壳元进行分析的,数值积分的方式是在单元面内 4 ×4 的完全高斯积分,而沿厚度 z 方向是用 Simpson 积分来完成的.

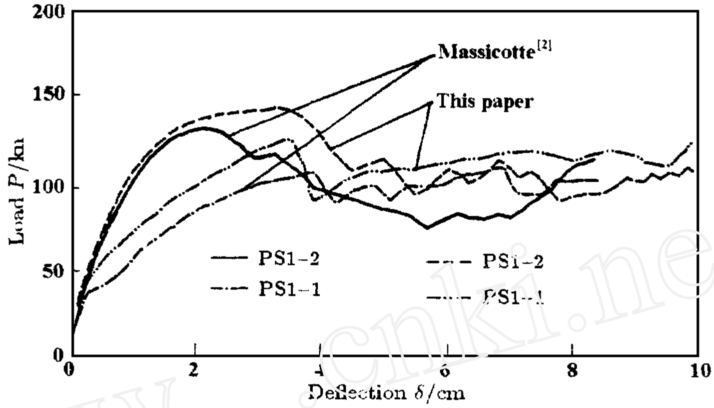

图 2 横向均布荷载作用下 PS1 板的荷载-挠度曲线

Fig. 2 Load-deflection curves of panel PS1 under lateral uniform loads

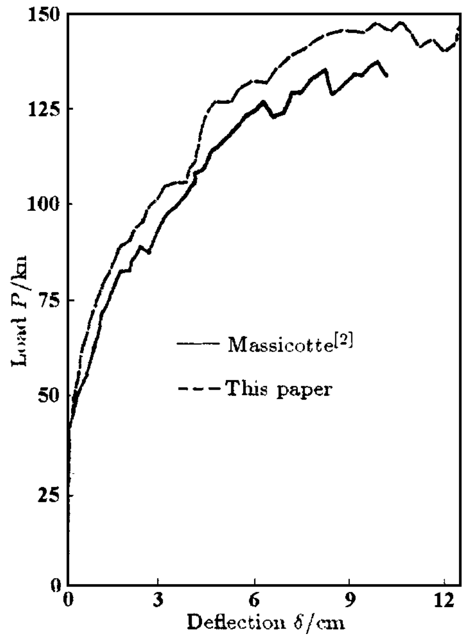

图 3 均布荷载作用下 PS2 板的载荷-挠度曲线

Fig. 3 Load-deflection curves of panel PS2 under uniform loads

算例 2 Saheb 和 Desayi 对开孔的墙体试件进行了竖向偏心加载试验,有关试件及材料参数请见文献[5],本文的计算结果与 Saheb 和 Desayi 的试验结果列于表 1,可见两者吻合较好.

4 结 语

本文克服高阶非线性有限元列式复杂,迭代求解难于收敛等困难获得了钢筋混凝土剪力

墙和开孔墙板的载荷-挠度曲线等 ,与前人比较 , 本文的主要特点是考虑了几何和材料双重非线性 ,并且获得了载荷-挠度曲线的下降段.

表 1　开孔墙板在竖向荷载作用下的开裂和极限荷载的试验结果和计算结果

Table 1 Experimental and computational results for crack and ultimate loads of RC panel walls with openings under eccentric compressive loads

Test specimen	Crack loads[5]	Crack loads	Ultimate loads[5]	Ultimate loads
WW0 - 1	483.97	480.02	672.56	658.31
WW0 - 2	193.19	217.72	568.90	570.02
WW0 - 3	143.18	167.73	433.47	438.82
WW0 - 4	-	490.31	652.65	650.70
WW0 - 5	229.97	261.10	548.02	561.37
WW0 - 1(P)	554.09	488.39	692.47	661.77
WW0 - 2(P)	207.42	232.67	592.83	570.83
WW0 - 3(P)	154.46	168.92	448.38	478.10
WW0 - 4(P)	547.23	492.66	697.47	700.31
WW0 - 5(P)	267.24	269.03	587.83	563.56

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A HIGHER ORDER NONLINEAR FEM FOR RC PLATES

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Abstract Based on the high order nonlinear membrane elements presented in Reference [1], the formulation of a higher order rectangular 14-noded RC Plate element is proposed for the analysis of RC panel walls. The nodal DOF of the element presented are assumed to be in-plane displacements u, v; transverse deflection w and cross-section rotations ψ_x, ψ_y, and their derivatives (with respect to x). Of the element displacements consequently are 3rd order Hermite polynomial in the x-direction and 6th order Lagrangian polynomial in y-direction along the element for transverse deflection w, and the full cubic polynomials for all others.

Green's strain-displacement relations are used to take into account geometric nonlinear behaviour of the panel walls. The plate element is considered to be composed of a series of layers. Material behaviour of concrete layers and steel reinforcement layers are defined separately on the basis of the smeared, rotating crack model proposed by Vecchio and Collins for RC membrane elements. Usually, according to Di and Cheung, a selective reduced order of integration is employed to improve the shear locking in thin RC panel walls.

Based on the constant are length method presented by Crisfield et al., a new constraint equation is developed in this study to trace the complete nonlinear equilibrium paths of RC panel walls under nonproportional loadings. This procedure employs projected length other than arc length as a control variable. The iterative procedures are carried out in the space R^{n+1}. Both the load factor and displacements are used here as iterative parameters. Hence whether or not the structural failures occur the displacements and associated loads can be calculated without any diffculties.

Numerical examples are given in the paper to demonstrate the applications of the proposed method to practical RC panel walls. Comparisons between present results and previous work have shown that the present higher order finite element formulation, including the material models and solution procedures, are more efficient than lower order elements by others, especially for the postfailure behaviours of RC plates.

Key words higher order plate element, smeared rotating crack model, geometric nonlinearity

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