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一类非圆空腔的冲击响应问题

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提要 本文用 Савин. Г. Н 扰动形式的映射函数对非圆空腔在冲击载荷作用下的动态响应问题进行了摄动求解，给出了“0”级及“1”级摄动解的具体算式及阶跃脉冲载荷作用下椭圆腔的数值算例.

关键词 非圆空腔，Савин 映射函数，冲击载荷，摄动解，椭圆腔数例

腔体在冲击载荷作用下的响应问题是弹性动力学中一个比较经典的问题. 这一问题曾为许多作者所研究，例如，A. Kromm[1], H. L. Selberg[2] Y. L. Luke[3], C. J. Tranter[4]. 松本[5]等. 然而，所有这些研究涉及的都是圆腔问题，本文则来讨论非圆腔问题. 文中用摄动方法对它进行了求解，并把其结果具体地应用到阶跃脉冲载荷作用下的椭圆腔体上，获得了计算腔体周边环向应力的十分简洁的解检算式.

一、基本关系式

在处理非圆空腔的二维弹性动力学问题时为适应复杂的边界形式，我们通过映射函数 $z=\omega(\zeta)$. 把物理平面(即 z 平面)上非圆腔体的外域映射到映射平面(即 ζ 平面)上单位圆的外域. 同时，映射函数 $\omega(\zeta)\equiv\omega(\xi e^{i\eta})$ ($\xi=|\zeta|$, $\eta=\arg\zeta$) 又可视为物理平面上的直角坐标 x, y 到某种正交曲线坐标 ξ, η 的坐标变换. 因此，把 ξ, η 视为 ζ 平面上的极坐标与视为 z 平面上的正交曲线坐标是完全等同的. 于是用复变函数在映射平面 ζ 上描述二维弹性动力学问题，其基本关系可写为[6]

波动方程

$$\left.\begin{aligned}\nabla^2\phi&=\frac{1}{c_p^2}\frac{\partial^2\phi}{\partial t^2}\\ \nabla^2\psi&=\frac{1}{c_s^2}\frac{\partial^2\psi}{\partial t^2}\end{aligned}\right\} \tag{1.1}$$

位移场

$$u_\xi+iu_\eta=2\frac{\zeta}{|\zeta|}\cdot\frac{1}{|\omega'|}\frac{\partial}{\partial\zeta}(\phi-i\psi) \tag{1.2}$$

应力场

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$$\left.\begin{aligned}&\sigma_\xi+\sigma_\eta=2(\lambda+\mu)\nabla^2\phi\\&\sigma_\eta-\sigma_\xi+2i\sigma_{\xi\eta}=-8\mu\frac{\zeta}{|\zeta|}\cdot\frac{1}{\bar\omega'}\frac{\partial}{\partial\zeta}\left(\frac{1}{\omega'}\frac{\partial}{\partial\zeta}(\phi+i\psi)\right)\\&\sigma_\xi-i\sigma_{\xi\eta}=(\lambda+\mu)\nabla^2\phi+4\mu\frac{\zeta}{\bar\zeta}\frac{1}{\bar\omega'}\frac{\partial}{\partial\zeta}\left(\frac{1}{\omega'}\frac{\partial}{\partial\zeta}(\phi+i\psi)\right)\end{aligned}\right\}\tag{1.3}$$

边界条件

$$\left.\begin{aligned}&(\lambda+\mu)\nabla^2\phi+4\mu\frac{\zeta}{\bar\zeta}\cdot\frac{1}{\bar\omega'}\frac{\partial}{\partial\zeta}\left(\frac{1}{\omega'}\frac{\partial}{\partial\zeta}(\phi+i\psi)\right)=f_\xi-if_\eta\\&(\lambda+\mu)\nabla^2\phi+4\mu\frac{\bar\zeta}{\zeta}\frac{1}{\omega'}\frac{\partial}{\partial\bar\zeta}\left(\frac{1}{\bar\omega'}\frac{\partial}{\partial\bar\zeta}(\phi-i\psi)\right)=f_\xi+if_\eta\end{aligned}\right\}\tag{1.4}$$

辐射条件

$$\left.\begin{aligned}&\frac{\partial\phi}{\partial r}+\frac{1}{c_p}\frac{\partial\phi}{\partial t}=o(r^{-\frac12});\ \phi=O(r^{-\frac12})\\&\frac{\partial\psi}{\partial r}+\frac{1}{c_s}\frac{\partial\psi}{\partial t}=o(r^{-\frac12});\ \psi=O(r^{-\frac12})\end{aligned}\right\}\tag{1.5}$$

式中，λ,μ 为腔体介质的 Lame 常数；c_p,c_s 为腔体介质的纵波波速与横波波速；t 为时间；ϕ,ψ 为波函数；u_ξ,u_η 为位移；$\sigma_\xi,\sigma_\eta,\sigma_{\xi\eta}$ 为应力；f_ξ,f_η 为外应力；$'$ 表示对其宗量的导数；$r=\sqrt{x^2+y^2}$；$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}=\frac{1}{\omega'\bar\omega'}\frac{\partial^2}{\partial\zeta\partial\bar\zeta}$ 为二维 Laplale 算子；对于映射函数 $\omega(\zeta)$，我们取，

$$\omega=R\Omega(\zeta)\equiv R(\zeta+\varepsilon\zeta^{-m})\tag{1.6}$$

它是一种按小参数 $0\leqslant\varepsilon<1$ 给出的边界扰动形式的映射函数，其中 R 为腔体 R 度参数；m 为常数。这种形式的映射函数为 Савин. Г. Н. 在处理弹性静力问题时首先使用[7]。

二、象域内的表述

把 (1.1)—(1.5) 式对无量纲时间 $\tau=c_pt/R$ 作 Laplace 变换，可得，

$$\left.\begin{aligned}&\nabla^2\Phi=\alpha^2\Phi\\&\nabla^2\Psi=\beta^2\Psi\end{aligned}\right\}\tag{2.1}$$

$$U_\xi+iU_\eta=2\frac{\zeta}{|\zeta|}\cdot\frac{1}{|\Omega'|}\frac{\partial}{\partial\bar\zeta}(\Phi-i\Psi)\tag{2.2}$$

$$\left.\begin{aligned}&\Sigma_\xi+\Sigma_\eta=2(\kappa^2-1)\alpha^2\Phi\\&\Sigma_\eta-\Sigma_\xi+2i\Sigma_{\xi\eta}=-8\frac{\zeta}{\bar\zeta}\cdot\frac{1}{\bar\Omega'}\frac{\partial}{\partial\zeta}\left(\frac{1}{\Omega'}\frac{\partial}{\partial\zeta}(\Phi+i\Psi)\right)\\&\Sigma_\xi-i\Sigma_{\xi\eta}=(\kappa^2-1)\alpha^2\Phi+4\frac{\zeta}{\bar\zeta}\cdot\frac{1}{\bar\Omega'}\frac{\partial}{\partial\zeta}\left(\frac{1}{\Omega'}\frac{\partial}{\partial\zeta}(\Phi+i\Psi)\right)\end{aligned}\right\}\tag{2.3}$$

$$\left.\begin{aligned}&(\kappa^2-1)\alpha^2\Phi+4\frac{\zeta}{\bar\zeta}\cdot\frac{1}{\bar\Omega'}\frac{\partial}{\partial\zeta}\left(\frac{1}{\Omega'}\frac{\partial}{\partial\zeta}(\Phi+i\Psi)\right)=F_\xi-iF_\eta\\&(\kappa^2-1)\alpha^2\Phi+4\frac{\bar\zeta}{\zeta}\cdot\frac{1}{\Omega'}\frac{\partial}{\partial\bar\zeta}\left(\frac{1}{\bar\Omega'}\frac{\partial}{\partial\bar\zeta}(\Phi-i\Psi)\right)=F_\xi+iF_\eta\end{aligned}\right\}\tag{2.4}$$

$$\left.\begin{aligned}\frac{\partial\Phi}{\partial \mathrm{R}}+\frac{\partial\Phi}{\partial\tau}=o(\mathrm{R}^{-\frac{1}{2}});\ \ \Phi=O(\mathrm{R}^{-\frac{1}{2}})\\ \frac{\partial\Psi}{\partial \mathrm{R}}+\frac{\partial\Psi}{\partial\tau}=o(\mathrm{R}^{-\frac{1}{2}});\ \ \Psi=O(\mathrm{R}^{-\frac{1}{2}})\end{aligned}\right\} \tag{2.5}$$

式中，$\kappa=c_p/c_s$; $\beta=\kappa\alpha$; $\mathrm{R}=r/R$; $\alpha\equiv s$ 为无量纲 Laplace 变换参数; $|\nabla^2=R^2\nabla^2$ 为无量纲 Laplace 算子;

$$(\Phi,\Psi)=\frac{1}{R^2}\int_0^\infty(\phi,\psi)e^{-s\tau}\mathrm{d}\tau;\ \ (U_\xi,U_\eta)=\frac{1}{R}\int_0^\infty(u_\xi,u_\eta)e^{-\zeta\tau}\mathrm{d}\tau;$$

$$(\Sigma_\xi,\Sigma_\eta,\Sigma_{\xi\eta})=\frac{1}{\mu}\int_0^\infty(\sigma_\xi,\sigma_\eta,\sigma_{\xi\eta})e^{-\zeta\tau}\mathrm{d}\tau;\ \ (F_\xi,F_\eta)=\frac{1}{\mu}\int_0^\infty(f_\xi,f_\eta)e^{-\zeta\tau}\mathrm{d}\tau.$$

方程 (2.1) 的一般解是函数 $I_n(\cdot)\left(\frac{\Omega}{|\Omega|}\right)^n$, $K_n(\cdot)\left(\frac{\Omega}{|\Omega|}\right)^n$ 的线性组合，但从修正 Bessel 函数 $I_n(\cdot)$, $K_n(\cdot)$ $(n=0,\pm1,\pm2\cdots\cdots)$ 对其宗量很大时的渐近展开式易知，只有 $K_n(\cdot)$ 能够满足辐射条件 (2.5)，于是我们有:

$$\left.\begin{aligned}\Phi=\sum_{n=-\infty}^{+\infty}A_nK_n(\alpha|\Omega|)\left(\frac{\Omega}{|\Omega|}\right)^n\\ \Psi=\sum_{n=-\infty}^{+\infty}B_nK_n(\beta|\Omega|)\left(\frac{\Omega}{|\Omega|}\right)^n\end{aligned}\right\} \tag{2.6}$$

式中，A_n,B_n $(n=0,\pm1,\pm2\cdots)$ 为待定常数，把 (2.6) 式代入 (2.4) 式，并运用如下关系

$$\left.\begin{aligned}\frac{\partial}{\partial\zeta}\left(K_n(\alpha|\Omega|)\left(\frac{\Omega}{|\Omega|}\right)^n\right)=-\frac{\alpha}{2}\,\Omega'K_{n-1}(\alpha|\Omega|)\left(\frac{\Omega}{|\Omega|}\right)^{n-1}\\ \frac{\partial}{\partial\bar\zeta}\left(K_n(\alpha|\Omega|)\left(\frac{\Omega}{|\Omega|}\right)^n\right)=-\frac{\alpha}{2}\,\bar\Omega'K_{n+1}(\alpha|\Omega|)\left(\frac{\Omega}{|\Omega|}\right)^{n+1}\end{aligned}\right\} \tag{2.7}$$

则有，

$$\left.\begin{aligned}&\sum_{n=-\infty}^{+\infty}\{\alpha^2[(\kappa^2-1)H(n,\alpha,\Omega)+\Lambda H((n-2),\alpha,\Omega)]A_n\\ &\qquad+i\beta^2\Lambda H((n-2),\beta,\Omega)B_n\}=F_\xi-iF_\eta\\ &\sum_{n=-\infty}^{+\infty}\{\alpha^2[(\kappa^2-1)H(n,\alpha,\Omega)+\bar\Lambda H((n+2),\alpha,\Omega)]A_n\\ &\qquad-i\beta^2\bar\Lambda H((n+2),\beta,\Omega)B_n\}=F_\xi+iF_\eta\end{aligned}\right\} \tag{2.8}$$

式中，$\Lambda=\zeta\Omega'/\bar\zeta\bar\Omega'$; $H(n,\alpha,\Omega)\equiv K_n(\alpha|\Omega|)\left(\frac{\Omega}{|\Omega|}\right)^n$ (2.9)

三、摄 动 求 解

令，

$$\left.\begin{aligned}A_n=\overset{(0)}{A}_n+\varepsilon\overset{(1)}{A}_n+\cdots,\ \ B_n=\overset{(0)}{B}_n+\varepsilon\overset{(1)}{B}_n+\cdots\\ F_\xi=\overset{(0)}{F}_\xi+\varepsilon\overset{(1)}{F}_\xi+\cdots,\ \ F_\eta=\overset{(0)}{F}_\eta+\varepsilon\overset{(1)}{F}_\eta+\cdots\end{aligned}\right\} \tag{3.1}$$

把 (1.6),(3.1) 式代入 (2.8) 式，对 ε 展开并比较系数，则有，

$$\left.\begin{aligned}&\sum_{n=-\infty}^{+\infty}\left\{\alpha^2\left[(\kappa^2-1)H(n,\alpha,\zeta)+\frac{\zeta}{\bar{\zeta}}H((n-2),\alpha,\zeta)\right]\overset{(0)}{A}_n\right.\\&\qquad\left.+i\beta^2\frac{\zeta}{\bar{\zeta}}H((n-2),\beta,\zeta)\overset{(0)}{B}_n\right\}=\overset{(0)}{F}_\xi-i\overset{(0)}{F}_\eta+\overset{(0)}{F}_-\\&\sum_{n=-\infty}^{+\infty}\left\{\alpha^2\left[(\kappa^2-1)H(n,\alpha,\zeta)+\frac{\bar{\zeta}}{\zeta}H((n+2),\alpha,\zeta)\right]\overset{(0)}{A}_n\right.\\&\qquad\left.-i\beta^2\frac{\bar{\zeta}}{\zeta}H((n+2),\beta,\zeta)\overset{(0)}{B}_n\right\}=\overset{(0)}{F}_\xi+i\overset{(0)}{F}_\eta+\overset{(0)}{F}_+\end{aligned}\right\}\tag{3.2}$$

$$\left.\begin{aligned}&\sum_{n=-\infty}^{+\infty}\left\{\alpha^2\left[(\kappa^2-1)H(n,\alpha,\zeta)+\frac{\zeta}{\bar{\zeta}}H((n-2),\alpha,\zeta)\right]\overset{(1)}{A}_n\right.\\&\qquad\left.+i\beta^2\frac{\zeta}{\bar{\zeta}}H((n-2),\beta,\zeta)\overset{(1)}{B}_n\right\}=\overset{(1)}{F}_\xi-i\overset{(1)}{F}_\eta+\overset{(1)}{F}_-\\&\sum_{n=-\infty}^{+\infty}\left\{\alpha^2\left[(\kappa^2-1)H(n,\alpha,\zeta)+\frac{\bar{\zeta}}{\zeta}H((n+2),\alpha,\zeta)\right]\overset{(1)}{A}_n\right.\\&\qquad\left.-i\beta^2\frac{\bar{\zeta}}{\zeta}H((n+2),\beta,\zeta)\overset{(1)}{B}_n\right\}=\overset{(1)}{F}_\xi+i\overset{(1)}{F}_\eta+\overset{(1)}{F}_+\end{aligned}\right\}\tag{3.3}$$

式中，　$\overset{0}{F}_-=0,\quad \overset{0}{F}_+=0$　　(3.4)

$$\left.\begin{aligned}\overset{(1)}{F}_-=\sum_{n=-\infty}^{+\infty}&\left\{\left[\frac{1}{2}(\kappa^2-1)\alpha H((n-1),\alpha,\zeta)\zeta^{-m}\right.\right.\\&-\frac{\zeta}{\bar{\zeta}}(m(\bar{\zeta}^{-m-1}-\zeta^{-m-1})H((n-2),\alpha,\zeta)\\&\left.-\frac{\alpha}{2}H((n-3),\alpha,\zeta)\zeta^{-m})\right]\alpha^2\overset{(0)}{A}_n\\&-\frac{\zeta}{\bar{\zeta}}\left[m(\bar{\zeta}^{-m-1}-\zeta^{-m-1})H((n-2),\beta,\zeta)\right.\\&\left.\left.-\frac{\beta}{2}H((n-3),\beta,\zeta)\zeta^{-m}\right]\beta\overset{(0)}{B}_n\right\}\end{aligned}\right\}\tag{3.5}$$

$$\begin{aligned}\overset{(1)}{F}_+=\sum_{n=-\infty}^{+\infty}&\left\{\left[\frac{1}{2}(\kappa^2-1)\alpha H((n-1),\alpha,\zeta)\zeta^{-m}\right.\right.\\&-\frac{\bar{\zeta}}{\zeta}(m(\zeta^{-m-1}-\zeta^{-m-1})H((n+2),\alpha,\zeta)\\&\left.-\frac{\alpha}{2}H((n+1),\alpha,\zeta)\zeta^{-m})\right]\alpha^2\overset{(0)}{A}_n\\&+i\frac{\bar{\zeta}}{\zeta}\left[m(\zeta^{-m-1}-\zeta^{-m-1})H((n+2),\beta,\zeta)\right.\\&\left.\left.-\frac{\beta}{2}H((n+1),\beta,\zeta)\zeta^{-m}\right]\beta\overset{(0)}{B}_n\right\}\end{aligned}$$

………………

把外载荷作 Fourier 展开，有

$$\left.\begin{aligned}\overset{(0)}{F}_\xi = \sum_{n=-\infty}^{+\infty}\binom{0}{n}_\xi\sigma^n, \quad \overset{(0)}{F}_\eta = \sum_{n=-\infty}^{+\infty}\binom{0}{n}_\eta\sigma^n\\ \overset{(1)}{F}_\xi = \sum_{n=-\infty}^{+\infty}\binom{1}{n}_\xi\sigma^n, \quad \overset{(1)}{F}_\eta = \sum_{n=-\infty}^{+\infty}\binom{1}{n}_\eta\sigma^n\end{aligned}\right\} \tag{3.6}$$

....

式中，$\sigma = e^{i\theta}$; $\theta \equiv \eta$;

$$\left.\begin{aligned}\binom{0}{n}_\xi = \frac{1}{2\pi i}\oint \overset{(0)}{F}_\xi\sigma^{-n}d\sigma, \quad \binom{0}{n}_\eta = \frac{1}{2\pi i}\oint \overset{(0)}{F}_\eta\sigma^{-n}\mathrm{d}\sigma\\ \binom{1}{n}_\xi = \frac{1}{2\pi i}\oint \overset{(1)}{F}_\xi\sigma^{-n}\mathrm{d}\sigma, \quad \binom{1}{n}_\eta = \frac{1}{2\pi i}\oint \overset{(1)}{F}_\eta\sigma^{-n}\mathrm{d}\sigma\end{aligned}\right\} \tag{3.7}$$

....

把 (3.4),(3.6) 式代入 (3.2) 式，并注意到在边界上 $\zeta \equiv \sigma$ 及 $\sigma^n(n = 0, \pm1, \pm2\cdots)$ 的正交性，则有，

$$\left.\begin{aligned}\alpha^2[(\kappa^2-1)K_n(\alpha)+K_{n-2}(\alpha)]\overset{(0)}{A}_n + i\beta^2K_{n-2}(\beta)\overset{(0)}{B}_n = \binom{0}{n}_\xi - i\binom{0}{n}_\eta\\ \alpha^2[(\kappa^2-1)K_n(\alpha)+K_{n+2}(\alpha)]\overset{(0)}{A}_n - i\beta^2K_{n+2}(\beta)\overset{(0)}{B}_n = \binom{0}{n}_\xi + i\binom{0}{n}_\eta\end{aligned}\right\} \tag{3.8}$$

$$(n = 0, \pm1, \pm2\cdots\cdots)$$

解之，得，

$$\overset{(0)}{A}_n = \frac{\overset{(0)}{\mathscr{D}}_{An}}{\mathscr{D}}, \quad \overset{(0)}{B}_n = \frac{\overset{(0)}{\mathscr{D}}_{Bn}}{\mathscr{D}} \tag{3.9}$$

式中，

$$\overset{(0)}{\mathscr{D}}_{An} = i\beta^2\left[\left(\binom{0}{n}_\xi - i\binom{0}{n}_\eta\right)K_{n+2}(\beta) + \left(\binom{0}{n}_\xi + i\binom{0}{n}_\eta\right)K_{n-2}(\beta)\right]$$

$$\left.\begin{aligned}\overset{(0)}{\mathscr{D}}_{Bn} = \alpha^2\Bigg[&\left(\binom{0}{n}_\xi - i\binom{0}{n}_\eta\right)((\kappa^2-1)K_n(\alpha)+K_{n+2}(\alpha))\\ &-\left(\binom{0}{n}_\xi - i\binom{0}{n}_\eta\right)((\kappa^2-1)\kappa_n(\alpha)+K_{n-2}(\alpha))\Bigg]\\ \mathscr{D} = i\alpha^2\beta^2[&((\kappa^2-1)K_n(\alpha)+K_{n+2}(\alpha))K_{n-2}(\beta)\\ &-((\kappa^2-1)K_n(\alpha)+K_{n-2}(\alpha))K_{n+2}(\beta)]\end{aligned}\right\} \tag{3.10}$$

同样，从 (3.3),(3.5),(3.6),(3.9) 式，我们可求得，

$$\overset{(1)}{A}_n = \frac{\overset{(1)}{\mathscr{D}}_{An}}{\mathscr{D}}, \quad \overset{(1)}{B}_n = \frac{\overset{(1)}{\mathscr{D}}_{Bn}}{\mathscr{D}} \tag{3.11}$$

式中，

$$\left.\begin{aligned}\overset{(1)}{\mathscr{D}}_{An}&=i\beta^2\Big[\Big(\binom{1}{n}_\xi-i\binom{1}{n}_\eta+\binom{1}{n}_-\Big)K_{n+2}(\beta)+\Big(\binom{1}{n}_\xi\\&\quad+i\binom{1}{n}_\eta+\binom{1}{n}_+\Big)K_{n-2}(\beta)\Big]\\\overset{(1)}{\mathscr{D}}_{Bn}&=\alpha^2\Big[\Big(\binom{1}{n}_\xi-i\binom{1}{n}_\eta+\binom{1}{n}_-\Big)((\kappa^2-1)K_n(\alpha)\\&\quad+K_{n+2}(\alpha))-\Big(\binom{1}{n}_\xi+i\binom{1}{n}_\eta+\binom{1}{n}_+\Big)\\&\quad\times((\kappa^2-1)K_n(\alpha)+K_{n-2}(\alpha))\Big]\end{aligned}\right\}\tag{3.12}$$

$$\binom{1}{n}_+=\frac{1}{2\pi i}\oint\overset{(1)}{F}_+\sigma^{-n}d\sigma,\ \binom{1}{n}_-=\frac{1}{2\pi i}\oint\overset{(1)}{F}_-\sigma^{-n}\mathrm{d}\sigma\tag{3.13}$$

把 (3.5) 式代入 (3.13) 式，可具体求得，

$$\binom{1}{-m}_-=\frac{1}{2}\alpha^2(\kappa^2-1)K_{m-1}(\alpha)\overset{(0)}{A}_{-m}$$

$$\left.\begin{aligned}\binom{1}{m+3}_-&=-m\alpha^2(K_{m+1}(\alpha)\overset{(0)}{A}_{m+3}+i\kappa^2K_{m+1}(\beta)\overset{(0)}{B}_{m+3})\\\binom{1}{-m+1}_-&=m\alpha^2(K_{m-1}(\alpha)\overset{(0)}{A}_{-m+1}+i\kappa^2K_{-m-1}(\beta)\overset{(0)}{B}_{-m+1})\\\binom{1}{-m+2}_-&=\frac{1}{2}\alpha^2(K_{-m-1}(\alpha)\overset{(0)}{A}_{-m+2}+i\kappa^3K_{-m-1}(\beta)\overset{(0)}{B}_{-m+2})\\\binom{1}{-m}&=\frac{1}{2}\alpha^2(\kappa^2-1)K_{-m-1}(\alpha)\overset{(0)}{A}_{-m}\\\binom{1}{-m-3}_+&=-m\alpha^2(K_{-m-1}(\alpha)\overset{(0)}{A}_{-m+3}-i\kappa^2K_{-m-1}(\beta)\overset{(0)}{B}_{-m-3})\\\binom{1}{m-1}_+&=m\alpha^2(K_{m+1}(\alpha)\overset{(0)}{A}_{m-1}-i\kappa^2K_{m+1}(\beta)\overset{(0)}{B}_{m-1})\\\binom{1}{-m-2}_+&=\frac{1}{2}\alpha^2(K_{-m-1}(\alpha)\overset{(0)}{A}_{-m-2}-i\kappa^3K_{-m-1}(\beta)\overset{(0)}{B}_{-m-2})\end{aligned}\right\}\tag{3.14}$$

其余 $\binom{1}{n}_-=\binom{1}{n}_+=0$

把 (3.9)，(3.11) 式代入 (2.6) 式，Φ,Ψ 的各级摄动解可近似地写为，

"0" 级

$$\left.\begin{aligned}\Phi&=\sum_{n=-\infty}^{+\infty}\overset{(0)}{A}_nK_n(\alpha|\Omega|)\left(\frac{\Omega}{|\Omega|}\right)^n\\\Psi&=\sum_{n=-\infty}^{+\infty}\overset{(0)}{B}_nK_n(\beta|\Omega|)\left(\frac{\Omega}{|\Omega|}\right)^n\end{aligned}\right\}\tag{3.15}$$

"1" 级

$$\left.\begin{aligned}\Phi &= \sum_{n=-\infty}^{+\infty}(\overset{(0)}{A}_n+\varepsilon\overset{(1)}{A}_n)K_n(\alpha|\Omega|)\left(\frac{\Omega}{|\Omega|}\right)^n\\ \Psi &= \sum_{n=-\infty}^{+\infty}(\overset{(0)}{B}_n+\varepsilon\overset{(1)}{B}_n)K_n(\beta|\Omega|)\left(\frac{\Omega}{|\Omega|}\right)^n\end{aligned}\right\} \tag{3.16}$$

四、解的反演

从 (2.3) 式,我们得腔边应力为

$$\Sigma_\eta = 2(\kappa^2-1)\alpha^2\Phi-\Sigma_\xi \tag{4.1}$$

用 Laguere 多项式, $\mathrm{Ln}(\tau)=\sum_{j=0}^{n}\theta_j^n\tau^n$ 逼近时,(4.1) 式的反演解为,

$$\frac{\sigma_\eta}{\mu} = 2(\kappa^2-1)e^{-\chi\tau}\sum_{n=0}^{+\infty}\phi_nL_n(\tau)-\frac{f_\xi}{\mu} \tag{4.2}$$

式中,θ_j^n 为 n 次 Laguere 多项式的第 j 项系数; χ 为常数;

$$\phi_n = \frac{1}{(n!)^2}\sum_{j=0}^{n}(-1)^j\theta_j^n\left.\frac{d^j(\alpha^2\Phi)}{d\alpha^j}\right|_{\alpha=1-\chi} \tag{4.3}$$

作为一个具体算例,下面我们来考虑受阶跃脉冲作用的椭圆形空腔,此时

$$f_\xi = -p_0H(t),\ f_\eta=0,\ m=1 \tag{4.4}$$

式中,p_0 为压力幅; $H(\cdot)$ 为 Heaviside 函数。于是从 (3.7) 式,我们有,

$$\binom{0}{0}_\xi = -p_0/\mu\alpha = -P_0/\alpha,\ \text{其余}\ \binom{k}{n}_\xi = \binom{k}{n}_\eta = 0 \tag{4.5}$$

进而从 (3.9),(3.10) 式,求得,

$$\overset{(0)}{A}_0 = -\frac{P_0}{\alpha^3[(\kappa^2-1)K_0(\alpha)+K_2(\alpha)]},\ \text{其余}\ \overset{(0)}{A}_n = 0 \tag{4.6}$$

又从 (3.14) 式,我们有,

$$\left.\begin{aligned}&\binom{1}{0}_- = \alpha^2K_0(\alpha)\overset{(0)}{A}_0,\ \binom{1}{0}_+ = \alpha^2K_2(\alpha)\overset{(0)}{A}_0\\ &\text{其余}\ \binom{1}{n}_{-1} = \binom{1}{n}_+ = 0\end{aligned}\right\} \tag{4.7}$$

进而从 (3.11),(3.12) 式,求得,

$$\overset{(1)}{A}_0 = -\frac{[K_0(\alpha)+K_2(\alpha)]P_0}{\alpha^3[(\kappa^2-1)K_0(\alpha)+K_2(\alpha)]^2},\ \text{其余}\ \overset{(1)}{A}_n = 0 \tag{4.8}$$

于是从 (3.16),(4.2),(4.3),(4.6),(4.8) 式,我们得到椭圆腔边环向应力分布的一次摄动解为,

$$\frac{\sigma_\theta}{p_0} = 1-\{a_1+\varepsilon(a_2-a_3\cos 2\theta)-[b_1+\varepsilon(b_2-b_3\cos 2\theta)]\tau\}\times\exp(-0.49\tau) \tag{4.9}$$

式中,$a_1, a_2, a_3, b_1, b_2, b_3$ 是与腔体介质的 Possion 比 ν 有关的常数,其值如表 1 所示。

$\frac{\sigma_\theta}{p_0}$ 随时间 τ，椭圆度 ε 及边界点 θ 的变化情况分别如图 1,2,3 所示。

表 1　$a_1, a_2, a_3, b_1, b_2, b_3$ 之值

ν	a_1	a_2	a_3	b_1	b_2	b_3
0.24	1.48254	1.20878	1.09255	0.72056	0.51787	0.74700
0.27	1.59490	1.22972	1.18658	0.75482	0.48682	0.80561
0.30	1.72285	1.23465	1.29751	0.78684	0.43268	0.87303
0.33	1.86803	1.21098	1.42989	0.81134	0.34221	0.95069
0.36	2.03003	1.13637	1.58961	0.81696	0.19347	1.03949
0.39	2.20163	0.97047	1.78364	0.77784	−0.05011	1.13796
0.42	2.35328	0.64107	2.01749	0.63017	−0.44606	1.23607

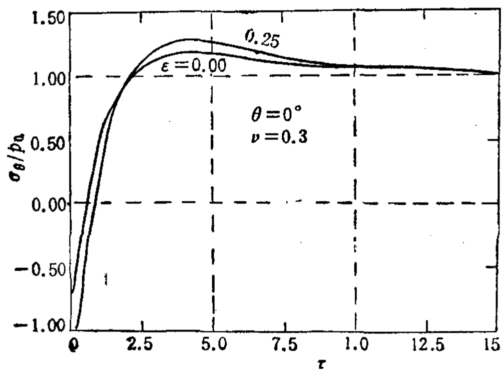

图 1　腔边应力 $\frac{\sigma_\theta}{p_0}$ 随时间 τ 的变化

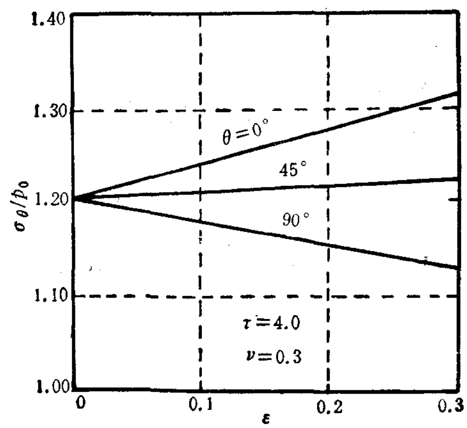

图 2　腔边应力 $\frac{\sigma_\theta}{p_0}$ 随腔的椭圆度 ε 的变化

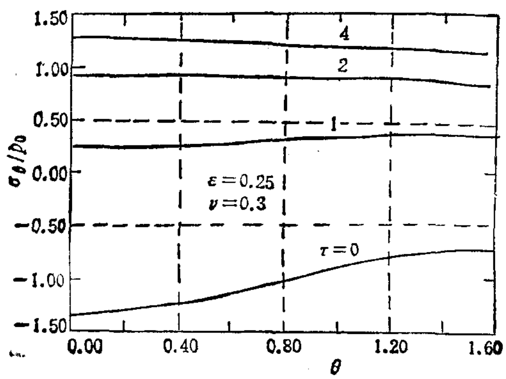

图 3　腔边应力 $\frac{\rho_\theta}{p_0}$ 随腔边位置 θ 的变化

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IMPULSIVE RESPONSE PROBLEMS FOR A CLASS OF NON-CIRCULAR CAVITY

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Abstract In this paper, by using the Savin's mapping function in perturbed form, the dynamic response problems of non-circular cavity under impulsive load are solved. Formulas for calculating "0"-order and "1"-order perturbational solutions and numerical examples for elliptical cavity under step impulsive load are given.

Key words non-circular cavity, Savin's mapping function, impulsive load, perturbational solution, elliptical cavity