区间参数振动系统的动力优化

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摘要 对具有区间参数的多自由度振动系统的不确定性优化问题，提出一种新的区间优化方法. 利用泰勒展开和函数的区间扩张，将区间优化问题转化为近似的确定性优化问题. 该方法应用于多自由度线性扭振系统，并把区间设计变量的中值和不确定性半径取作优化参数. 算例表明该方法是有效的.

关键词 区间参数，区间优化，近似的确定性优化，线性扭振系统

引 言

确定性优化方法已经很好地用于具有确定性参数和载荷的结构 [1~3]. 但是, 在许多工程问题中, 结构的参数和载荷具有某种程度的误差或不确定性, 若不将它们定量化或模型化加以考虑，就不能做出合理的分析和设计. 尽管概率统计方法在描述结构力学中的不确定性现象方面获得了很大成功，但是，在没有足够的实验数据证明由联合概率密度 (联合概率密度函数) 所给出的假设正确的情况下, 这种方法并不能给出满足某种规定的可靠结果. 因此, 概率统计方法不应该是研究不确定性问题的唯一方法.

自 20 世纪 60 年代以来，出现了一种新的所谓的区间分析方法. Moore[4] 和他的合作者 Alefeld 和 Herzberger[5] 已经作了许多开创性的工作. Hansen 在他的书中 [6] 基于区间分析讨论了全局优化问题. 但是由于这些区间算法的复杂性，很难用它们来解决实际的工程问题. 最近，区间分析方法被用于具有区间参数的不确定性结构的静态位移和特征值的分析 [7,8]. 但是, 很少有文献涉及到具有区间参数结构的结构优化问题. 因此，很有必要提出一种有效的方法，用于解决具有区间参数结构的结构优化问题. 本文基于区间分析提出了一种区间优化方法. 有关区间分析的基本算法请参考文献 [4~6].

1 区间优化模型

如果结构参数是区间变量，那么，目标函数和约束条件都是区间的. 因此，区间优化问题可以描述为

$$\left.\begin{aligned} &\min\ f(\boldsymbol{X}^I)=F(X_1^I,X_2^I,\cdots,X_n^I)\\ &\text{s.t.}\left\{\begin{aligned} &p_i(\boldsymbol{X}^I)\le 0\,,\quad i=1,2,\cdots,m\\ &q_j(\boldsymbol{X}^I)=0\,,\quad j=1,2,\cdots,l\end{aligned}\right.\end{aligned}\right\}\tag{1}$$

其中 $\boldsymbol{X}^I=(X_1^I,X_2^I,\cdots,X_n^I)$ 是结构的区间参数向量.

直接求解区间优化问题是非常困难的，本文将区间优化问题转化为近似的确定性优化问题. 为此, 将 $f(\boldsymbol{X}^I)$ 在区间向量 $\boldsymbol{X}^I$ 的中点附近泰勒展开，并忽略高阶项，得到

$$\begin{aligned} f(\boldsymbol{X}^I)=\,&f(\boldsymbol{X}^C)+\sum_{i=1}^n\frac{\partial f(\boldsymbol{X}^C)}{\partial x_i}(X_i^I-X_i^C)=\\ &f(\boldsymbol{X}^C)+\sum_{i=1}^n\left|\frac{\partial f(\boldsymbol{X}^C)}{\partial x_i}\Delta X_i\right|e_\Delta=\\ &\left[f(\boldsymbol{X}^C)-\sum_{i=1}^n\left|\frac{\partial f(\boldsymbol{X}^C)}{\partial x_i}\Delta X_i\right|,\right.\\ &\left.f(\boldsymbol{X}^C)+\sum_{i=1}^n\left|\frac{\partial f(\boldsymbol{X}^C)}{\partial x_i}\Delta X_i\right|\right]\end{aligned}\tag{2}$$

约束条件可以同样处理.

因此，我们把区间优化问题近似为确定性优化

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问题

$$
\left.\begin{array}{l}
\min\left[f(\boldsymbol{X}^C)+\sum\limits_{i=1}^{n}\left|\dfrac{\partial f(\boldsymbol{X}^C)}{\partial x_i}\right|\Delta X_i\right] \\
\text{s.t.}\left\{\begin{array}{l}
p_i(\boldsymbol{X}^C)+\sum\limits_{i=1}^{n}\left|\dfrac{\partial p_i(\boldsymbol{X}^C)}{\partial x_k}\right|\Delta X_k\le 0 \\
\qquad\qquad i=1,2,\cdots,m \\
q_j(\boldsymbol{X}^C)=0\,,\qquad j=1,2,\cdots,l \\
\sum\limits_{k=1}^{n}\left|\dfrac{\partial q_j(\boldsymbol{X}^C)}{\partial X_k}\right|\Delta X_k=0 \\
\qquad\qquad j=1,2,\cdots,l \\
-\Delta X_k\le 0\,,\qquad k=1,2,\cdots,n \\
\delta-\delta_0\le 0
\end{array}\right.
\end{array}\right\}\qquad(3)
$$

其中，$\delta=\dfrac{\sum\limits_{i=1}^{n}\left|\dfrac{\partial f(\boldsymbol{X}^C)}{\partial x_i}\right|\Delta X_i}{|f(\boldsymbol{X}^C)|}$ 是相对不确定量，δ_0 是指定的常数.

2 在线性扭振系统中的应用

将前面给出的区间优化方法应用于图 1 所示的 n 自由度线性扭振系统.

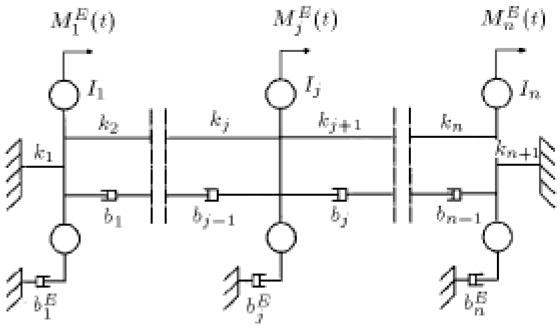

图 1 n 自由度线性扭振系统

Fig.1 Linear torsional vibration system with n degrees of freedom

每个圆盘受谐波激励为 $M_i^E(t)=M_{ic}\cos\omega t+M_{is}\sin\omega t(i=1,2,\cdots,n)$.

区间优化的目的是确定区间设计变量 $\{I_i,K_i,b_i,b_i^E\}$ 的中点值和不确定性半径，使位移幅值之和 $\sum\limits_{i=1}^{n}|q_i(t)|$ 为最小且有区间值相对比较窄. 将所有设计变量写成一个 N 维向量

$$
\boldsymbol{X}=\{x_1,x_2,\cdots,x_N\}^{\mathrm{T}}=
\{I_1,\cdots,I_n,K_1,\cdots,K_{n+1},b_1,\cdots,b_{n-1},b_1^E,\cdots,b_n^E\}^{\mathrm{T}}
$$

其中，I_i 是转动惯量，K_i 是扭转刚度，b_i 和 b_i^E 分别是内、外阻尼系数.

确定性优化问题为

$$
\left.\begin{array}{l}
\min\ \psi(\boldsymbol{X})=\sum\limits_{j=1}^{n}|q_j(\boldsymbol{X})| \\
\text{s.t.}\left\{\begin{array}{l}
g_i(\boldsymbol{X})=x_i^L-x_i\le 0 \\
\qquad\qquad i=1,2,\cdots,N \\
g_{i+N}(\boldsymbol{X})=x_i-x_i^U\le 0 \\
\qquad\qquad i=1,2,\cdots,N \\
g_{i+2N}(\boldsymbol{X})=|q_i(\boldsymbol{X})|-q_i^A\le 0 \\
\qquad\qquad i=1,2,\cdots,n
\end{array}\right.
\end{array}\right\}\qquad(4)
$$

其中 x_i^L 和 x_i^U 分别是设计变量 x_i 的上、下界，$|q_i(\boldsymbol{X})|$ 是 $q_i(\boldsymbol{X})$ 的幅值，q_i^A 是位移允许的最大偏离值.

区间优化问题为

$$
\left.\begin{array}{l}
\min\ \psi(\boldsymbol{X}^I)=\sum\limits_{j=1}^{n}|q_j(\boldsymbol{X}^I)| \\
\text{s.t. } g_i(\boldsymbol{X}^I)\le 0\,,\quad i=1,2,\cdots,2N+n
\end{array}\right\}\qquad(5)
$$

根据方程 (3)，我们可得到近似的确定性优化问题为

$$
\min\left[\sum_{i=1}^{n}|q_i(\boldsymbol{X}^C)|+\sum_{i=1}^{n}\sum_{k=1}^{N}\left|\frac{\partial|q_i(\boldsymbol{X}^C)|}{\partial x_k}\right|\Delta X_k\right]\qquad(6)
$$

$$
\text{s.t.}\left\{\begin{array}{ll}
x_i^L-X_i^C+\Delta X_i\le 0\,, & i=1,2,\cdots,N \\
X_i^C-x_i^U+\Delta X_i\le 0\,, & i=1,2,\cdots,N \\
|q_i(\boldsymbol{X}^C)|+\sum\limits_{k=1}^{N}\left|\dfrac{\partial|q_i(\boldsymbol{X}^C)|}{\partial x_k}\right|\Delta X_k-q_i^A\le 0 & \\
 & i=1,2,\cdots,n \\
-\Delta X_k\le 0\,, & k=1,2,\cdots,N \\
\delta-\delta_0\le 0 &
\end{array}\right.\qquad(7)
$$

其中，$\delta=\sum\limits_{i=1}^{n}\sum\limits_{k=1}^{N}\left|\dfrac{\partial|q_i(\boldsymbol{X}^C)|}{\partial x_k}\right|\Delta X_k\Big/\sum\limits_{i=1}^{n}|q_i(\boldsymbol{X}^C)|$.

图 1 所示的扭转振动系统的运动微分方程为

$$
\boldsymbol{M}(I_i)\ddot{\boldsymbol{q}}(t)+\boldsymbol{B}(b_i,b_i^E)\dot{\boldsymbol{q}}(t)+\boldsymbol{K}(K_i)\boldsymbol{q}(t)=
\boldsymbol{F}_c\cos\omega t+\boldsymbol{F}_s\sin\omega t\qquad(8)
$$

其中，$\boldsymbol{F}_c=\{M_{ic}\}^{\mathrm{T}}, \boldsymbol{F}_s=\{M_{is}\}^{\mathrm{T}}$. 方程 (8) 的稳态解复幅值为

$$\boldsymbol{q}(\boldsymbol{X})=\boldsymbol{G}(\boldsymbol{X})(\boldsymbol{F}_c-\mathrm{i}\boldsymbol{F}_s)=\{q_i(\boldsymbol{X})\} \tag{9}$$

其中

$$\boldsymbol{G}(\boldsymbol{X})=[-\boldsymbol{M}(I_i)\omega^2+\mathrm{i}\omega\boldsymbol{B}(b_i,b_i^E)+\boldsymbol{K}(k_i)]^{-1} \tag{10}$$

是系统的复频响应矩阵.

3 算　例

考虑如图 2 所示的动态减振器.

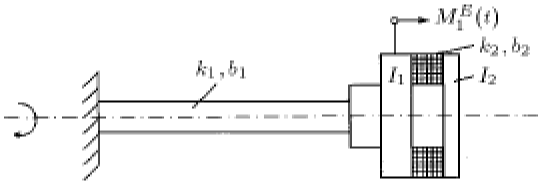

图 2 动态减振器

Fig.2 Dynamic absorber

设圆盘 I_1 受谐波激励

$$M_1^E(t)=M_{1c}\cos\omega t$$

这里有 6 个系统参数 $(N=6)$

$$\boldsymbol{X}=\{I_1,I_2,K_1,K_2,b_1,b_2\}^{\mathrm{T}}$$

优化变量有 12 个：$I_1^C, I_2^C, K_1^C, K_2^C, b_1^C, b_2^C, \Delta I_1, \Delta I_2, \Delta K_1, \Delta K_2, \Delta b_1, \Delta b_2$；其近似的确定性优化问题 (6) 中只对圆盘 I_1 的幅值 $|q_1(t)|$ 进行优化.

谐波激励的幅值和激励频率分别为 $M_{1c}=100\,\mathrm{kg{\cdot}m}$，$\omega=20\,\mathrm{rad/s}$，优化参数 I_1^C，I_2^C，K_1^C，K_2^C，b_1^C，b_2^C，ΔI_1，ΔI_2，ΔK_1，ΔK_2，Δb_1，Δb_2 的变化范围分别为 50~500 kg·m^2，5~50 kg·m^2，5 000~20 000 kg·m/rad，100~800 kg·m/rad，10~100，5~50，1.0~5.0 kg·m^2，0.2~1.0 kg·m^2，10.0~50.0 kg·m/rad，1.0~5.0 kg·m/rad，0.1~1.0，0.1~1.0. $|q_1(\boldsymbol{X})|$ 的最大允许偏离值取为 $q_1^A=0.017$ rad. 利用 SQP(序列二次规划) 算法求解近似的确定性非线性规划问题. 为了比较，确定性优化和区间优化的结果列于表 1.

表 1　确定性优化与区间优化结果比较 $(\delta_0=0.01)$

Table 1　Comparison of results of deterministic and interval optimization

X_i^c	Initial values	Derterministic optimization values	Interval optimization Mid-point values
I_1^C	200.000 000	500.000 000 0	499.000 000 0
I_2^C	10.000 000 0	10.249 096 6	10.842 631 9
K_1^C	5 000.000 000 0	5 000.000 000 0	5 051.573 933 0
K_2^C	300.000 000 0	299.130 853 4	299.130 409 2
b_1^C	10.000 000 0	10.087 593 8	32.774 925 4
b_2^C	10.000 000 0	11.985 563 1	12.012 511 1
ΔX_i	initial values	derterministic optimization values	uncertainties
ΔI_1	1.000 000 0	0	1.000 000 0
ΔI_2	0.200 000 0	0	0.255 862 0
ΔK_1	10.000 000 0	0	10.256 623 2
ΔK_2	1.000 000 0	0	1.000 000 0
Δb_1	0.100 000 0	0	0.100 097 8
Δb_2	0.100 000 0	0	0.100 000 0
$\min\psi(\boldsymbol{X})$		5.1362×10^{-4}	$5.1372\times10^{-4}, 5.1590\times10^{-4}$

4 结　论

本文针对具有区间参数的振动结构提出了一种新的区间优化算法. 由于区间优化问题的求解一般要比确定性优化问题的求解复杂得多，所以我们将区间优化问题转化为近似的确定性优化问题从而可利用现有标准的非线性规划算法来求解. 本文把该方法应用到多自由度线性扭振系统. 从算例中可以看出，区间优化比确定性优化可得到更多的信息. 区间优化不仅给出了设计参数的中值，而且也给出了相应的优化误差. 此外，目标函数值也被包含在最窄的区间内. 算例表明本方法是有效的.

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DYNAMIC OPTIMIZATION FOR VIBRATION SYSTEMS WITH INTERVAL PARAMETERS

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Abstract The deterministic optimization of structural behavior has been well developed for specified structural parameters and loading conditions. However, in most practical situations, the structural parameters and loads are uncertain, for example, there may be measurement inaccuracy or errors in the manufacturing process. Therefore, the concept of uncertainty plays an important role in the investigation of various engineering problems. The most common approach to problems of uncertainty is to model the structural parameters as random variables or fields. Unfortunately, probabilistic model is not the only way one could describe the uncertainty, and uncertainty does not equal randomness.

Since the mid-1960s, a new method called the interval analysis has appeared. Recently, the interval analysis method has been used to deal with the static displacement and eigenvalue analysis of the uncertain structures with interval parameters. However, few papers can be found about the optimization of structures with interval parameters in engineering. Hence, it is necessary to develop an effective method to solve the optimal problems of structures with interval parameters.

This paper presents an interval optimization method to solve the uncertain problems of the vibration systems with multi-degrees of freedom, where the structural characteristics are assumed to be expressed as interval parameters. Using the Taylor expansion and interval extension of functions, the interval optimization problem can be transformed into the approximate deterministic optimization one, so we can use the standard algorithm of the optimization to solve the interval optimization problem. It can be seen that, using the interval optimization method, more information for the optimal structures can be obtained, such as how the optimization results change if the uncertainties of structural parameters are imposed on the structures. The present method is implemented for a torsional vibration system. A numerical example, the optimization of a dynamic absorber with interval parameters, is given. The numerical results are compared with those obtained by the deterministic optimization method. The numerical results show that the present method is effective for dealing with the optimal problems of structures with interval parameters.

Key words interval parameter, interval optimization, approximate deterministic optimization, linear torsional vibration system

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