Calculation of Aberrations Introduced by a Crystal Components

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Abstract A birefringent ray-tracing package based on the mathematical software is established. The calculating method of wave aberration is also presented. Using the package, various aberrations for bifocus lens and the relationship between the aberrations and titled angle of incident beam are calculated. The advantage of the package is that the calculations of various physical quantities are quite distinct. These calculations provide a theoretical base for improving design of crystal element.

Keywords Aberration; Ray-tracing packages; Birefringent components
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0 Introduction

Lenses made from birefringent materials have found a wide variety of use, including common-path profilometer systems[1], compact-disk[2], image processing[3]. Zhang presents a ray-tracing for crystal[4,5] and has designed a bifocus lens[6]. The lens[7] is used in the profilometer with non-contact testing ultra-smooth surface.

There are some difficulties in designing the crystal components. First, the refractive index dependents on propagation direction. Secondly, the a-ray direction is given by the Poynting vector, rather than by wave vector. Hence, commercially available ray-tracing packages are not accurate for birefringent materials. This means usual calculation of aberrations are not valid either. J. Poul[8] has presented the package, but it is not quite distinct how some physical quantities in the package (example $\boldsymbol{k}_r$, Δw) are calculated.

In this paper we incorporate an algorithm for calculating the wave aberration and imaging aberration into a ray-tracing package. These calculations are helpful for the decrease of aberration and optimum design of crystal component.

1 Ray tracing in crystal media

Snell's law of wavevector is

$$n_i \sin \theta_i = n_r \sin \phi \tag{1}$$

where n_i, n_r are the refractive indices of incident and refractive media, θ_i and φ are respectively the incident angle and the refracted angle. Using this equation at successive surface, the paths of rays through isotropic media can be determined. However, ray tracing in crystal media differs in two ways:

1) The refractive index n_r depends on the direction of wavevector $\boldsymbol{k}_r$[4]

$$n_r = [1/n_e^2 + (1/n_o^2 - 1/n_e^2) k_{rz}^2]^{-1/2} \tag{2}$$

where k_{rz} is the direction cosines of refracted wavevector in the principal axis coordinate XYZ of crystal, and

$$k_{rz} = \cos \alpha \cos \beta \sin \phi - \cos \phi \sin \beta \tag{3}$$

where α is the angle formed between the incident plane and the principal section of crystal. β is the angle formed between the interface and the optic axis. Given the normal $\boldsymbol{n}$ to the interface at the incident point, then[6]

$$\cos \alpha = (\boldsymbol{n} \times \boldsymbol{k}_i) \cdot (\boldsymbol{n} \times \boldsymbol{z}) / |\boldsymbol{n} \times \boldsymbol{k}_i| / |\boldsymbol{n} \times \boldsymbol{z}|$$
$$\cos \beta = \boldsymbol{n} \cdot (\boldsymbol{z} \times \boldsymbol{n} \times \boldsymbol{z}) / |\boldsymbol{z} \times \boldsymbol{n} \times \boldsymbol{z}| \tag{4}$$

where $\boldsymbol{k}_i$ is incident wavevector, $\boldsymbol{z}$ is a unit vector of optic axis.

2) A further complication is that the ray direction $\boldsymbol{S}$ does not coincide with the normal $\boldsymbol{k}_r$ to wavefront. Once $\boldsymbol{k}_r$ has been determined, the ray direction $\boldsymbol{S}$ is given by[6]

$$\boldsymbol{S} = n_r n [1/n_e^2 \boldsymbol{k}_r + (1/n_o^2 - 1/n_e^2) k_{rz} \boldsymbol{z}] \tag{5}$$

where n_r is the refractive index of the ray.

3) In the coordinate system $X_1 Y_1 Z$ of lens, Y_1 is the principal axis of the lens and Z is the optic axis of crystal. In calculation it must be noted that all direction cosines are finally transformed into the coordinate system of lens.

We have incorporated their algorithm into a ray-tracing package of birefringent materials and developed the software application. As a example we have made a software of the bifocus lens. The lens is a triplet lens that is composed of two glass bicovex lenses with a calcite biconcave lens between them. (Fig. 1) The refractive index n_1 of glass should satisfy $n_e < n_1 < n_o$. If the spherical radii are properly selected, the e-ray is focused on the sample surface and o-ray is a parallel beam. After the o and e beams are reflected on the sample and again pass through the lens, they become all parallel beams[6].

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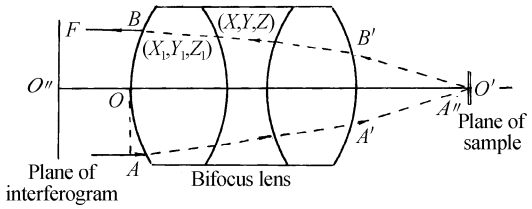


Fig. 1 Instruct of the bifocus lens, the calculations of the optical path length and the wave aberration

2 Wave aberration

Once both $\boldsymbol{k}_r$ and $\boldsymbol{S}$ are known, the optical path length(OPL)in one component can be calculated.

$$\mathrm{OPL}=n(\boldsymbol{k}_r)l_s \quad (6)$$

where l_s is the geometric path length taken by ray and $l_s=[(x_1-x)^2+(y_1-y)^2+(z_1-z)^2]^{1/2}$. x_1, y_1, z_1 are the coordinates of the incident point, x, y, z are the coordinates of the emergent point. And

$$x=(y-y_1)S_x/S_y+x_1,\ z=(y-y_1)S_z/S_y+z_1,\ (x^2+y^2+z^2)=R^2 \quad (7)$$

where R is the spherical radius. The wavefront of incident parallel beam is a plane OA. For the ray returned to the plane of interfrogram, total OPL of the ray along principal axis of the lens is

$$(\mathrm{OPL})_o=n(OO')+n(O'OO'') \quad (8)$$

Total OPL of the other ray is (see Fig. 1)

$$\mathrm{OPL}=n(AA'O')+n(O'B'BF) \quad (9)$$

The coordinate of the point F of equal OPL for every ray can be found from $\mathrm{OPL}=(\mathrm{OPL})_o$. The assemblage of the points of all rays equal OPL forms a real wavefornt. Ideal wavefront is a plane for parallel beam. Hence, the wave aberration Δw is the distance from the F point to the plane of interferogram.

3 Calculation of imaging aberrations

Imperfection in the performance of a lens are usefully characterized in terms of the five Seidel aberrations: spherical aberration, coma, astigmatism, field flatness and distortion. These aberrations can be calculated by the deviation from ideal wavefront.

After through the lens the deviation from ideal wavefront can be represented by the wavefront aberration function[8]

$$\Delta w=A(x^2+z^2)^2+Bz(x^2+z^2)+C(x^2+3z^2)+D(x^2+z^2)+Ez+Fx \quad (10)$$

where the coefficient A represents the amount of spherical aberration, B is the coma, C is the astigmatism, D is the amount of defocus, and E and F represent tilts about the z and x axes, respectively.

The incident beam is a rhomb about length 1.5 mm of side. We divide incident beam into several rays and get the wave aberration after the pass through the lens. Using fitting method the best-fit coefficients in Eq. (10) and corresponding aberrations can be determined. Based on the method, we establish the birefringent ray-tracing software about calculation of imaging aberrations. We discuss the relationship between the various aberrations and tilted angle of incident beam. Fig. 2～6 show these relationship. For the lens, we get the following conclusions:

1) The coma which only appears for object point off the axis can be neglected.

2) The spherical aberration appears when object points are on the axis and result in the off-axis ray experiencing a different focal power. From

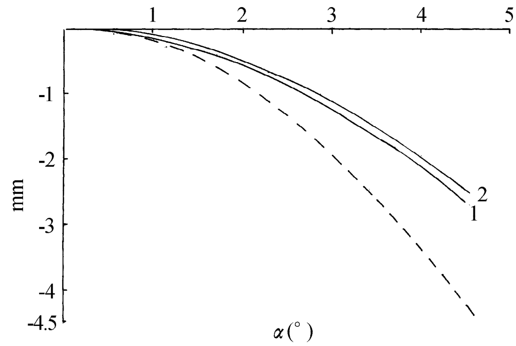


Fig. 2 Variation of the largest spherical aberration $[A(x^2+z^2)^2]_{max}$ with tilted angle for e-ray (solid curves), for o-ray (dashed curve), 1 and 2 correspond to $S_{ix}=0$ and $S_{iz}=0$, respectively

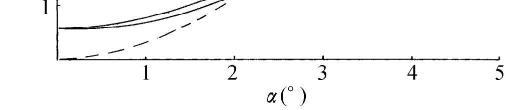


Fig. 3 Variation of the largest astigmatism $[C(x^2+3z^2)]_{max}$ with tilted angle for e-ray (solid curves), for o-ray (dashed curve), 1 and 2 correspond to $S_{ix}=0$ and $S_{iz}=0$, respectively

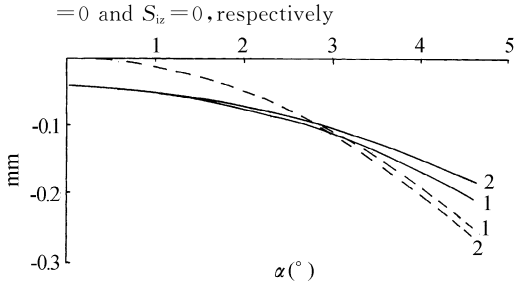


Fig. 4 Variation of the largest defocus $[D(x^2+z^2)]_{max}$ with tilted angle for e-ray (solid curves), for o-ray (dashed curve), 1 and 2 correspond to $S_{ix}=0$ and $S_{iz}=0$, respectively

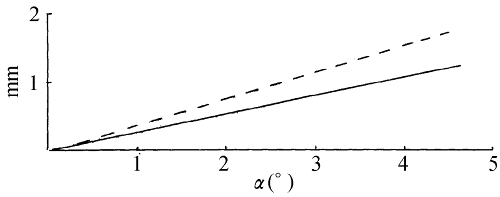


Fig. 5 Variation of the largest tilted aberrations with tilted angle for e-ray, solid curves and dashed curve represent tilts about z and x axes, respectivily

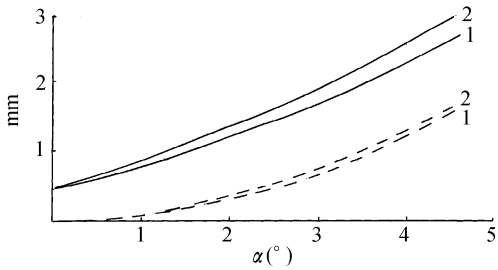


Fig. 6 Variation of the largest wave aberration with tilted angle for e-ray (solid curves), for o-ray (dashed curve), 1 and 2 correspond to $S_{ix}=0$ and $S_{iz}=0$, respectively

Fig. 2 at normal incident the spherical aberration is near zero.

3) The astigmatism and defocus are been combined since they depend on x and z in the same way. Astigmatism is an aberration that results in different focal lengths in the horizontal and vertical planes. Defocus shows that the focal length of the lens changes for off-axis rays. The rotational symmetry of an isotropic spherical lens means that astigmatism and defocus are not normally associated with on-axis ray. Hence, from Fig. 3, 4 at normal incident they are zero for o-polarization. However, the optic axis of a birefringent lens breaks this rotational symmetry and e-polarization is subject to astigmatism and defocus.

4) In Eq. (10) eighth Ez and ninth Fx terms are known as tilted aberrations about the z and x axes. For e-polarization they increase with tilted angle. (see Fig. 5) For o-polarization they can be neglected.

5) From Fig. 6 at normal incident the largest wave aberration of o-polarization is near zero and that of e-polarization is larger.

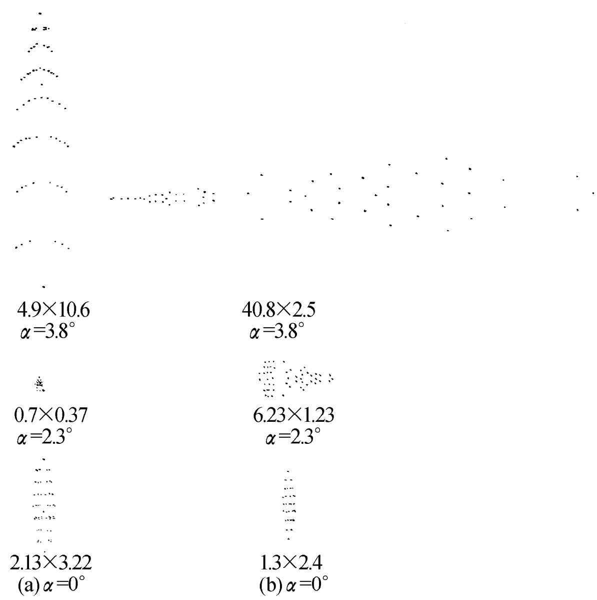
6) From Figs 2-6 the largest aberrations all increase with tilted angle of incident beam.

7) When incident beam tilts along x and z two different directions, for o-polarization changes of the aberrations with tilted angle are nearly same, because the lens have a rotational symmetry. But for e-polarization that are different, because the symmetry is broken.

It is a useful aid to design that the imaging imperfection is separated into the Seidel aberrations. According to these aberrations our design can be improved. Beside, when we applied the lens, alignment of incident beam is very important. It will be guaranteed that the aberrations reach the smallest.

5 Spot diagram

In order to visualize in terms of the overall lens performance, it is often more intuitive to consider the point-spread function. Figs. 7, 8 show the point-spread functions on the plane of interferogram for collimated bundles of on-axis and off-axis rays with incident angle of 0°, 2. 3°, 3. 8° along x and z two directions, respectively. For the o-ray the lens is rotational symmetry, and so the spot diagrams for two directions are identical. However, for e-ray the rotational symmetry is broken, that of two directions need to be considered separately. At normal incident, comparing with incident beam, for o-ray the spot diagram still is a rhomb similar to incident aperture because the aberrations are near zero, for e-ray its shape is changed because there are lager aberrations. At tilted incident all spot diagrams are confused



(a) $S_{ix}=0, S_{iz}=\sin\alpha, f=63.3$ mm, f is a focal line on sample
(b) $S_{ix}=\sin\alpha, S_{iz}=0, f=64.7$ mm, f is the other focal line. Unit of dimension is mm

Fig. 7 Geometrical spot diagrams for e-ray on the plane of interferogram.

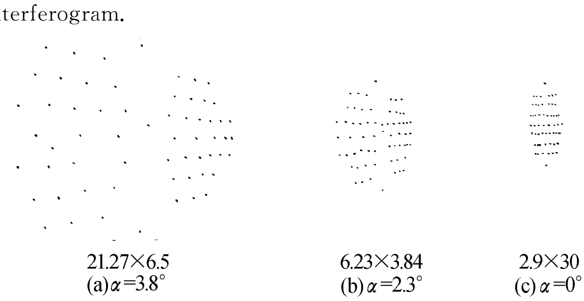


Fig. 8 Geometrical spot diagrams for o-ray on the plane of interferogram. $S_{ix}=\sin\alpha$, $S_{iz}=0$, $f=64.7$ mm, unit of dimension is mm

rapidly because the aberrations increase rapidly.

6 Conclusion

Using the software package based on the birefringent ray - tracing we discuss the wave aberration and the Seidel aberrations on the plane of interferogram for the bifocus lens. At normal incident the aberrations of o-polarization is near zero. However, for e-polarization these aberrations are dominated by astigmatism and defocus. This arises from the breaking of rotational symmetry within the birefringent material. The aberrations increase with tilted angle of incident beam, rapidly. Hence, alignment is important for using the lens. The computer package that we have developed in this study is applied to the triplet lens. It is emphasized that the computer package can also be applied to arbitrarily complex systems. According to the aberrations of element we can present a approach for improving design of system.

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晶体元件的象差计算

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摘　要　建立了折射光线轨迹的软件包和提出了波象差的计算方法.利用这个软件包,分析了双焦透镜的各类象差和象差与入射角的关系.这个软件包的特点是各种物理量的计算十分清晰,它为改进晶体元件的设计提供了理论依据.

关键词　象差;光线轨迹包;双折射元件

Zhang Weiquan is a professor. He is engaged in the research of crystal optics and has published over 30 scientific papers. The abstracts of 23 papers are published on international authority journals SCI or EI. Now, he is a member of the Optical Society of America and its reviewer. He also is a member of American Institute of Physics.