Agreement to Disagree on a Common Signal

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Abstract

This paper develops a model in which investors agree to disagree on the precision of a publicly observed signal and are prohibited from short selling. In equilibrium, a very positive (negative) signal crowds out low (high) precision investors. The equilibrium asset price is a convex function of the signal. The model implies that market confidence increases with the asset price and tends to be higher than the average confidence of the investor pool. The testable prediction is that skewness increases with intensity of disagreement and cost of short selling. Supportive evidence is found.

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This paper develops a model in which investors agree to disagree on the precision of a publicly observed signal and are prohibited from short selling. In equilibrium, a very positive (negative) signal crowds out low (high) precision investors. Consequently, the equilibrium asset price is a convex function of the signal. The model implies that market confidence increases with the asset price and tends to be higher than the average confidence of the investor pool. The testable prediction is that skewness increases with intensity of disagreement and cost of short selling. Supportive evidence is found.

The basic idea can be illustrated by the following example. Consider a stock market where short sales are not allowed. Investors initially agree on the value of a stock but disagree on the precision of a publicly observed signal (for example, an earning announcement): some investors think its precision is higher than other investors think. High precision investors will adjust their valuation more according to this signal. If the signal is positive, high precision investors will value the stock higher than low precision investors. The latter may wish to short sell the asset and end up being crowded out of the market if deterred by the cost and risk of short selling. So the market actually reacts to the positive signal through the reaction of high precision investors. On the other hand, if the signal is negative, high precision investors may be crowded out of the market. In this case the market reacts to the negative signal through the reaction of low precision investors. Therefore, the market reacts more to a positive signal than to a negative signal, resulting in a convex function of price in information: positive and negative price movements are asymmetric.

Central to the model is that investors agree to disagree on the precision of the public signal. This assumption is motivated by the observation that sometimes people reacts to the same signal to different extent. For example, after earning announcements analysts adjust their estimates by different amount, which suggests that analysts disagree on how to interpret this public signal. A straightforward way to catch this heterogeneity is to assume that analysts hold different opinions about the precision of this public information. This assumption does not lose generality due to the obvious possibility that analysts have heterogeneous prior beliefs. First, analysts may disagree in their pre-announcement estimates. But if analysts update their beliefs in a Bayesian way, disagreement in prior means cannot explain why they update their estimates by different amount. Second, analysts' confidence in their own pre-announcement beliefs also differs. However, modeling heterogeneous prior confidence and modeling heterogeneous confidence in a new public signal is equivalent. To say that one is more confident in her prior beliefs is equivalent to say that she is less confident in the new signal. If we assume instead that investors have heterogeneous confidence in their priors but agree on the precision of the new signal, the model leads to the same set of results. A second key assumption is short sale constraints, which can be justified by the cost, risk and prohibiting rules of short selling.

The paper's contribution is threefold. First, it adds new insights to the heterogeneous beliefs and short sale constraints literature. It has long been recognized that in the presence of heterogeneous beliefs and short sale constraints, risky assets are held by the most optimistic investors (Miller (1977), Harrison and Kreps (1978)). Furthermore, the opportunity to resell the asset to somebody else with higher asset valuation in the future can create bubbles (Harrison and Kreps (1978), Morris (1996), Scheinkman and Xiong (2003)). Novel in the current paper is that the asset price is asymmetric on the up and down sides. This is obtained by assuming that investors agree to disagree on the precision of a public signal.

Second, the model produces testable implications about the skewness of market returns. Since the equilibrium price is a convex function of information and a convex function makes a distribution skew to the right, *ceteris paribus*, market returns should be more positively skewed when the price function is more convex. In this model, price convexity arises from disagreement over information precision and short sale constraints, thus skewness should increase with disagreement intensity and short selling cost. We empirically test these predictions and find confirmative evidence. First, skewness increases with trading volume, which is positively correlated with disagreement[2]. Second, skewness decreases with stock size, institutional ownership and ownership breadth. This is because bigger stocks, stocks of higher institutional ownership and broader ownership are easier to short sell[3], thus their price functions are less convex.

These predictions about skewness do not contradict the existing evidence on negative skewness. First, these predictions indicate that skewness should be more positive for some stocks and less so for other stocks. Nothing is said about whether skewness should be positive or negative, though. If the sign of skewness is to be determined, numerous other skewness factors will have to be exclusively taken into consideration and their relative strength will have to be weighted[4], which is far beyond the scope of this paper. Second, existing evidence on skewness of individual stocks is mixed. Although market indices tend to be negatively skewed, individual stock skewness is often positive (Harvey and Siddique (1999, 2000), Chen, Hong and Stein (2001)).

The paper adds to the reviving literature on skewness in two aspects. First, it correlates skewness with variables measuring disagreement and short selling cost. It differs from Chen, Hong and Stein (2001) in that their paper conditions skewness on lagged information while this paper correlates skewness with contemporaneous information. Second, it produces new insight into the effect of short sale constraints on skewness. Diamond and Verrecchia (1987), Hong and Stein (2003), among others, argue that short sale constraints may be responsible for negative

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[2] See Varian (1989), Harris and Raviv (1993), Kandel and Pearson (1995), Odean (1998), Chen, Hong and Stein (2001), and Hong and Stein (2003) for models with this feature and for empirical evidence.

[3] See D'Avolio (2002) and Duffie, Garleanu, and Pedersen (2002) for discussions on the specialness of stocks.

[4] Proposed skewness factors include "leverage effect" (Black (1976), Christie (1982), Schwert (1989), Bekaert and Wu (2000)), "volatility feedback" (Pindyck (1984), Poterba and Summers (1986), French et al. (1987), Campbell and Hentschell (1992)), "stochastic bubbles" (Blanchard and Watson (1989)), "short sale constraints" (Diamond and Verrecchia (1987), Hong and Stein (2003)), "negative news threshold" (Ekholm and Pasternack (2004)) and "state-switching" (Veronesi (1999)).

skewness because bad news cannot be incorporated into the price quick enough. Chen, Hong and Stein (2001) find supportive evidence for their 2003 paper. Surprisingly, we find that short sale constraints, when combined with heterogeneous beliefs on the precision of a common signal, can have the opposite effect on skewness[5].

Third, the paper proposes a possible reconciliation for the conflict between rationality and overconfidence. A growing body of literature assumes that investors are overconfident[6]. Overconfidence is in conflict with the classical economics assumption of rationality. If investors are rational, their beliefs should be well calibrated. In this paper, high precision investors become more confident in their posterior beliefs than low precision investors through Bayesian updating. They demand less risk premium; thus tend to value the asset higher. Consequently, they are less likely to be crowded out of the market; the market tends to be overconfident rather than underconfident. One way to interpret this reconciliation is that investors make random errors about the precision of the new signal, but their average opinion can still be correct. According to Muth (1961), these random errors do not contradict rationality[7]. Therefore, the primitive assumption of heterogeneous beliefs in information precision is compatible with both rationality and overconfidence in financial markets.

This paper differs from the existing overconfidence literature in there aspects. First, we do not assume private information. In stead, we assume agreement to disagree on the precision of

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[5] Veronesi (1999) and Cao, Coval and Hirshleifer (2002) represents two other recent efforts in explaining skewness. Veronesi (1999) explains negative skewness through overreaction to bad news in good times and underreaction to good news in bad times. Cao, Coval and Hirshleifer (2002) employs information blockage to examine skewness conditional on past price movements, due to "sidelined" investors delaying trading until price movements validate their private information.

[6] For a recent review of the literature on cognitive biases and behavioral economics, see Barberis and Thaler (2001) and Hirshleifer (2001).

[7] According to Muth (1961), rationality does not require individuals to be exactly correct or identical in their beliefs. In Muth (1961), rationality means that "the subjective probability distributions of outcomes tend to be distributed about the objective probability distribution of outcomes." Muth further classifies that "… expectations of a single firm may still be subject to greater error than the theory." Therefore, random errors in individual investor beliefs do not contradict rationality.

a public signal. This feature is responsible for all the results of this paper. It differentiates this paper from other papers where investors are overconfident in their favored signals (Daniel, Hirshleifer and Subrahmanyam (1998, 2001), Odean (1998), Hong and Stein (2003), Scheinkman and Xiong (2003)). Second, in this paper we assume short sale constraints together with the agreement to disagree on information precision. In contrast, in Daniel, Hirshleifer and Subrahmanyam (1998, 2001), Odean (1998), Gervais and Odean (2001), there are no short sale constraints. Finally, in our model overconfidence is endogenous. In contrast, most existing overconfidence models take overconfidence from the psychology literature and use it as a modeling device.

The term "overconfidence" has slightly different meanings when used by different authors. It usually means that one attaches too much precision to currently held beliefs and can generally be attributed to too much precision attached to the information used to form the beliefs. Sometimes the information is restricted to be private[8]. In this paper we do not impose this restriction[9]. The signal is publicly observed. This paper adopts the generic definition of too precise posterior beliefs. This ingredient of overconfidence is shared by most overconfidence models[10]. It is also in alignment with the definition of overconfidence in the calibration literature, from where the term comes[11].

The rest of this paper is organized as follows. Section 1 sets up the model. Section 2

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8 See, for instance, Daniel, Hirshleifer and Subrahmanyam (1998, 2001), Odean (1998), and Gervais and Odean (2001), Hong and Stein (2003).

9 We are not the first to notice that overconfidence does not necessarily involve private information. Odean (1998) notices that "Traders could, instead, be overconfident about they way they interpret information rather than about the information itself."

10 For example, in Odean (1998), overconfidence means that "traders (1) hold posterior beliefs that are too precise and (2) overweight their own information relative to that of others" (Page 1894, Odean (1998)). We keep the first ingredient but relax the second. See the previous footnote about Odean (1998)'s discussion about overconfidence.

11 In the calibration literature (see, for instance, Lichtenstein et al. (1982) and Yates (1990)), overconfidence refers to the phenomenon that people are too certain about their judgment based on given information. The information is "public" in the sense that experimental subjects are given the same information and that subjects know that other subjects are given the same information.

solves the equilibrium. Section 3 explains how the crowding out endogenously generates market overconfidence. Section 4 shows that asset prices are convex functions of information. This convexity leads to testable predictions about skewness of market returns. Section 5 empirically tests these predictions. Confirmative evidence is found. Section 6 briefly summarizes the paper and discusses directions for future research. All proofs are presented in the appendix.

1. The Model

Consider a competitive market with a risky asset and a riskless bond of infinite supply and demand at the gross interest rate *R*. There are three periods, 0, 1, and 2. At time 0, investors of exponential utility, $U(w) = -e^{-aw}$, enter the market each with a unit endowment of the risky asset. At time 1, a signal about the payoff of the risky asset is publicly observed. At time 2 the payoff is realized.

The time 1 signal s equals the asset payoff x plus noise ε:

$$s = x + \varepsilon . \tag{1}$$

The random variables follow independent normal distributions: $x \sim N(\mu_x, \sigma_x^2)$, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. Investors agree over the distribution of *x*. But they disagree over the distribution of ε. More precisely, they disagree over its variance σ_ε^2. For expositional convenience, we use precision τ, which is the reciprocal of variance. τ_ε denotes the true precision of ε.

Assumption 1: Investors disagree over the precision of ε.

Investors are classified into τ_L and τ_H traders accordingly: τ_L (τ_H) traders believe the precision is τ_L (τ_H), $\tau_L < \tau_H$. A proportion λ of investors are τ_L traders; the rest of them are τ_H

traders[12]. By definition, τ_L traders associate lower quality to the signal than τ_H traders do, because they think the signal contains more noise. From now on τ_L (τ_H) traders will also be referred to as the "low (high) precision investors". Disagreement over information precision is a direct application of "difference of opinions" (Harris and Raviv (1993), Kandel and Pearson (1995))[13] to information precision. One origin of this disagreement is noise. Black (1986) forcefully argues that the world we are dealing with is full of noise, thus it is implausible that people always agree on how to differentiate information from noise.

Assumption 2: Short sales of the risky asset are prohibited.

This assumption serves the purpose of crowding potential short sellers out of the market. It is justified by the cost and risk associated with equity short selling. If short selling is allowed but incurs higher costs and risk, our results still hold qualitatively. For simplicity, we assume that short sales are prohibited[14].

2. The Equilibrium

The equilibrium in a competitive market is composed of a set of price and demand functions under which 1) traders maximize their expected utility, and 2) the market clears. Because investors' current decisions depend on future information, backward induction is employed. Investors update their beliefs when new information comes, which is summarized in lemma 1.

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[12] If we further assume that $\lambda\tau_L + (1-\lambda)\tau_H = \tau_\varepsilon$, i.e., the average belief about information precision is correct, then the "average" investor neither overestimates nor underestimates the information precision. This assumption is not necessary for selective market participation and convex price function to be derived. Under this further assumption, τ_H investors are actually overconfident and τ_L investors are actually underconfident. Without this assumption, over- and under- confidence is relative to the average confidence of the two types of investors.

[13] Kandel and Pearson (1995) provide evidence that investors interpret public announcements differently.

[14] For a discussion on short sale constraints, see Scheinkman and Xiong (2003). Also notice that the expectation of asset payoff can be negative if the signal is very negative. To avoid dealing with conditional distributions, we assume unlimited liability and allow asset price to go negative. None of our results depends on this simplifying assumption.

Lemma 1: *The posterior mean and precision of type* $\theta\,(\theta = L, H)$ *traders are:*

$$\hat{\tau}_\theta = \tau_x + \tau_\theta. \tag{2}$$

$$\hat{\mu}_\theta = \frac{\tau_x}{\hat{\tau}_\theta}\mu_x + \frac{\tau_\theta}{\hat{\tau}_\theta}s. \tag{3}$$

First notice from equation (2) that the posterior precision of low (high) precision investors is also lower (higher). Since posterior precision measures how confident one is in his posterior beliefs, low precision traders are less confident than high precision investors after the signal is observed. In other words, the confidence in the signal transforms into confidence in posterior beliefs. For this reason, τ_L (τ_H) investors will also be referred as "less (more) confident investors". Second notice from equation (3) that when updating their beliefs, τ_L traders put less weight on the signal than τ_H traders and more weight on their priors. Intuitively, those who think the information is of higher quality are more sensitive to the information.

2.A Time 1

At time 2, the payoff is realized; thus there is no Pareto improving trading. At time 1, the decision problem is essentially a static one because there is no more chance for trading. Type $\theta\,(\theta = L, H)$ traders' optimization problem is:

$$J_{1,\theta}(W_{1,\theta}, I_1) = U_{1,\theta}V_{1,\theta}.$$

Where,

$$U_{1,\theta} \equiv -\exp(-aRW_{1,\theta}),$$

$$V_{1,\theta} \equiv Max\, E_{1,\theta}[\exp(-ay_{1,\theta}(x - RP_1))].$$

The value function at time 1 equals the multiplication of two parts: the risk free part, which is the utility if all wealth is invested in the risk-free bond, and the risky part, which is the maximum expected utility from investing in the risky asset. For expositional convenience, we first examine the case without short sale constraints. The equilibrium is presented in proposition 1.

Proposition 1: *In the absence of short sale constraints, the equilibrium at time 1 is:*

$$P_{1,m} = \frac{\lambda \hat{\tau}_L \hat{\mu}_L + (1-\lambda)\hat{\tau}_H \hat{\mu}_H - a}{R(\lambda \hat{\tau}_L + (1-\lambda)\hat{\tau}_H)}, \tag{4}$$

$$y_{1,L} = \frac{b(1-\lambda)\hat{\tau}_L \hat{\tau}_H (\hat{\mu}_L - \hat{\mu}_H) + \hat{\tau}_L}{\lambda \hat{\tau}_L + (1-\lambda)\hat{\tau}_H}, \tag{5}$$

$$y_{1,H} = \frac{b\lambda \hat{\tau}_L \hat{\tau}_H (\hat{\mu}_H - \hat{\mu}_L) + \hat{\tau}_H}{\lambda \hat{\tau}_L + (1-\lambda)\hat{\tau}_H}. \tag{6}$$

Where $b = \frac{1}{a}$.

Proposition 1 says that in the absence of short sale constraints, the equilibrium price is a weighted average of the opinions of both types of traders and optimal asset demands are determined by difference of opinions. This is consistent with the result in Varian (1989). Notice that $y_{1,L}$ and $y_{1,H}$ in (5) and (6) could be negative, if the difference of opinions is big enough. Under short sale constraints this could not happen. Proposition 2 examines the equilibrium under short sale constraints.

Proposition 2: *Under short sale constraints, the equilibrium at time 1 is:*

1. If condition (a): $\hat{\mu}_H - \hat{\mu}_L > \frac{a\hat{\sigma}_H^2}{1-\lambda}$ *holds, only* τ_H *traders hold the risky asset.* τ_L *traders sell all their risky assets. The equilibrium price is* $P_{1,H} = \frac{1}{R}(\hat{\mu}_H - \frac{a\hat{\sigma}_H^2}{1-\lambda})$.

2. If condition (b): $\hat{\mu}_L - \hat{\mu}_H > \frac{a\hat{\sigma}_L^2}{\lambda}$ *holds, only* τ_L *traders hold the risky asset.* τ_H *traders sell all their risky assets. The equilibrium price is* $P_{1,L} = \frac{1}{R}(\hat{\mu}_L - \frac{a\hat{\sigma}_L^2}{\lambda})$.

3. If conditions (a) and (b) do not hold, the equilibrium in proposition 1 is still valid.

Conditions (a) and (b) can be rewritten as:

Condition (a'): $s_1 > s_1^h = \mu_x + \dfrac{a\hat{\tau}_L}{(1-\lambda)(\tau_H - \tau_L)\tau_x}$

Condition (b'): $s_1 < s_1^l = \mu_x - \dfrac{a\hat{\tau}_H}{\lambda(\tau_H - \tau_L)\tau_x}$

The two critical values, s^l and s^h, together divide the real line into three parts: A) $s \le s^l$, τ_H traders crowded out; B) $s^l < s < s^h$, nobody crowded out; C) $s \ge s^h$, τ_L traders crowded out. The equilibrium price functions in these 3 cases are $P_{1,L}$, $P_{1,m}$ and $P_{1,H}$, respectively.

2.B Time 0

At time 0 there are three cases to consider, depending on whether people know the coming of information at time 1 and whether trader types are known at time 0. If the information is foreseen, investors may or may not know their own types at time 0, depending on whether people have a judgment of the information quality in advance. Trader types are probably unknown for information of completely new types, such as a new technology, and probably known for information that can be repeatedly observed, such as economic and accounting data releases.

If information is not foreseen, no trade can be foreseen, and thus investors are homogenous at time 0. In this case no trade will happen and the equilibrium price is $P_0 = \dfrac{\mu_x - a\sigma_x^2}{R^2}$. If information is foreseen but types are unknown at time 0, investors are still homogenous. Each individual will be a τ_L trader with probability λ and a τ_H trader with probability 1-λ. The value function is:

$$J_0(W_0, I_0) = U_0 V_0,$$

$$U_0 \equiv -\exp(-aR^2 W_0),$$

$$V_0 \equiv Max\, E_0[(\lambda V_{1,L} + (1-\lambda)V_{1,H})\exp(-aRy_0(P_1 - RP_0))].$$

If types are known at time 0, each type of trader maximizes their next period's expected utility. The value function of type θ ($\theta = L, H$) trader is:

$$J_{0,\theta}(W_{0,\theta}, I_0) = U_{0,\theta}V_{0,\theta},$$

$$U_{0,\theta} \equiv -\exp(-aR^2 W_{0,\theta}),$$

$$V_{0,\theta} \equiv Max\, E_{0,\theta}[V_{1,\theta}\exp(-aRy_{0,\theta}(P_1 - RP_0))].$$

Both $V_{1,\theta}$ and P_1 in the value functions are stochastic functions of future information. There are two sources of randomness: x and ε. They are mixed in the single signal s_1 and investors cannot differentiate them. Furthermore $V_{1,\theta}$ and P_1 are nonlinear in the signal. There are kinks on these two functions as established in proposition 2. Therefore we do not have a closed form solution for the equilibrium. Numerical method is needed to solve for the equilibrium. Because all our results are derived from time 1 equilibrium, whether information is foreseen and whether types are known in advance does not change our results, as will be made clear shortly.

3. Endogenous Market Confidence

Proposition 2 establishes a "crowding out" mechanism. When the signal is bigger than the critical value s^h, τ_L traders are crowded out. When the signal is smaller than the critical value s^l, τ_H traders are crowded out. If on average τ_L traders are more likely to be crowded out than τ_H traders, the market tend to be more confident than the "average" investor.

Proposition 3: 1). Market confidence increases with the signal. 2). The probability of τ_L and τ_H traders being crowded out decreases with their proportion in the aggregate investor population, λ and 1-λ, respectively. 3). If $\frac{\hat{\tau}_L}{1-\lambda} < \frac{\hat{\tau}_H}{\lambda}$, especially, if $\lambda = 0.5$, τ_L traders are more likely to be crowded out.

Claim 1) implies higher market confidence in a bull market than in a bear market. Intuitively, this is because a positive signal simultaneously increases the asset price and crowds out less confident traders, while a negative signal simultaneously decreases the asset price and crowds out more confident traders. It is interesting to compare this implication with the existing overconfidence dynamics based on biased self-attribution, such as in Daniel, Hirshleifer and Subrahmanyam (1998) and Gervais and Odean (2001). According to biased self-attribution, investors take too much credit for their successes and less than enough blames for their failures. This leads them to become overconfident. This dynamics of overconfidence implies that investors are most overconfident early in their careers because for experienced investors self-assessment becomes more realistic (Gervais and Odean (2001)). It is clear that in biased self-attribution confidence fluctuates with "successes" and failures, while in our model market confidence moves with price moves up or down. Nonetheless, biased self-attribution may also imply higher market confidence in bull markets because market participants in bull markets are more likely to consider themselves as "successful".

Claim 2) implies a *majority effect*. It is easier for the majority group to crowd out the *minority* group than vice versa. For the majority group to crowd out the minority group, each member in the majority group needs to bear a smaller share of the gross risk than vice versa. *Ceteris paribus*, the majority group has a bigger chance of staying in the market. This result is very intuitive. The belief shared by more people is more likely to be reflected in the market.

Claim 3) suggests that more confident investors are more likely to stay in the market. When a new signal comes, investors adjust their estimation of both the mean and the variance of future payoff. The effect of the new signal on the means is symmetric in the sense that after positive signals high precision investors have higher mean while after negative signals low precision investors have higher mean. But the effect on the variances is asymmetric. High precision investors always have smaller posterior variance, regardless of the sign of the signal. Therefore, they always demand less risk premium. Their average asset valuation tends to be higher; thus are less likely to be crowded out. Consequently, on average market confidence is higher than the confidence of the "average" investors, because less confident investors are more likely to stay out of the market. Mathematically, the distance between s^l and μ_x is longer than that between s^h and μ_x when $\lambda = \frac{1}{2}$.

The asymmetric crowding out in proposition 3 is derived from the equilibrium at time 1. It does not depend on the assumption that there are no more chances for trading, nor does it depend on previous states. What drives the result is the less risk premium demanded by more confident traders. In cases of further chances for trading, hedge demand is present and no analytical solution exists. The result is still valid qualitatively because hedge demand does not change the relative risk premium of the two types of traders. An alternative interpretation for this result is that investors have heterogeneous risk tolerance. Less risk averse investors are more likely to stay in the market. In other words, more confidence is observationally equivalent to more risk tolerance. But this alternative interpretation cannot explain the other main results of this paper, namely price convexity and the variation of skewness with disagreement intensity of short selling cost.

4. Price Convexity and Skewness

Crowding out of less confident investors by a very positive signal and more confident investors by a very negative signal implies that price is a convex function of the signal. Mathematically, when $s > s^h$, $P_{1,H} > P_{1,m}$; when $s < s^l$, $P_{1,L} > P_{1,m}$. Graphically, the price function is steeper when the signal exceeds the upper critical value and flatter when the signal is lower than the lower critical value (figure 1). The kinks on the price function come from the assumption that the asset payoff will be realized in the next period thus there are no more chances for trading. For assets such as common stocks, people have more chances to trade and will hedge against future price movements. The hedge motivation will smooth the kinks on the price function, making it a smooth convex curve.

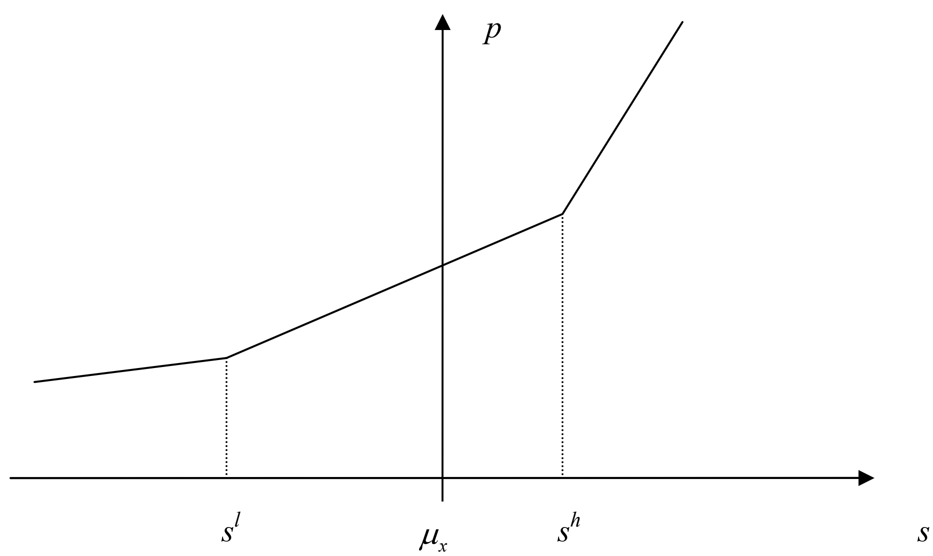

Fig. 1: Price Convex in Signal

When the signal is positive (negative) enough, only high (low) precision investors participate in the market. The market's reaction to signals depends on who is in the market.

Price convexity has immediate implications. First, expectation of the price is higher than the price at the expectation of the signal. Therefore, if it is known at time 0 that a signal is going to be observed at time 1 and people are going to interpret it differently, the time 0 expectation of time 1 price is higher than the time 1 price at the expectation of the signal. The higher expectation for price from convexity raises the equilibrium price at time 0 even without the actual coming of the information[15]. This also implies that the equilibrium price is higher when

[15] This is consistent with the old Wall Street saying "buy on rumors".

the information is foreseen than when it is not foreseen. In other words, unanticipated news has a greater impact on equilibrium price than anticipated news.

Second, a convex price function tend to make returns skew to the right. To get the intuition of this result, suppose that the signal is symmetrically distributed, so that a positive signal is equally probable with a negative signal of the same size, and that there are no other factors affecting the skewness of market returns. The positive signal increases the asset price more than the negative signal decreases the asset price. This asymmetry reflected in market returns–the percentage of price changes–is that the positive return is bigger than negative return in absolute value. Correspondingly, the right tail for the distribution of market returns is long and thin, while the left tail is fat and short. If we calculate the skewness of such a distribution, it should be positive. Furthermore, when the price function is more convex, the right tail is longer and thinner while the left tail is fatter and shorter, so the skewness should be more positive.

The above argument connects price convexity to skewness for the highly simplified case where the distribution of the signal is symmetric and there are no other factors affecting the skewness of market returns. Both assumptions could be violated. Fir the first assumption, Ekholm and Pasternack (2004) argue that the distribution of new information might be negatively skewed; while Chen, Hong and Stein (2001) suggests that signals could be positively skewed due to discretionary behavior of managers. For the second assumption, numerous hypotheses have been proposed to explain the observed negative skewness of market indices. The "leverage effect" (Black (1976), Christie (1982), Bekaert and Wu (2000)) and "volatility feedback" (Pindyck (1984), French et al. (1987), Campbell and Hentschell (1992)) explain negative skewness through asymmetric volatility; the "stochastic bubble" hypothesis (Blanchard and Watson (1989)) explains negative skewness though popping of bubbles; "short sale constraints" (Diamond and Verrecchia (1987), Hong and Stein (2003)) explains negative skewness through the revelation of bad news hidden by short sale constraints.

Despite the existence of all these skewness factors, the connection between price convexity and skewness is still valid. In particular, the property that a more convex price function leads to more positively skewed returns is still valid. This property does not depend on the symmetry of the signals, nor does it depend on the absence of other skewness factors. If the signal is asymmetric, or other skewness factors such as leverage effect exist, the overall skewness is affected, but the relative skewness due to the degree of price convexity is not.

More importantly, this property leads to testable predictions for our model. In our model, price convexity is derived from disagreement over information quality and short sale constraints. The price function is more convex when disagreement is more intense, or when short sale constraints are more binding. Therefore, skewness should increases with intensity of disagreement and effectiveness of short sale constraints. Factors other than disagreement and short sale constraints, which nonetheless interact with the crowding out mechanism to make price convexity more observable, should also be positively correlated with skewness.

Existing studies provide fruitful results on measures of disagreement. Volume is generally considered as a good proxy for disagreement. Varian (1989), Harris and Raviv (1993), Kandel and Pearson (1995), Odean (1998), and Hong and Stein (2003) all imply that trading volume is a measure of disagreement. However, the positive correlation between volume and disagreement may be blurred by other factors. For instance, Xu (2004a) find that volume is heavier after price decreases than after price increases and label this as “asymmetric volume”. Thus heavier volume can be due to negative previous price movements rather than more disagreement. To control for this possible “asymmetric volume” effect, lagged returns should be controlled for.

Hypothesis 1: Skewness increases with trading volume.

Volatility may also interact with skewness though multiple effects. First of all, it is well known that volatility and volume are positively correlated (Karpoff (1987)). Through this positive correlation volatility is also positively correlated with disagreement. But this positive correlation also suffers from the "asymmetric volatility" effect, which is used to explain negative skewness in "leverage effect". Inclusion of lagged returns should also be able to control for this effect. When a long time window, such as one year or a half year, is used to calculate skewness, contemporaneous returns should also be controlled. Second, in the volatility feedback model of Campbell and Hentschel (1992), higher volatility is associated with more negative skewness. Existence of such correlations suggests that we should include volatility in our empirical analysis.

Existing studies also produced fruitful results on effectiveness of short sale constraints. D'Avolio (2002) finds that bigger stocks and stocks with higher institutional ownership are easier to short sell. According to our model, such stocks should have more negatively skewed returns. A third closely related factor, ownership breadth, should also be negatively correlated with short sale cost because a stock owned by more investors is easy to borrow due to the following considerations. First, the "searching" cost (Duffie, Garleanu, and Pedersen (2002)) when locating a lender is lower. Second, broader ownership potentially implies more competition on the lender side and this competition tends to squeeze the asking price of lenders. Furthermore, broader ownership also implies more diversified and continuously distributed opinions. The crowding out of some investors may leave investors of "adjacent" opinions in the market. Price is thus less significantly influenced than in the case where there are jumps in opinions because not many people are looking at the stock. *Ceteris paribus*, stocks of broader ownership should exhibit less convex price function and less positive skewness.

Hypothesis 2: Skewness decreases with stock size, institutional ownership, and ownership breadth.

The three factors, size, institutional ownership, and ownership breadth, are not independent. Bigger stocks are more likely to be owned by institutional investors; bigger stocks also tend to have more investors thus broader ownership; larger institutional ownership is at least plausible to be associated with broader ownership[16]. The correlations among these three variables bring forward the possibility that institutional ownership and ownership breadth serve as the channel through which size is linked to skewness. That is, the effect of size on skewness may be indirectly obtained from its correlation with institutional ownership and ownership breadth, both of which are directly correlated with short sale cost and thus have direct effects on skewness. If this is true, inclusion of institutional ownership and ownership breadth should reduce the explanatory power of size on skewness.

Hypothesis 3: Inclusion of institutional ownership and ownership breadth reduces the negative correlation between skewness and size.

This hypothesis provides a possible explanation for the phenomenon identified by Harvey and Siddique (2000) and Chen, Hong and Stein (2001). Both papers find that bigger stocks tend to be more negatively skewed, but the reason is not clear yet. After finding this negative correlation, Chen, Hong and Stein (2001) propose a hypothesis of discretionary information disclosure by firm managers, i.e., good news is released right away but bad news dribbles out slowly, which causes positive skewness in firm level returns. If it is easier for managers of small firms to temporarily hide bad news, returns of small firms are more positively skewed. Effectively, this explanation assumes that information for small stocks is more positively skewed. Whether this is true is unclear yet, but opposite arguments exist. For example, Ekholm and Pasternack (2004), in their "negative threshold hypothesis", argue that new information is negatively skewed.

It is interesting to consider a stronger version of hypothesis 3. If size obtains all its correlation with skewness from institutional ownership and ownership breadth, inclusion of these two variables should reduce the correlation to zero. If this is true, it lends stronger support to our

[16] Chen, Hong and Stein (2002) find that the correlation between stock size and ownership breadth is 0.691. The correlation between ownership breadth and institutional ownership is especially true if one uses institutional stock holding data to measure ownership breadth, which is the case in our empirical study that follows.

model, by showing that the size effect on skewness is obtained exclusively through the two proxies for short sale constraints.

Hypothesis 3′: Skewness is not correlated with size after controlling for institutional ownership and ownership breadth.

5. Empirical Evidence

5.A Data

We have two data sources: the Center for Research in Security Prices (CRSP) at the University of Chicago and the CDA/Spectrum Institutional 13(f) Common Stock Holdings database. CRSP contains daily stock returns, prices, trading volume and shares outstanding for individual stocks from 07/03/1962 to 12/31/2003. We include all NYSE stocks except observations with missing values or negative prices. We follow tradition by excluding ADRs, REITs, close-end funds and other exotica and focus on ordinary common shares, to which our model is more applicable. We exclude NASDAQ stocks because the dealership structure of NASDAQ makes its trading volume, one of our key analysis variables, incomparable to that of NYSE. We exclude AMEX stocks because AMEX includes mainly small stocks for which volume is a noisier measure of disagreement than that of big stocks[17].

From CRSP data, we calculate annual returns, volatility, skewness, volume, and size. First, we calculate daily log returns, from which we calculate annual volatility and skewness using one year's daily data. One year is selected as a compromise between accuracy and degree of freedom. High order moments such as skewness need more observations to achieve accurate estimations. For stocks with less than 40 valid observations in a year, we exclude that year from the calculation. This exclusion guarantees all skewness coefficients are calculated at least from

[17] We redo our analysis using both NYSE and AMEX stocks. The result is qualitatively the same. All explanatory variables still have the same predicted signs. The single most noticeable difference is that the parameter estimates of volume is positive but insignificant, confirming our choice of NYSE stocks only.

40 observations. It also helps reduce IPO anomalies. Log returns are used instead of simple returns because simple returns are obviously more positively skewed. Second, we calculate daily turnover (TO) as the ratio of daily volume to shares outstanding and detrend the turnover series using a moving average of 20 trading days[18]. Annual turnover is calculated as the mean detrended daily turnover times 250, which is approximately the number of trading days in one year. We also use 250 to annualize return and volatility. Third, we calculate annual average market capitalization, the product of daily price and shares outstanding, as a measure of stock size. We use annual average instead of market capitalization on a selected day because skewness is measured over the same period. Average capitalization smoothes out random errors due to date selection and allows us to achieve more accuracy.

Our second data source, the Spectrum 13(f) database, contains quarterly institutional stock holdings of 13(f) institutions 1980-2003. We use the institutional 13(f) database because of its broader coverage than the CDA/spectrum S12 database[19]. For each stock in the 13(f) database, we calculate two variables: institutional ownership (IO) and ownership breadth (OB). Institutional ownership is the percentage of total shares outstanding owned by 13(f) institutions. Ownership breadth is the percentage of total institutions holding a positive[20] position of the stock. Total institutions are all institutions that report any positions in that quarter. Ownership breadth thus defined is actually "institutional ownership breadth", and is a little mislabeled. Without ownership data on all investors, this is the best measure of ownership breadth we can have.

Table 1 presents some summary statistics of the variables we use in the analysis. Because volatilities, capitalization, and institutional ownership are nonnegative, we take natural

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[18] Sometimes logarithm volume is used because volume cannot be negative and tends to be positively skewed. In our case, annual average of daily volume is used; thus this transformation is not necessary. A visual check finds no obvious skewness in turnover.

[19] The 13(f) database covers entire investment companies, including banks, insurance companies, parents of mutual funds, pension funds, university endowments, and numerous other types of professional investment advisors, while the S12 database covers only individual mutual funds.

[20] One exception is that positions with less than 10,000 shares or $200,000 value are not required to report.

logarithms for these variables. A visual check confirms that the logarithm transformation makes their distributions more symmetric. We present both original variables and the transformed variables to facilitate comparison. Because we use lagged returns of up to 3 periods as control variables, our sample starts from 1965. The availability of 13(f) data naturally defines two sub samples, 1965-1979 and 1980-2003. We present descriptive statistics for the whole sample and the two sub samples. For the 13(f) data, we have fewer observations because not all stocks in the CRSP database have institutional holding records in the 13(f) database.

Among the most noticeable in table 1 is the positive average skewness of individual stock returns. This is consistent with the previous finding that individual stock returns are often positively skewed. The high standard deviation of skewness indicates that this positiveness is not robust, though. Volatility is slightly higher in the second sub sample (1980-2203) than in the first sub sample (1965-1979). This is consistent with the finding of Campbell, Lettau, Malkiel and Xu (2001) that firm level variance displays a significantly positive trend between 1962 and 1997. Turnover is also higher in the second sub period, which could possibly be attributed to factors such as innovations in trading technology and wider participation of equity markets. The average institutional ownership is 1.8%. The average ownership breadth is 11.5%.

Motivated by existing evidence on the correlation between size, institutional ownership and ownership breadth, we cross tabulate (table 2) skewness, institutional ownership and ownership breadth with market capitalization. Consistent with earlier findings of Harvey and Siddique (2000), Chen, Hong and Stein (2001, 2002), and D'Avolio (2002), table 2 shows that skewness decreases with stock and that institutional ownership and ownership breadth increase with stock size.

The correlations among these variables reported in table 3 cast light on the quality of our proxies. All correlations are significant at the 5% or higher levels, except between skewness and turnover, and between turnover and lagged returns beyond 1 period. Consistent with table 2, size is highly positively correlated with institutional ownership and ownership breadth. The correlations are 0.893 and 0.533, respectively.

Table 3 also brings forward intriguing results. First, although skewness is positively correlated with volume and volume is positively correlated with volatility, as expected, the correlation between skewness and volatility is negative. Second, the correlation between skewness and volume is very small and statistically insignificant. The correlation coefficient is only 0.006 with a *p* value of 0.19. We have expected turnover to be a good proxy for disagreement and it should exhibit a significantly positive correlation with skewness. These intriguing correlations suggest the existence of potential counteracting factors that cannot be identified by simple correlations. Several possibilities exist. First of all, the asymmetric volatility effect may counteract the possible positive co-movements of volatility and skewness. Higher volatility resulting from bad lagged returns leads to negative skewness, resulting in negative correlation between skewness and volatility. This is confirmed by the significantly negative correlation of volatility with current and lagged returns. The correlation coefficients up to lagged 3 years are -0.274, -0.321, -0.217 and -0.143, respectively. The negative correlation between volatility and return is due to the long time window selected (one year); thus the asymmetric volatility effect creeps into contemporaneous returns[21]. These highly negative and significant correlations suggest a strong asymmetric volatility effect. Second, an "asymmetric volume" effect may be at work, reducing the correlation between skewness and volume. This is confirmed by the negative correlation between turnover and return. The correlation coefficient is -0.029. This correlation is highly significant. Finally, the "bull market effect" on trading volume may also in play. This effect states that volume is heavier in a bull market than in a bear market (Karpoff (1987)). This effect suggests a positive correlation between volume and contemporaneous returns. All these effects suggest that in our empirical analysis, returns and

[21] Black (1976) reports asymmetric volatility in weekly data.

lagged returns should be controlled for.

The asymmetric volatility and volume effects are confirmed by the partial correlations in table 4. To facilitate comparison, the raw correlations are also reported. After controlling for returns and lagged returns, volatility and volume become significantly positively correlated with skewness. Before the control, they are negatively correlated or uncorrelated with skewness, respectively.

Skewness is negatively correlated with size, institutional ownership and ownership breadth, as expected. Also noticeable is the positive correlations between size and institutional ownership and ownership breadth. These are the largest correlations in table 3 and remind us that institutional ownership and ownership breadth could be responsible for the correlation between skewness and size. Partial correlation analysis in table 4 lends support to this conjecture. After controlling for institutional ownership and ownership breadth, the correlation between size and skewness become insignificantly positive, keeping in mind that before the controlling it is significantly negative.

Finally, the correlation between skewness and contemporaneous returns is significantly positive, while those between skewness and lagged returns are significantly negative. While the negative correlations can potentially be attributed to asymmetrically volatility, the positive correlation is intriguing. One possible origin is the limited number of observations used to calculate the means and skewness, which can numerically lead to a positive correlation between the first and the third order moments. Intuitively, if in a particular year a stock has several big daily returns, these big returns raise both the mean return and skewness of that year. If a big enough sample is used so the estimation of the mean and skewness are both very accurate, this possibility does not exist. Unfortunately, this is not our case. In our sample, at most 260 observations are used. The standard error of the skewness coefficients calculated by this number of observations is at least 0.15, theoretically. From table 1 the standard deviation is much bigger.

This correlation between return and skewness provides another justification for the average return to be included to avoid the possible "omitted variable bias". If not included, higher skewness due to higher return may be incorrectly attributed to other variables correlated with return, such as size, institutional ownership and ownership breadth (see table 3).

5.B Disagreement Effect: Trading Volume

Table 5 reports some of the regression results. We add regressors step by step to separate out the interactions. Columns 1-3 regress skewness on size, turnover and volatility, without controlling for current and lagged returns. Size has the expected negative sign, and it is highly significant. Turnover has the expected positive sign, but only becomes significant after we add volatility into the regression. Volatility has negative coefficient estimations in all 3 regressions, which are all highly significant.

Motivated by the partial correlation results in table 4, in regression 4 current and lagged returns are included to control for the asymmetric volatility and volume effects. The difference is strikingly clear. Both turnover and volatility now have significantly positive coefficients. The estimated turnover and volatility coefficients are 0.0057 and 0.054, respectively. The t values are 5.1 and 3.9, respectively. These are significant at any usually used significance levels. The change in the size coefficient is barely noticeable, indicating that the size effect on skewness is not due to difference in returns. Lagged returns have the expected negative signs that are highly significant. Current return is also significant, but has a positive sign. From our earlier discussion, this is not surprising. To confirm the necessity of including the contemporaneous return, regression 5 omits contemporaneous return. As a result, the coefficients on volatility, volume and lagged returns all significantly decrease, although the volume coefficient is still positive and significant. This comparison indicates that lagged returns are not sufficient to catch the asymmetric volatility and volume effects[22].

The result in regression (4) is robust across sub samples (regressions 6-7). All the estimations still have the predicted sign. The only difference is that for the 80-03 sub period, the coefficient of volatility is not significant, while that of the 65-79 period is highly significant. This can possibly be explained by the higher volatility in the later period, which can lead to stronger asymmetric volatility effect. Modifying regression (7) by increasing lagged returns to 5 periods (unreported) raises the coefficient of volatility to 0.059 (t=3.07), without noticeably changing other estimates.

The above regressions strongly support our model's prediction that skewness increases with disagreement. The weaker correlation between skewness and volatility, compared with that between skewness and volume, lends further support to the model by showing that proxies strongly correlated with disagreement have a stronger effect on skewness. The above evidence also supports the short sale constraints effect. Consistent with part of hypothesis (3), we find that size is strongly and negatively correlated with skewness. We turn to ownership data to further investigate the origin of the size effect.

5.C Short Sale Constraints Effect: Size or Ownership

Regressions (8)-(10) add institutional ownership and ownership breadth into the regression. In all 3 regressions, institutional ownership and ownership breadth have the predicted negative coefficients that are highly significant. In regressions (8) and (10), where institutional ownership is included, the intercept changes from significantly positive to negative. Noticeably,

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[22] We also tried running the regression without volatility. All coefficients remain approximately the same. The coefficient of volume is slightly bigger while those of size, current and lagged returns are slightly smaller. These changes are consistent with the correlations of volatility with these variables. The big t values for contemporaneous returns may seem troublesome. It is because the sample size is big thus the standard error is small. We carried out a Belsley, Kuh, and Welsch (1980) test for multicollinearity. All condition numbers are smaller than 2, indicating the absence of multicollinearity.

the turnover coefficient increases from 0.0062 to 0.0081-0.0084, a 33% increase. This is due to the positive correlation of turnover with IO and OB, so when the ownership variables are absent, turnover catches part of their negative effects on skewness.

The most striking change is found in the coefficient of size. Adding ownership breadth alone increases the size coefficient significantly, but it is still negative. Adding institutional ownership alone makes it significantly positive. When both are added, the size coefficient becomes insignificant! The disappearing of the size effect on skewness is consistent with the insignificant partial correlation between skewness and size after controlling for institutional ownership and ownership breadth. Put together, these several pieces of evidence suggest that the effect of size on skewness originates from the correlation of size with institutional ownership and ownership breadth. Because institutional ownership and ownership breadth proxy for effectiveness of short sale constraints and depth of the investor pool, size effect on skewness actually represents the effect of short sale constraints and investor pool depth on price convexity.

To check for robustness, we further split the 1980-2003 sub sample into two sub-sub samples of equal length, 1980-1991 and 1992-2003, each with 12 years of coverage (regressions 11 and 12). The pattern remains unchanged in both sub-sub samples. The size coefficient is positive in both sub samples. And it is even significant for the 1980-1991 sub period.

Why size should have a positive coefficient is not clear to us; but since it is only significant for one sub sample, it could be due to reasons idiosyncratic to that time period. One possible origin is an "errors-in-variables" problem. Precisely, it arises from an "overestimation" bias. Ownership breadth as defined is actually "institutional ownership breadth" that does not include investors that do not meet the 13(f) criterion, including individual investors and small institutional investors with less than $100 million under control. This is not a big problem if institutional ownership breadth is approximately proportional to the true ownership breadth, defined as the percentage of investors holding a positive position of a stock as a percentage of all potential investors, in which case institutional ownership breadth is approximately a linear transformation of the true ownership breadth. Unfortunately, this is unlikely to be true. If individual investors tend to own more small stocks than big stocks, ownership breadth of big stocks is relatively overestimated. Because ownership breadth has a negative effect on skewness, the overestimation exaggerates its negative effect on skewness, and the exaggeration increases with stock size. The size coefficient increases to compensate for this exaggeration. The larger size coefficient in the 1980s suggests that during that time period the overestimation is bigger. In the 1990s, there are more institutional investors, and they extend their coverage to smaller stocks that they previously do not invest in. Consequently, the overestimation problem becomes less severe and the size coefficient becomes smaller.

Table 6 supports this conjecture by comparing the two sub samples over which we have ownership data: 1980-1991 and 1991-2003. During the first period there are on average 735 institutions reported in our database each year. In the second period, the number more than doubled to 1519.8. The table has 3 panels that compare number of institutions, institutional ownership and ownership breadth for stocks divided into 10 deciles based on market capitalization. First, columns 1-3 compare the average number of institutions owning a stock. Although the number of owners increases for all stocks, that of smaller stocks increases faster. Deciles 2 and 3 stocks get the largest increases, 150% and 159%, while deciles 8 and 9 stocks get the smallest increases, 63% and 54%. The average increase is 84%. Second, columns 4-6 present enlightening evidence about institutional ownership. Although we have more institutions in the second period, the average institutional ownership decreases from 11.4% to 10.3%, which suggests that institutions diversify their holdings to smaller stocks that they previously do not hold. Consistent with the overall image, the smaller 5 deciles' institutional ownership increases by 12%, 26%, 28%, 16% and 4%, respectively, while that of the larger 5 deciles decreases by 5%, 13%, 19%, 24%, and 18%. Finally, columns 7-9 show relatively more direct evidence. The average breadth of ownership increases by 49%. Interestingly, the increases in ownership breadth monotonically decrease from 94% of decile 1 to 22% of decile 10. All 3 comparisons consistently suggest that smaller stocks receive more attention from institutional investors in the second period. As a result, the overestimation problem becomes less severe and the size coefficient becomes smaller.

The estimated volatility coefficient is also noticeably different between the 80s and 90s. It is significantly positive in the 1980s and significantly negative in the 1990s. This can be explained by higher volatility in the second period. The average volatilities for the two periods are 0.378 and 0.413, respectively. Therefore, the asymmetric volatility effect is likely to be stronger in the second period. If we compare the two periods 65-79 and 80-03 in regressions (6) and (7), volatility is also more significant in the lower volatility 65-79 period.

6. Summary

This paper develops a model in which investors initially agree on asset payoff but agree to disagree about the precision of a publicly observed signal. Investors are prohibited from short selling. In equilibrium, a very positive signal crowds out low precision investors and a very negative signal crowds out high precision investors. The equilibrium asset price is a convex function of the signal, due to the heterogeneous sensitivity of high and low precision investors to the signal. The model implies that market confidence increases with asset price. It also implies that the average market confidence tends to be higher than the average confidence of the investor pool, because more confident investors are more likely to participate in the market than less confident investors. This is because they demand less risk premium thus value the asset higher. The testable prediction is that skewness increases with intensity of disagreement and cost of short selling. Empirical tests find confirmative evidence. Specifically, individual stock returns are more positively skewed for stocks of smaller capitalization, heavier trading volume, lower institutional ownership and narrower ownership breadth. After controlling for institutional ownership and ownership breadth, the effect of size on skewness disappears, suggesting that stock size works on skewness through the channel of the ownership variables, which proxies for short selling costs.

The paper has three major contributions. First, it identifies that heterogeneous investor confidence and short sale constraints imply convexity of asset prices in information, giving rise to asymmetric price changes on the up and down sides. Second, it predicts how skewness of individual stock returns should correlate with trading volume, stock size, institutional ownership and ownership breadth. Confirmative evidence is found. Finally, it proposes a possible reconciliation for the conflict between overconfidence and rationality.

These findings are robust to maintained assumptions. We have assumed that initially investors are equally confident in their priors but have different confidence in the new information. Investors may differ in their prior confidence as well. But as long as the confidence in the prior and the confidence in the new signal are sufficiently independent, all our results are still qualitatively valid. Specifically, asset prices are still convex in the signal due to the different sensitivity and short sale constraints and more confident investors are still more likely to participate. What happens in reality is that those who think they know more about the market are more likely to participate in the market, no matter whether this belief comes from more confident prior or higher precision attached to new information. We have also assumed that investors share a common prior on the expectation of asset payoff. Investors could also disagree over the expectation of asset payoff[23]. Relaxing this assumption does not change the result of our model, as long as the beliefs over the expectation and the confidence in new information are sufficiently independent.

In addition to the findings, the paper also brings forward many unanswered questions.

[23] Varian (1989) models disagreement on the mean of asset returns.

We leave them for future research. First of all, this paper cannot answer why market indices are generally negatively skewed. This is particularly intriguing when taking into consideration that individual stocks are often positively skewed. To fully answer this question, a multiple asset framework is called for. Second, the determination of investor and market confidence is only partially answered in this paper. In the simple model presented, investor confidence in their prior beliefs and in the new signal is exogenously given. The distribution of beliefs is also exogenously given. Market confidence is then determined by who stay in the market. But what if the quality of the signals is determined endogenously? What other factors may influence the over all quality of the signals? What factors may influence the distribution of beliefs? What welfare effect dos it imply? For example, if the overall quality of information is higher, investor and market confidence also tend to be higher. This simple observation can potentially explain the higher risk of emerging financial markets. Third, the implications of investor and market confidence reach more than discussed in this paper. Confidence can be immediately linked to the sensitivity to new information. Straightforward implications of this property include that market confidence can influence volatility and volume and that heterogeneous investor confidence can influence investment decisions[24]. Finally, and on more general level, this paper calls for more attention to beliefs on higher order moments. The existing literature examining the beliefs of economic agents has largely focused on the first order moment. Higher order moments have largely been overlooked. This paper demonstrates that heterogeneous beliefs about the second order moments, when joint with market frictions such as short sale constraint, lead to interesting implications.

[24] Xu (2004a, b) is pursuing research in these directions and finds some confirmative results.

Appendix: Proof of propositions.

A. Proof of Proposition 1.

Because x is normally distributed, $V_{1,\theta} = Max \exp[-ay_{1,\theta}(\hat{\mu}_\theta - RP_1) + \frac{(ay_{1,\theta})^2}{2}\hat{\sigma}_\theta^2]$. Solving this maximization problem gives the optimal demand, $y_{1,\theta} = \frac{\hat{\mu}_\theta - RP_1}{a\hat{\sigma}_\theta^2}$. The market clear condition is $\lambda y_{1,L} + (1-\lambda)y_{1,H} = 1$. Substituting in $y_{1,L}$ and $y_{1,H}$ gives the equilibrium price. Substituting this price into the demand functions gives the equilibrium demands.

B. Proof of Proposition 2.

Condition (a) follows from $y_{1,L}$<0. In this case τ_L traders wish to hold a short position on the risky asset but the best thing they can do is to sell out. Condition (b) follows from $y_{1,H}$<0. In this case τ_H traders wish to hold a short position but the best thing they can do is to sell out. If none of these two conditions hold, the short sale constraint does not bind and the equilibrium in proposition 1 is still valid.

C. Proof of Proposition 3.

Claim 1) follows from the fact that less confident traders are crowded out when the signal is positive enough and more confident traders are crowded out when the signal is negative enough. τ_L traders are crowded out with the probability:

$$P(s > s^h) = \Phi(-\frac{a\hat{\tau}_L}{(1-\lambda)(\tau_H - \tau_L)\tau_x\sigma_s})$$

τ_H traders are crowded out with the probability:

$$P(s < s^l) = \Phi(-\frac{a\hat{\tau}_H}{\lambda(\tau_H - \tau_L)\tau_x\sigma_s})$$

$$P(s < s^l) - P(s > s^h) = \Phi(\frac{a\hat{\tau}_L}{(1-\lambda)(\tau_H - \tau_L)\tau_x\sigma_s}) - \Phi(\frac{a\hat{\tau}_H}{\lambda(\tau_H - \tau_L)\tau_x\sigma_s}) .$$

Where $\sigma_s = \sqrt{\sigma_x^2 + \sigma_\varepsilon^2}$ is the standard deviation of the signal, and $\Phi(\bullet)$ is the normal cumulative distribution function. Claim 2) follows from that $\Phi(\bullet)$ is a monotone increasing function. If $\frac{\hat{\tau}_L}{1-\lambda} < \frac{\hat{\tau}_H}{\lambda}$, $\frac{a\hat{\tau}_L}{(1-\lambda)(\tau_H - \tau_L)\tau_x\sigma_s} < \frac{a\hat{\tau}_H}{\lambda(\tau_H - \tau_L)\tau_x\sigma_s}$, thus $P(s < s^l) - P(s > s^h) < 0$. Claim 3) follows.

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Table 1: Summary Statistics

Period	1965-2003		1965-1979		1980-2003	
No. of Obs.	48701		17867		30834 (24904)	
	Mean	Std.	Mean	Std.	Mean	Std.
Skewness	0.253	1.156	0.318	0.821	0.216	1.310
Capitalization (Billions)	2.261	10.259	0.516	1.848	3.273	12.707
Log Capitalization	12.805	1.824	11.912	1.466	13.322	1.812
Turnover (Raw) (%)	64.057	66.996	33.283	34.289	81.889	74.441
Turnover (Detrended) (%)	0.226	4.445	0.099	2.482	0.299	5.256
Return (%)	0.070	0.522	0.073	0.414	0.069	0.575
Volatility (Std.) (%)	0.376	0.237	0.345	0.158	0.394	0.271
Volatility (Log Std.)	-1.099	0.462	-1.151	0.408	-1.069	0.488
Institutional Ownership					0.108	0.451
Institutional Ownership (Log)					-2.764	1.117
Ownership Breadth					0.115	0.224

Skewness is calculated using one year's daily returns for each stock. If a stock has less than 40 valid observations in a given year, skewness is not calculated. Capitalization is the mean of share price multiplied by shares outstanding in a given year. Returns, turnover, and volatility are annualized. Institutional Ownership (IO) is the percentage of shares owned by institutional investors. Ownership Breadth (OB) is the percentage of institutions owning the stock in the 13(f) data base. Natural logs are taken on capitalization, volatility and IO. For the sub period 1980~2003, Skewness, (Log) Capitalization, Return, Turnover, and (Log) volatility have 32961 observations. (Log) IO and ownership breadth have 26714 observations.

Table 2: Skewness, Institutional Ownership, and Ownership Breadth with Market Capitalization

Period	1965-2003 Skewness		1965-1979 Skewness		1980-2003 Skewness		1980-2003 IO		1980-2003 OB	
No. of Obs.	48701		17867		30834		24904		24904	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
Smallest	0.353	1.243	0.462	0.919	0.285	1.402	0.011	0.221	0.006	0.170
2	0.427	1.230	0.475	0.828	0.397	1.422	0.022	0.331	0.010	0.203
3	0.376	1.266	0.477	0.914	0.313	1.441	0.033	0.390	0.013	0.222
4	0.308	1.255	0.421	0.923	0.237	1.420	0.043	0.432	0.015	0.222
5	0.264	1.246	0.351	0.868	0.210	1.432	0.057	0.453	0.019	0.219
6	0.235	1.185	0.314	0.857	0.186	1.346	0.075	0.480	0.024	0.215
7	0.207	1.184	0.249	0.824	0.181	1.361	0.098	0.509	0.030	0.202
8	0.166	1.075	0.211	0.711	0.138	1.244	0.132	0.547	0.038	0.198
9	0.126	0.950	0.171	0.557	0.099	1.124	0.189	0.558	0.056	0.178
Largest	0.086	0.851	0.137	0.488	0.055	1.009	0.351	0.547	0.131	0.153

IO is institutional ownership, percentage of shares owned by institutional investor. OB is ownership breadth, percentage of institutions owning the stock in the 13(f) data base. Each year, stocks are grouped into 10 deciles according to annual average market capitalization. Mean and standard deviations for skewness, IO, and OB are calculated for each of the sub samples for the time periods indicated.

Table 3: Pearson Correlation

	Cap	TO	Std.	Ret	Ret(-1)	Ret(-2)	Ret(-3)	Log IO	OB
Skew	-0.113	0.006	-0.011	0.316	-0.018	-0.033	-0.011	-0.101	-0.124
Cap		0.019	-0.366	0.123	0.204	0.196	0.182	0.893	0.533
TO			0.070	-0.028	0.015	0.002	0.003	0.021	0.056
Std.				-0.274	-0.321	-0.217	-0.143	-0.366	-0.124
Ret					0.126	-0.014	0.043	0.142	0.078
Ret(-1)						0.095	-0.071	0.196	0.085
Ret(-2)							0.070	0.200	0.093
Ret(-3)								0.188	0.075
Log IO									0.556

See note under table 1 for descriptions of variables. Correlations between skewness, Log Cap, return, lagged returns (Ret(-1), Ret(-2), Ret(-3)), Log Std., Turnover are calculated using data from 1965, totally 48701 observations. Correlations with IO (Institutional Ownership) and OB (Ownership Breadth) are calculated using data from 1980, totally 24904 observations. All correlations are significant at 5% or higher level except between skewness and turnover, between turnover and lagged returns beyond 1 period.

Table 4: Pearson Partial Correlation with Skewness

Correlation Variable	Partial Variables	Partial Correlation	Correlation
Log Std.	Ret, Ret(-1), Ret (-2), Ret (-3)	0.060 (<0.0001)	-0.011 (0.012)
Turnover	Same as above	0.017 (0.0002)	0.006 (0.19)
Log Cap	Log IO, Breadth	0.001 (0.86)	-0.101 (<0.0001)

Partial correlations are calculated using the same samples to calculate correlations. See note under table 3 for detailed information. In parentheses are *p* values testing the null hypothesis of zero correlation.

Table 5: Determination of Skewness

Reg. ID	1	2	3	4	5	6	7	8	9	10	11	12
Period	65-03	65-03	65-03	65-03	65-03	65-79	80-03	80-03	80-03	80-03	80-91	92-03
Intercept	1.196	1.186	1.194	1.341	1.116	1.239	1.373	-1.072	1.157	-0.334	-0.904	-0.746
	(23.55)	(23.36)	(23.53)	(27.28)	(21.41)	(23.74)	(21.73)	(-4.91)	(15.40)	(-1.37)	(-2.63)	(-1.86)
Log Cap.	-0.078	-0.063	-0.079	-0.084	-0.0723	-0.060	-0.091	0.051	-0.063	0.019	0.072	0.027
	(-21.63)	(-20.09)	(-21.69)	(-23.77)	(-19.34)	(-12.56)	(-19.28)	(3.86)	(-10.72)	(1.34)	(3.45)	(1.27)
Turnover		0.0016	0.0024	0.0057	0.003	0.0059	0.0062	0.0081	0.0081	0.0084	0.0095	0.0081
		(1.4)	(2.07)	(5.20)	(2.31)	(2.52)	(4.66)	(4.80)	(4.80)	(4.97)	(3.27)	(3.84)
Log Std.	-0.115		-0.117	0.054	-0.140	0.173	0.014	0.023	0.019	0.024	0.258	-0.110
	(-8.37)		(-8.51)	(3.93)	(-9.83)	(9.41)	(0.75)	(1.05)	(0.88)	(1.11)	(7.81)	(-3.71)
Ret				0.82		0.74	0.84	0.88	0.90	0.89	0.90	0.87
				(78.73)		(42.15)	(62.99)	(55.45)	(56.98)	(55.91)	(37.49)	(40.71)
Ret(-1)				-0.14	-0.052	-0.11	-0.15	-0.19	-0.15	-0.18	-0.15	-0.20
				(-9.88)	(-3.55)	(-6.01)	(-8.04)	(-9.36)	(-7.45)	(-8.40)	(-4.45)	(-7.22)
Ret(-2)				-0.044	-0.066	-0.028	-0.053	-0.060	-0.035	-0.047	-0.017	-0.067
				(-3.11)	(-4.36)	(-1.46)	(-2.80)	(-2.81)	(-1.64)	(-2.21)	(-0.48)	(-2.43)
Ret(-3)				-0.044	-0.057	-0.010	-0.060	-0.079	-0.063	-0.071	-0.032	-0.089
				(-3.18)	(-3.90)	(-0.56)	(-3.22)	(-3.74)	(-2.96)	(-3.34)	(-0.96)	(-3.25)
Log IO								-0.2289		-0.15	-0.18	-0.20
								(-11.83)		(-6.44)	(-6.05)	(-5.05)
OB									-0.55	-0.37	-0.22	-0.36
									(-12.07)	(-6.87)	(-2.49)	(-5.06)
No. of Obs.	48701	48701	48701	48071	48701	17867	30834	24904	24904	24904	10906	13998
Adj. R2	0.029	0.0276	0.029	0.1395	0.030	0.1357	0.1394	0.1471	0.1473	0.1487	0.1375	0.1483

The dependent variable is skewness calculated using one year's daily observations. Log Cap is logarithm market capitalization. Turnover is average daily turnover in a given year detrended using 20 days MA. Log Std. is logarithm standard deviation of returns. Ret, Ret(-1), Ret(-2), Ret(-3) are returns of the current year the 3 previous years. Log IO is logarithm institutional ownership (percentage of shares held by 13(f) institutional investors. OB is ownership breadth, defined as the number of institutions holding the stock as a percentage of all 13(f) institutions filing with SEC that quarter. Annual log IO and OB are the mean of the four quarters. All regressions include time dummies (unreported) for each year. In parentheses are *t* values.

Table 6: Institutional Ownership: 80-91 and 92-03

	No. of Institutions			Institutional Ownership			Ownership Breadth		
	80-91	92-03	ratio	80-91	92-03	ratio	80-91	92-03	ratio
All	84.5	155.3	1.84	0.114	0.103	0.91	0.353	0.527	1.49
Smallest	8.3	17.7	2.14	0.010	0.012	1.12	0.143	0.277	1.94
2	15.1	37.6	2.50	0.019	0.024	1.26	0.221	0.418	1.89
3	21.4	55.5	2.59	0.028	0.036	1.28	0.267	0.495	1.85
4	30.0	70.7	2.36	0.040	0.046	1.16	0.319	0.531	1.66
5	41.9	89.2	2.13	0.056	0.058	1.04	0.347	0.543	1.56
6	57.4	112.0	1.95	0.077	0.074	0.95	0.382	0.561	1.47
7	78.8	138.1	1.75	0.105	0.092	0.87	0.417	0.584	1.40
8	110.3	179.7	1.63	0.148	0.120	0.81	0.445	0.621	1.40
9	162.2	249.3	1.54	0.220	0.167	0.76	0.473	0.618	1.31
Largest	290.5	482.7	1.66	0.392	0.323	0.82	0.485	0.590	1.22
Average No. of institutions each year:				735 (80-91)			1519.8 (92-03)		

This table compares the two sub samples: 1980-1991 and 1992-2003. Columns 1-3 show that the second period has larger number of institutional investors, spreading over all stock size deciles. Columns 4-6 show that institutional ownership of smaller stocks increases while that of bigger stocks decreases. Columns 7-9 show that ownership breadth of smaller increases faster than that of bigger stocks.