第27卷第1期 上海师范大学学报(自然科学版) Vol. 27, No. 1
1998年3月 J. of Shanghai Teachers Univ. (Natural Sciences) Mar, 1998

Iterative Algorithms for Finding Approximate Solutions to Generalized Strongly Nonlinear Complementarity Problems and Quasi-complementarity Problems *

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Abstract We study iterative algorithms for finding approximate solutions of generalized strongly nonlinear complementarity problems and quasi-complementarity problems which include, as special cases, some known results in this field. Our results extend, improve and complement the earlier and recent results obtained by several authors including Noor, Chang and Huang, and Li and Ding.

Key words complementarity problem; quasi-complementarity problem; closed convex cone; H-Lipschitz continuous mapping with respect to g; Ψ-strongly monotone mapping with respect to g

0 Introduction

The complementarity theory introduced by Lemke[1] and Cottle and Dantzig[2] in the early 1960s and later developed by others is a very powerful tool of the current mathematical technology in the study of a wide class of problems arising in control and optimization, economics and transportation equilibrium, contact problems in elasticity, fluid flow through porous media and many other branches of physics, mathematics, and engineering sciences. It has been proved by Karamardian[3] that if the convex set involved in a variational inqauality problem and a complementarity problem is a convex cone, then both problems are equivalent. In recent years the complementarity problems have been extended and generalized in several directions. Among these generalizations of the complementarity problem, the quasi-(implicit) complementarity problem considered and studied by Pang[5], Noor[4], Chang and Huang[12,13], Ding[14], Li and Ding[15],

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Received: 1996-11-11

* This project is supported by Shanghai Youth Scientific Fund for Institutions of Higher Learning

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and Zeng[9~11], and the class of mildly nonlinear complementarity problems introduced and considered by Noor[16] are important and useful generalizations.

Motivated and inspired by recent research work in this field, in this paper, we study iterative algorithms for finding approximate solutions of generalized strongly nonlinear complementarity problems, and quasi-complementarity problems which include, as special cases, some known results. Our results extend, improve and implement the earlier and recent results obtained previously by some authors including Noor, Chang and Huang, and Li and Ding.

2 Preliminaries

Let H be a Hilbert space and $(\cdot,\cdot)$ and $\|\cdot\|$ denote the inner product and norm on H, respectively. If $K \subset H$ is a closed convex cone, we use K^* to denote the polar cone of K, i.e.,

$$K* = \{u \in H : (u,v) \geqslant 0, \text{ for each } v \in K.$$

Let $D \subset H$ be a nonempty subset. Given set-valued mappings $K, T, A : D \to 2^H$ and a single valued mapping $g : D \to H$, we consider the problem of finding $x^* \in D, u^* \in T(x^*)$ and $v^* \in A(x^*)$ such that

$$g(x^*) \in K(x^*),\ u^* + v^* \in K^*(x^*) \text{ and } (g(x^*) - m(x^*), u^* + v^*) = 0, \quad (2.1)$$

which is said to be the generalized strongly nonlinear quasi-complementarity problem introduced and studied by Li and Ding[15]. In many important applications K(x) has the form

$$K(x) = m(x) + K,$$

where $m : D \to K$ is a given single valued mapping, $K^*(x)$ is the polar cone of $K(x)$, i e,

$$\{K^*(x) = w \in H : (w,z) \geqslant 0 \text{ for each } z \in K(x)\}$$

and the equality holds: $K^*(x) = (m(x) + K)^* = m^*(x) \cap K^*$.

Definition 2.1 Let $D \subset H$ be a given nonempty subset, $g : D \to H$ and $F : D \to 2^H$ be two given mappings, and $\Phi, \Psi : [0,\infty) \to (0,\infty)$. We say that

(1) F is Φ-Lipschitz continuous with respect to g, if

$$\|u - v\| \leqslant \Phi(\|g(x) - g(y)\|)\|g(x) - g(y)\|$$

for each $x, y \in D, u \in F(x)$ and $v \in F(y)$;

(2) F is Ψ-strongly monotone with respect to g, if

$$(u - v, g(x) - g(y)) \geqslant \Psi(\|g(x) - g(y)\|)\|g(x) - g(y)\|^2$$

for each $x, y \in D, u \in F(x)$ and $v \in F(y)$.

Definition 2.2[13] Let $D \subset H$ be a given nonempty subset, $g : D \to H$ be a single valued mapping, and $F : D \to C(H)$ be set-valued mapping, where $C(H)$ stands for a family of all nonempty compact subsets of H, F is said to be H-Lipschitz continuous with respect to g if there is a constant $\gamma > 0$ such taht

$$H(F(x), F(y)) \leqslant \gamma\|g(x) - g(y)\| \text{ for all } x, y \in D,$$

where H(• , •) is the Hausdorff metric on C(H).

Lemma 2.1[6,9] If $K \subset H$ is a closed convex subset and $z \in H$ is a given point, then $u \in K$ satisfies the inequality $(u - z, v - u) \geqslant 0$ for all $v \in K$ if and only if

$$u = P_K z, \tag{2.2}$$

where P_K is the projection of H onto K.

Lemma 2.2[6,9] The mapping P_K defined by (2.2) is nonexpansive, that is,

$$\| P_K u - P_K v \| \leqslant \| u - v \| \text{ for all } u, v \in H.$$

Lemma 2.3[12] If $K(u) = m(u) + K$ and $K \subset H$ is a closed convex subset, then for any $u, v \in H$, we have

$$P_{K(u)} v = m(u) + P_K(v - m(u)). \tag{2.3}$$

Lemma 2.4[15] If $K \subset H$ is a closed convex cone and $K(x) = m(x) + K$ for each $x \in D$, where $m: D \to K$ is a single valued mapping and $K \subset g(D)$, then $x^* \in D$, $u^* \in T(x^*)$ and $v^* \in A(x^*)$ are a solution of the generalized strongly nonlinear quasi-complementarity problem $(T, A, g; K(x))$ (2.1), if and only if $x^* \in D$, $u^* \in T(x^*)$ and $v^* \in A(x^*)$ are a solution of the generalized strongly nonlinear quasivariational inequality $(T, A, g; K(x))$.

Lemma 2.5[15] Let $D \subset H$ be a nonempty subset, and $K \subset g(D)$. Then $x \in D$, $u \in T(x)$ and $v \in A(x)$ satisfy $g(x) \in K(x)$ and

$$(u + v, g(y) - g(x)) \geqslant 0 \text{ for all } g(y) \in K(x),$$

if and only if $x \in D$, $u \in T(x)$ and $v \in A(x)$ satisfy the relation

$$g(x) = m(x) + P_K(g(x) - \rho(u + v) - m(x)) \tag{2.4}$$

where $\rho > 0$ is a constant.

Lemma 2.6[15] Let D be a nonempty subset of H, $g: D \to H$, $m: D \to K$ and $T, A: D \to C(H)$ where K is a closed convex cone of H and $K \subset g(D)$. Assume that $T: D \to C(H)$ is Ψ-strongly monotone with respect to g and H-Lipschitz continuous with respect to g where the H-Lipschitz constant is $\beta > 0$, $A: D \to C(H)$ is H-Lipschitz continuous with respect to g where the H-Lipschitz constant is $\gamma > 0$ and $m: D \to K$ is μ-Lipschitz continuous with respext to g. If there exist $\rho > 0$ and $k \in (0,1)$ such that for each $t \in (0, \infty)$,

$$0 \leqslant (1 - 2\rho\Psi(t) + \rho^2\beta^2)^{\frac{1}{2}} < k - \rho\gamma - 2\mu(t),$$
$$\Psi(t) > \gamma(1 - 2\mu(t)), \rho\gamma < k - 2\mu(t) \tag{2.5}$$

then there exist $x^* \in D$, $u^* \in T(x^*)$ and $v^* \in A(x^*)$ which form a solution of the generalized strongly nonlinear quasi-complementarity problem (2.1).

3 Iterative algorithm for the generalized strongly nonlinear quasi-complementarity problem

In this section we give the following general and unified algorithms for the generalized strongly nonlinear quasi-complementarity problem (2.1), and study the conditions under which the approximate solution obtained from the iterative algorithms converges strongly to the exact solution of the generalized strongly nonlinear quasi-complementarity problem (2.1).

Algorithm 3.1[15] Let $D \subset H$ be a nonempty subset, $g: D \to H$, $m: D \to K$ and $T, A: D \to C(H)$ where $K \subset g(D)$ and K is a closed convex cone of H. For any given $x_0 \in D$, we take

$u_0 \in T(x_0)$ and $v_0 \in A(x_0)$. Let

$$w_1 = m(x_0) + P_K[g(x_0) - \rho(u_0 + v_0) - m(x_0)] \in g(D).$$

Since $m(x_0) \in K \subset g(D)$, there exists $x_1 \in D$ such that $g(x_1) = w_1$. Since

$$u_0 \in T(x_0) \in C(H) \text{ and } v_0 \in A(x_0) \in C(H),$$

by [7] there exist $u_1 \in T(x_1)$ and $v_1 \in A(x_1)$ such that $\| u_1 - u_0 \| \leqslant H(T(x_0), T(x_1))$ and $\| v_1 - v_0 \| \leqslant H(A(x_0), A(x_1))$. Let

$$w_2 = m(x_1) + P_K[g(x_1) - \rho(u_1 + v_1) - m(x_1)] \in g(D).$$

Then there exists $x_2 \in D$ such that $g(x_2) = w_2$. By induction, we can obtain three sequences $\{u_n\}$, $\{v_n\}$ and $\{x_n\}$:

$$u_{\in} T(x_n),\ v_n A(x_n),$$
$$\| u_n - u_{n-1} \| \leqslant H(T(x_n), T(x_{n-1})),\ \| v_n - v_{n-1} \| \leqslant H(A(x_n), A(x_{n-1})),$$
$$g(x_{n+1}) = m(x_n) + P_K[g(x_n) - \rho(u_n + v_n) - m(x_n)],\ n = 0,1,2,\cdots,$$

where $\rho > 0$ is a constant.

Theorem 3.1 Let D be a nonempty subset of H, $g: D \to H$, $m: D \to K$ and $T, A: D \to C(H)$ where K is a closed convex cone of H and $K \subset g(D)$. Assume that $T: D \to C(H)$ is Ψ-strongly monotone with respect to $(g - m)$ and H-Lipschitz continuous with respect to $(g - m)$ where the H-Lipschitz constant is $\beta > 0$, $A: D \to C(H)$ is H-Lipschitz continuous with respect to g where the H-Lipschitz constant is $\gamma > 0$ and $m: D \to K$ is μ-Lipschitz contiouous with respect to g and λ-strongly monotone with respect to g where μ and λ are positive constants. If there exist $\rho > 0$ and k in $(0,1)$ such that for each $t \in [0,\infty)$,

$$\left.\begin{aligned} &0 \leqslant (1 - 2\rho\Psi(t) + \rho^2\beta^2)^{\frac{1}{2}} < k - \rho\gamma h - \mu h, \\ &\Psi(t) > h\gamma(1 - h\mu),\ \rho h\gamma < k - \mu h, \end{aligned}\right\} \tag{3.1}$$

where $h = (1 - 2\lambda + \mu^2)^{-\frac{1}{2}} \geqslant 1$, there exist $x^* \in D$, $u^* \in T(x^*)$ and $v^* \in A(x^*)$ which are a solution of the generalized strongly nonlinear quasi-complementarity problem (2.1) such that $g(x_n) \to g(x^*)$, $u_n \to u^*$ and $v_n \to v^*$, where $\{x_n\}$, $\{u_n\}$ and $\{v_n\}$ are the sequences generated by Algorithm 3.1.

Proof By Algorithm 3.1 and Lemma 2.2, we have

$$\begin{aligned} &\| w_{n+1} - w_n \| = \| g(x_{n+1}) - g(x_n) \| = \\ &\| m(x_n) + P_K[g(x_n) - \rho(u_n + v_n) - m(x_n)] - m(x_{n-1}) - \\ &P_K[g(x_{n-1}) - \rho(u_{n-1} + v_{n-1}) - m(x_{n-1})] \| \leqslant \\ &\| m(x_n) - m(x_{n-1}) \| + \| g(x_n) - \rho(u_n + v_n) - m(x_n) - \\ &g(x_{n-1}) + \rho(u_{n-1} + v_{n-1}) + m(x_{n-1}) \| \leqslant \\ &\| m(x_n) - m(x_{n-1}) \| + \| g(x_n) - m(x_n) - g(x_{n-1}) + m(x_{n-1}) - \\ &\rho(u_n - u_{n-1}) \| + \rho \| v_n - v_{n-1} \|. \end{aligned}$$

By using the method of Li and Ding[15] and the H-Lipschitz continuity and Ψ-strong monotonicity of T with respect to $(g - m)$, we obtain

$$\begin{aligned} &\| g(x_n) - m(x_n) - g(x_{n-1}) + m(x_{n-1}) - \rho(u_{n-1} - u_{n-1}) \|^2 = \\ &\| g(x_n) - m(x_n) - g(x_{n-1}) + m(x_{n-1}) \|^2 - \end{aligned}$$

$$2\rho(u_n - u_{n-1}, g(x_n) - m(x_n) - g(x_{n-1}) + m(x_{n-1})) + \rho^2 \| u_n - u_{n-1} \|^2 \leqslant$$
$$\| g(x_n) - m(x_n) - g(x_{n-1}) + m(x_{n-1}) \|^2 -$$
$$2\rho\Gamma(\| g(x_n) - m(x_n) - g(x_{n-1}) + m(x_{n-1}) \|) \cdot \| g(x_n) - m(x_n) -$$
$$g(x_{n-1}) + m(x_{n-1}) \|^2 + \rho^2(H(T(x_n), T(x_{n-1})))^2 \leqslant$$
$$(1 - 2\rho\Gamma(\| g(x_n) - m(x_n) - g(x_{n-1}) + m(x_{n-1}) \|) +$$
$$\rho^2\beta^2) \cdot \| g(x_n) - m(x_n) - g(x_{n-1}) + m(x_{n-1}) \|^2. \tag{3.2}$$

Since m is μ- Lipschitz continuous with respect to g and λ- strongly monotone with respect to g, we have

$$\| g(x_n) - m(x_n) - g(x_{n-1}) + m(x_{n-1}) \|^2 =$$
$$\| g(x_n) - g(x_{n-1}) \|^2 - 2(g(x_n) - g(x_{n-1}), m(x_n) - m(x_{n-1})) + \| m(x_n) - m(x_{n-1}) \|^2$$
$$\leqslant \| g(x_n) - g(x_{n-1}) \|^2 - 2\lambda \| g(x_n) - g(x_{n-1}) \|^2 + \mu^2 \| g(x_n) - g(x_{n-1}) \|^2 =$$
$$(1 - 2\lambda + \mu^2) \| g(x_n) - g(x_{n-1}) \|^2. \tag{3.3}$$

By using the H-Lipschitz continuity of A with respect to g, we get

$$\| v_n - v_{n-1} \| \leqslant H(A(x_n), A(x_{n-1})) \leqslant \gamma \| g(x_n) - g(x_{n-1}) \|. \tag{3.4}$$

Therefore, we imply by (3.2)~(3.4)

$$\| w_{n+1} - w_n \| \leqslant$$
$$\mu \| g(x_n) - g(x_{n-1}) \| + (1 - 2\rho\Gamma(\| g(x_n) - m(x_n) - g(x_{n-1}) + m(x_{n-1}) \|) +$$
$$\rho^2\beta^2)^{\frac{1}{2}} \| g(x_n) - m(x_n) - g(x_{n-1}) + m(x_{n-1}) \| + \rho\gamma \| g(x_n) - g(x_{n-1}) \| \leqslant$$
$$\{\mu + [(1 - 2\rho\Gamma(\| g(x_n) - m(x_n) - g(x_{n-1}) + m(x_{n-1}) \|) +$$
$$\rho^2\beta^2)(1 - 2\lambda + \mu^2)]^{\frac{1}{2}} + \rho\gamma\} \| g(x_n) - g(x_{n-1}) \| \leqslant$$
$$(1 - 2\lambda + \mu^2)]^{\frac{1}{2}} \theta(w_n - w_{n-1}) \| w_n - w_{n-1} \|. \tag{3.5}$$

Since the condition (3.1) implies that $0 < (1 - 2\lambda + \mu^2)^{\frac{1}{2}} \leqslant 1$ and

$$\theta(w_n - w_{n-1}) = \mu(1 - 2\lambda + \mu^2)^{-\frac{1}{2}} + (1 - 2\rho\Gamma(\| g(x_n) -$$
$$m(x_n) - g(x_{n-1}) + m(x_{n-1}) \|) + \rho^2\beta^2)^{\frac{1}{2}} +$$
$$\rho\gamma(1 - 2\lambda + \mu^2)^{-\frac{1}{2}} < k < 1,$$

it follows from (3.5) that $\{w_n\}$ is the Cauchy sequence in K. Hence, we deduce that $w_n \to w \in K$ as $n \to \infty$. In view of $K \subset g(D)$, there exists $x^* \in D$ such that $w = g(x^*)$. Thus, $g(x_n) \to g(x^*) \in K$. Since (3.4) implies

$$\| v_n - v_{n-1} \| \leqslant \gamma \| w_n - w_{n-1} \|,$$

$\{v_n\}$ is the Cauchy sequence in H. It is easily seen that $\{u_n\}$ is also the Cauchy sequence in H. Consequently, $u_n \to u^* \in H$ and $v_n \to v^* \in H$ as $n \to \infty$. Let

$$w' = m(x^*) + P_K[g(x^*) - \rho(u^* + v^*) - m(x^*)].$$

Since

$$\| w_{n+1} - w' \| = \| g(x_{n+1}) - w' \| =$$
$$\| m(x_n) + P_K[g(x_n) - \rho(u_n + v_n) - m(x_n)] - m(x^*) -$$
$$P_K[g(x^*) - \rho(u^* + v^*) - m(x^*)] \| \leqslant$$

$$2\|m(x_n)-m(x^*)\|+\|g(x_n)-g(x^*)-\rho(u_n-u^*)-\rho(v_n-v^*)\|\leqslant$$
$$2\mu\|g(x_n)-g(x^*)\|+\|g(x_n)-g(x^*)-\rho(u_n-u^*)-\rho(v_n-v^*)\|,$$

we know that $w_n\to w'$. So, $w=w'$, i e,

$$g(x^*)=m(x^*)+P_K[g(x^*)-\rho(u^*+v^*)-m(x^*)]\in K(x^*). \tag{3.6}$$

Now we have to show that $u^*\in T(x^*)$ and $v^*\in A(x^*)$. Indeed, note that

$$d(u^*,T(x^*))\leqslant\|u^*-u_n\|+d(u_n,T(x^*))\leqslant$$
$$\|u^*-u_n\|+H(T(x_n)-T(x^*))\text{ (derived by [10])}\leqslant$$
$$\|u^*-u_n\|+\beta\|g(x_n)-m(x_n)-g(x^*)+m(x^*)\|\leqslant$$
$$\|u^*-u_n\|+\beta(1-2\lambda+\mu^2\|)^{\frac{1}{2}}\|g(x_n)-g(x^*)\|\leqslant$$
$$\|u^*-u_n\|+\beta\|g(x_n)-g(x^*)\|,$$

where $d(u^*,T(x^*))=\inf\{(\|u^*-u\|$, for each $u\in T(x^*)\}$. Hence, $d(u^*,T(x^*))=0$, which implies that $x^*\in T(x^*)$. Similarly, we can prove that $v^*\in A(x^*)$.

Finally, by Lemma 2.4 and 2.5, and the equality (3.6), we know that $x^*\in D$, $u^*\in T(x^*)$ and $v^*\in A(x^*)$ supply a solution of the generalized strongly nonlinear quasi-complementarity problem (2.1), $g(x_n)\to g(x^*)$, $u_n\to u^*$, and $v_n\to v^*$ as $n\to\infty$.

Algorithm 3.2[15] Let D be a nonempty subset of H, K be a closed convex cone of H, and $g:D\to H$ such that $g(D)$ is a convex subset of H and satisfies $K\subset g(D)$. Suppose that $m:D\to K$, T, $A:D\to C(H)$, and the sequences $\{\alpha_n\}$ and $\{\beta_n\}$ satisfy the condition that $0\leqslant\alpha_n,\beta_n\leqslant 1$ for each $n\geqslant 0$ and $\sum_{n=0}^{\infty}\alpha_n$ diverges. For any given $x_0\in D$, we take $\bar{u}_0\in T(x_0)$ and $\bar{v}_0\in A(x_0)$. Let

$$w_1'=(1-\beta_0)g(x_0)+\beta_0[m(x_0)+P_K(g(x_0)-\rho(\bar{u}_0+\bar{V}_0)-m(x_0))]\in g(D).$$

Then there exists $y_0\in D$ such that $g(y_0)=w_1'$. Since $\bar{u}_0\in T(x_0)\in C(H)$ and $\bar{v}_0\in A(x_0)\in C(H)$, by [7], there exist $u_0\in T(y_0)$ and $v_0\in A(y_0)$ such that

$$\|u_0-\bar{u}_0\|\leqslant H(T(x_0),T(y_0)),\quad\|v_0-\bar{v}_0\|\leqslant H(A(x_0),A(y_0)).$$

Let

$$w_1=(1-\alpha_0)g(x_0)+\alpha_0[m(y_0)+P_K(g(y_0)-\rho(u_0+v_0)-m(y_0))]\in g(D).$$

Then there exists $x_1\in D$ such that $g(x_1)=w_1$. By induction, we can obtain three sequences $\{u_n\}$, $\{v_n\}$ amd $\{x_n\}$:

$$u_n\in T(y_n),\ v_n\in A(y_n),\ \bar{u}_n\in T(x_n),\ \bar{v}_n\in A(x_n),$$
$$\|u_n-\bar{u}_n\|\leqslant H(T(y_n),T(x_n)),\quad\|v_n-\bar{v}_n\|\leqslant H(A(y_n),A(x_n)).$$
$$g(x_{n+1})=(1-\alpha_n)g(x_n)+\alpha_n[m(y_n)+P_K(g(y_n)-\rho(u_n+v_n)-m(y_n))],$$
$$g(y_n)=(1-\beta_n)g(x_n)+\beta_n[m(x_n)+P_K(g(x_n)-\rho(\bar{u}_n+\bar{v}_n)-m(x_n))],$$

$n=0,1,2,\cdots$, where $\rho>0$ is a constant.

Theorem 3.2 Let D be a nonempty subset of H, K be a closed convex cone of H, $m:D\to K$, T, $A:D\to C(H)$ and $g:D\to H$ such that $g(D)$ is a convex subset of H and $K\subset g(D)$. Assume that $T:D\to C(H)$ is Ψ-strongly monotone with respect to $(g-m)$ and H-Lipschitz continuous with respect to $(g-m)$ where the H-Lipschitz constant is $\beta>0$, $A:D\to C(H)$ is H-Lipschitz continuous with respect to g where the H-Lipschitz constant is $\gamma>0$ and $m:D\to K$ is

μ- Lipschitz continuous with respect to g and λ- strongly monotone with respect to g where μ and λ are positive constants. Suppose the condition (3. 1) in Theorem 3. 1 holds. If (x^*, u^*, v^*) is a solution of the generalized strongly nonlinear quasi-complementarity problem $(T, A, g; K(x))$ (2. 1), and $\{x_n\}$, $\{u_n\}$ and $\{v_n\}$ are the sequences generated by Algorithm 3. 2, then $g(x_n) \to g(x^*)$, $u_n \to u^*$ and $v_n \to v^*$.

Remark Using the same method as in the proof of Theorem 3. 1, we can prove Theorem 3. 2.

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关于求广义强非线性补问题和拟补问题的逼近解的迭代算法

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提　要　研究求广义强非线性补问题和拟补问题的逼近解的迭代算法．作为特例，概括了该领域中一些熟知的结果．我们的结果推广、改进和补充了先前由几位作者包括 Noor，Chang 与 Huang 及 Li 与 Ding 得到的早期和最近的结果．

关键词　补问题；拟补问题；闭凸锥；关于 g 的 H- Lipschitz 连续映象；关于 g 的 Ψ-强单调映象

中图法分类号　O177.92；O178