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SCALARIZATION AND GROUP DECISION MAKING

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Scalarization means the replacement of a multiobjective optimization problem by a scalar objective optimization problem. It is known that even though the dominance structure is a complete ordering set(for example, lexicographic ordering [1]), the scalar objective may not exist. This paper will present the conditions of existence with general binary relation and vector ordering without any convexity assumptions for objectives or constraints which generalize the assumptions (closed convex constant dominance structure) of Jahn's results[2]. This paper utilizes this scalar objective to present two ideas of group decision making which include Tanino-Nakayama-Sawaragi's results as a special case[3].

SCALARIZATION

Let Y be a real linear space, $S\subset Y$. The core of S is given as cor $S=\{y\in S/$ for any $z\in Y$, there exists $\alpha_0>0$ such that $y+\alpha z\in S, \forall\alpha\in[0,\alpha_0]\}$. For any real scalar λ, let $\lambda S=\{\lambda y/y\in S\}$.

Lemma 1. *Suppose that* $S\subset Y, \lambda S\subset \text{cor}S, \forall\ \lambda\in[0,1)$. *Then*

$$u(y)=\inf\{\lambda/\lambda>0, y\in\lambda\ S\}\geqslant 0, \forall\ y\in Y,$$

and

$$\text{cor}\ S=\{y\in Y/u(y)<1\}\subset S\subset\{y\in Y/u(y)\leqslant 1\}.$$

Proof. Since $0\in$ cor S, so for any $y\in Y$, there exists $\alpha_0>0$ such that $y\in\frac{1}{\alpha}S$, $\forall\alpha\in(0,\alpha_0]$, $u(y)\geqslant 0$.

Let $y\in$ cor S. There exists $\alpha>0$, such that $y+\alpha y\in S, u(y)\leqslant\frac{1}{1+\alpha}<1$. Let $y\in Y$, $u(y)<1$. By definition of infinimum, there exists $\lambda\in(0,1)$ such that $y\in\lambda S\subset$ cor S. The inequality on the right hand side is obvious by definition of $u(y)$.

For two subsets $S, S'\subset Y$, let $S+S'=\{y+y'/y\in S, y'\in S'\}$. For any $y\in Y, S\subset Y$, let $y+S=\{y+z/z\in S\}$.

Lemma 2. *Let* $S\subset Y$, cor $S\neq\phi$.

(1) If $\lambda S\subset S, \forall\lambda>0$ *and* $y\in$ cor S, *then* $\lambda y\in \text{cor}S, \forall\lambda>0$.

(2) If $S+S\subset S$, *then* $S+\text{cor}S\subset\text{cor}S$.

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(3) *Let the assumptions of* (1),(2) *be satisfied. Then*

$$\mu[(-y+S)\cap(y-S)]\subset(-y+\mathrm{cor}S)\cap(y-\mathrm{cor}S),\forall\mu\in[0,1).$$

Proof. (1) Let $y_1\in S, y_2\in \mathrm{cor}S$. For any $z\in Y$, there exists $\alpha_0>0$ such that $y_2+\alpha z\in S, \forall\ \alpha\in[0,\alpha_0]$, and $y_1+y_2+\alpha z\in S+S\subset S, \forall\ \alpha\in[0,\alpha_0]$, i.e., $y_1+y_2\in\mathrm{cor}S$.

(2) Let $y\in \mathrm{cor}\ S$. For any $z\in Y$, there exists $\alpha_0>0$ such that $y+\alpha z\in S$, $\forall\alpha\in[0,\alpha_0]$. For any definite $\lambda>0, \lambda y+\lambda\alpha z\in\lambda S\subset S, \forall\lambda\alpha\in[0,\lambda\alpha_0]$. So $\lambda y\in\mathrm{cor}S$.

(3) Let $y\in\mathrm{cor}S$. For any $z\in S, \mu\in(0,1), \mu(y-z)=y-(1-\mu)y-\mu z\in y-\mathrm{cor}S-S\subset y-\mathrm{cor}S$. $\mu(z-y)\in -y+\mathrm{cor}S$. So(3) is obtained.

Lemma 3. *Suppose that* $S\subset Y$ *is a convex set*, $0\in\mathrm{cor}S$. *Then* $\lambda S\subset\mathrm{cor}S$, $\forall\lambda\in[0,1)$, *and* $u(y)$ *in Lemma* 1 *is a positive homogeneous convex function on* Y.

Proof. (1) For any $z\in Y$, there exists $\alpha_0>0$ such that $\alpha z\in S, \forall\alpha\in[0,\alpha_0]$. For any $y\in S$, $\lambda\in(0,1)$, we have $\lambda y+(1-\lambda)\alpha z\in S$, $\forall(1-\lambda)\alpha\in[0,(1-\lambda)\alpha_0]$. So $\lambda y\in\mathrm{cor}S, \forall\lambda\in[0,1)$.

(2) For any $\alpha>0, y\in Y$,

$$u(\alpha y)=\alpha\inf\left\{\frac{\lambda}{\alpha}\Big/\lambda>0,\ y\in\frac{\lambda}{\alpha}\right\}S=\alpha u(y).$$

For any $y,z\in Y$ and arbitrary definite $\varepsilon>0$, there exist $\lambda_1>0,\lambda_2>0$ such that $0<u(y)<\lambda_1<u(y)+\varepsilon$ and $0<u(z)<\lambda_2<u(z)+\varepsilon, y\in\lambda_1 S, z\in\lambda_2 S$. So

$$\frac{1}{\lambda_1+\lambda_2}(y+z)=\frac{\lambda_1}{\lambda_1+\lambda_2}\frac{y}{\lambda_1}+\frac{\lambda_2}{\lambda_1+\lambda_2}\frac{z}{\lambda_2}\in S,$$

$$u(y+z)\leqslant\lambda_1+\lambda_2<u(y)+u(z)+2\varepsilon.$$

Let $\varepsilon\to0$. Hence $u(y+z)\leqslant u(y)+u(z)$.

Let $\succ$ be a binary relation on Y, $F\subset Y$. $\bar y\in F$ is called a minimal point of F with $\succ$, if there does not exists any $y\in F$, $y\neq\bar y$, such that $\bar y-y\succ0$. Let X be a real linear space, $R\subset X$. Suppose there is a mapping $f:R\to Y$. The mapping set is $f(R)=\{f(x)/x\in R\}$. $\bar x\in R$ is called a minimal solution of f on R with $\succ$, if there does not exists any $x\in R, f(x)\neq f(\bar x)$, such that $f(\bar x)-f(x)\succ0$.

For any $a,b\in Y$, let

$$[a,b]=\{y\in Y/y-a\succcurlyeq0, b-y\succcurlyeq0\}.$$

Theorem 1. *Let* $\succcurlyeq$ *be a binary relation on* Y, *and* $\bar y\in F\subset Y$ *be a minimal point of* F *with* $\succ$. *If there exists* $\hat y\in Y$, $b=\bar y-\hat y\neq0$, *such that*

(1) $\lambda[-b,b]\subset\mathrm{cor}[-b,b]$, $\forall\lambda\in[0,1)$;

(2) $b\in[-b,b], b\notin\mathrm{cor}[-b,b]$,

then

$$\min_{y\in F}u(y-\hat y)=u(\bar y-\hat y)=1,$$

where

$$u(y) = \inf\{\lambda/\lambda > 0, y \in \lambda[-b, b]\}.$$

Proof. Let $S = [-b, b]$ in Lemma 1. Then $u(b) = 1$ by assumptions (1) and (2). Since $\bar{y}$ is a minimal point, so there is no $y \in F$, $y \neq \bar{y}$, such that $b - (y - \hat{y}) = \bar{y} - y \geqslant 0$, i.e., there is no $y \in F, y \neq \bar{y}$ such that $y - \hat{y} \in [-b, b]$. So $(F - \hat{y}) \cap \text{cor}[-b, b] = \phi$; $u(y - \hat{y}) \geqslant 1, \forall y \in F$ by Lemma 1.

Corollary 1. Let $\geqslant$ be a binary relation on Y. $\bar{x} \in R \subset X$ is a minimal solution of f on R with $\geqslant$. If there exists $\hat{y} \in Y$, $b = f(\bar{x}) - \hat{y} \neq 0$, such that the assumptions (1) and (2) in Theorem 1 are valid, then

$$\min_{x \in R} u(f(x) - \hat{y}) = u(f(\bar{x}) - \hat{y}) = 1.$$

For any $y \in Y$, the subset $D(y) = \{z - y/z \in Y, z - y \geqslant 0\}$ is called a domination structure of Y at y. Actually, for any $y, z \in Y, z - y \geqslant 0$ if and only if $z \in y + D(y)$. The minimal point (solution) of F with $\geqslant$ is also called nondominated point (solution) of F with $\bigcup_{y \in Y} D(y)$. Meanwhile,

$$\text{cor}[a, b] = \{y \in Y / y \in [a + \text{cor}D(a)] \cap [b - \text{cor}D(y)]\},$$

$$[a, b] = \{y \in Y / y \in [a + D(a)] \cap [b - D(y)]\}.$$

Lemma 4. *Let $a, b \in Y$.*

(1) *If $-a \in \text{cor}D(a), b \in \text{cor}D(0)$, then $0 \in \text{cor}[a, b]$.*

(2) *If $0 \in D(b), b - a \in D(a)$, then $b \in [a, b]$; if $0 \in D(a)$, $b - a \in D(a)$, then $a \in [a, b]$.*

(3) *If $0 \notin \text{cor}\ D(b)$, or $b - a \notin \text{cor}D(a)$, then $b \notin \text{cor}[a, b]$; if $0 \notin \text{cor}D(a)$, or $b - a \notin \text{cor}D(a)$, then $a \notin \text{cor}[a, b]$.*

(4) *If for any $y, z \in Y, \alpha D(y) + (1 - \alpha)D(z) \subset D(\alpha y + (1 - \alpha)z)$, $\forall \alpha \in [0, 1]$, then $[a, b]$ is a convex set.*

Proof. (1),(2),(3) are obvious by definitions of core and $D(y)$.

(4) For any $y_1, y_2 \in [a, b], \alpha \in [0, 1]$,

$$\alpha y_1 + (1 - \alpha)y_2 \in [a + D(a)] \cap [b - \alpha D(y_1) - (1 - \alpha)D(y_2)] \subset [a + D(a)] \cap [b - D(\alpha y_1 + (1 - \alpha)y_2)],$$

i.e., $\alpha y_1 + (1 - \alpha)y_2 \in [a, b]$.

Corollary 2. Let $\bar{y} \in F \subset Y$ be a nondominated point of F with $\bigcup_{y \in Y} D(y)$, and

(1) $\alpha D(y) + (1 - \alpha)D(z) \subset D(\alpha y + (1 - \alpha)z), \forall y, z \in \forall$, $\forall \alpha \in [0, 1]$;

(2) there exists $\hat{y} \in Y$, $b = \bar{y} - \hat{y} \neq 0$, such that $D(-b)$ is a cone, $0 \in D(b)$, $0 \notin \text{cor}\ D(b), b \in \text{cor}D(0) \cap \text{cor}D(-b)$. Then

$$\min_{y \in Y} u(y - \hat{y}) = u(\bar{y} - \hat{y}) = 1,$$

where

$$u(y) = \inf\{\lambda/\lambda > 0, y \in \lambda[(-b + D(-b)) \cap (b - D(y/\lambda))]\}.$$

Proof. Let $a = -b$ in Lemma 4 and $S = [-b, b]$ in lemma 3. Then the conclusion can be obtained easily.

Corollary 3. Suppose that $D(y)$ is independent of y, i.e., $D(y) \equiv D$, $\forall y \in Y$. $\bar{y} \in F$ is a minimal point of F with D, D being a convex cone, $0 \in D, 0 \notin \text{cor}D$. If there exists $\hat{y} \in Y, b = \bar{y} - \hat{y} \neq 0$, such that $b \in \text{cor}D$, then

$$\min_{y \in F} u(y - \hat{y}) = u(\bar{y} - \hat{y}) = 1,$$

where

$$u(y) = \inf\{\lambda/\lambda > 0, y \in \lambda[(-b + D) \cap (b - D)]\}$$

and $u(y)$ is a positive homogeneous convex function on Y. Furthermore, if $F \subset \hat{y} + D$, then for any y, $z \in Y$,

$$z \succ y (\text{i.e., } z \in y + D) \Rightarrow u(z - \hat{y}) \geqslant u(y - \hat{y}).$$

Proof. The first conclusion is a straight consequence of Corollary 2. For the second conclusion, we only need prove that when $y, z \in F, z \in y + D, z - \hat{y} \in \lambda \; [(-b + D) \cap (b - D), \lambda > 0$, we have $y - \hat{y} \in \lambda[(-b + D) \cap (b - D)]$. Since $b \in D, y - \hat{y} \in D = \lambda D = \lambda(-b + b + D) \subset \lambda(-b + D), y - \hat{y} \in z - \hat{y} - D \in \lambda(b - D) - D = \lambda(b - D) - \lambda D = \lambda(b - D)$, so $y - \hat{y} \in \lambda[(-b + D) \cap (b - D)]$.

Remark. The assumptions of Corollary 3 are weaker than Jahn's theorem (without closedness and pointedness of D).

The corollaries about the nondominated solution can be established in correspondence with Corollaries 2 and 3.

Suppose that the binary relation $\succ$ has the following properties:

(I) Addition invariant: For any $y_1, y_2, y_3 \in Y$,

$$y_1 \succ y_2 \Rightarrow y_1 + y_3 \succ y_1 + y_3.$$

(II) Multiple invariant: For any $y_1, y_2 \in Y$, and real scalar $\alpha \geqslant 0$,

$$y_1 \succ y_2 \Rightarrow \alpha y_1 \succ \alpha y_2.$$

$z \succ y$ may be denoted by $y \prec z$. Then for any $a, b \in y$, $b - a \succ 0$ is equivalent to $b \succ a$, and $[a, b] = \{y \in Y/a \prec y \prec b\}$.

Lemma 5. *Let* $b \in Y, b \succ 0, b \neq 0, 0 \in \text{cor}[-b, b]$. *Then*

(1) $[-b, b]$ *is a convex set, and* $u(y) = \inf\{\lambda/\lambda > 0, y \in \lambda[-b, b]\}$ *is a convex function on* Y;

(2) $\text{cor}[-b, b] = \{y \in Y/u(y) < 1\} \subset [-b, b] \subset \{y \in Y/u(y) \leqslant 1\}$;

(3) $u(b) = u(-b) = 1$ *when* $\succ$ *is reflexive* ($y \succ y$, $\forall y \in Y$) *and antisymmetric* ($y_1 \succ y_2, y_2 \succ y_1 \Rightarrow y_1 = y_2$);

(4) *for any* $y, z \in Y, 0 \prec y \prec z \Rightarrow u(y) \leqslant u(z)$ *when* $\succ$ *is transitive* ($y_1 \succ y_2, y_2 \succ y_3 \Rightarrow y_1 \succ y_3$).

Proof. (1), (2) are true by lemmas 1,3 and (I), (II). $\pm b \in [-b, b]$ by (I) and reflexivity. If $b \in \mathrm{cor}[-b,b]$, then there exists $\alpha > 0$ such that $b + \alpha b \preccurlyeq b$. Hence $b \preccurlyeq 0$, $b = 0$ by antisymmetry. This is a contradiction. So $b \notin \mathrm{cor}[-b,b]$, $u(b) = 1$. Similarly, $u(-b) = 1$. (3) holds.

Since $0 \in \mathrm{cor}[-b, b]$, for any $z \in Y$, there exists $\lambda > 0$ such that $z \in \lambda[-b, b]$. When $0 \preccurlyeq y \preccurlyeq z, \lambda > 0, z \in \lambda[-b, b]$, we have $y \preccurlyeq \lambda b$ by transitivity. So $-\lambda b \preccurlyeq 0 \preccurlyeq y \preccurlyeq \lambda b, y \in \lambda[-b,b]$. (4) holds.

Theorem 2. *Let* $\succcurlyeq$ *has properties* (I) *and* (II). $\bar{y} \in F \subset Y$. $\hat{y} \in Y$, $0 \in \mathrm{cor}[\hat{y} - \bar{y}, \bar{y} - \hat{y}]$. *Then*

(1) $u(y) = \inf\{\lambda / y \in \lambda[\hat{y} - \bar{y}, \bar{y} - \hat{y}], \lambda > 0\}$ *is convex on* Y.

(2) *If* $\succcurlyeq$ *is transitive, then* $u(y)$ *is monotone increasing* ($y, z \in Y$, $0 \preccurlyeq y \preccurlyeq z \Rightarrow u(y) \leqslant u(z)$); *if* $y \succcurlyeq \hat{y}, \forall y \in F$, then $y, z \in F, y \preccurlyeq z \Rightarrow u(y - \hat{y}) \leqslant u(z - \hat{y})$.

(3) *If* $\succcurlyeq$ *is reflexive and antisymmetric and* $\bar{y} \in F$ *is a minimal point, then*

$$\min_{y \in F} u(y - \hat{y}) = u(\bar{y} - \hat{y}) = 1.$$

Proof. Let $b = \bar{y} - \hat{y}$, (1) holds by lemma 5(1). Since $\bar{y}$ is a minimal point, so for any $y \in F$, $y \neq \bar{y}$, $y - \hat{y} \notin \mathrm{cor}[-b,b]$. Hence $u(y - \hat{y}) \geqslant 1 = u(\bar{y} - \hat{y})$, $\forall y \in F$. by Lemma 5(2),(3).(3) holds. (2) holds by Lemma 5(4).

Corollary 4. Let $D \subset Y$ be a pointed convex cone, cor $D \neq \phi$. $y \preccurlyeq z \Longleftrightarrow z \in y + D$. $\bar{y} \in F \subset Y$. $\hat{y} \in Y$. $\bar{y} \in \hat{y} + \mathrm{cor}D$. Then

(1) $u(y) = \inf\{\lambda / y \in \lambda[(\hat{y} - \bar{y} + D) \cap (\bar{y} - \hat{y} - D)], \lambda > 0\}$ is convex on Y.

(2) For any $y, z \in Y, y \in D \cap (z - D)$, there must be $u(y) \leqslant u(z)$; if $F \subset \hat{y} + D$, then for any $y, z \in F, z \in y + D$, there must be $u(y - \hat{y}) \leqslant u(z - \hat{y})$.

(3) If $\bar{y} \in F$ is a nondominated point of F with D, then

$$\min_{y \in F} u(y - \hat{y}) = u(\bar{y} - \hat{y}) = 1.$$

Theorem 3. (Converse of Theorem 2). *Let* $\succcurlyeq$ *be a binary relation on* Y. $\bar{y} \in F \subset T$, $\hat{y} \in Y, 0 \in \mathrm{cor}[\hat{y} - \bar{y}, \bar{y} - \hat{y}]$. $\min_{y \in F} u(y - \hat{y}) = u(\bar{y} - \hat{y})$, *where* $u(y) = \inf\{\lambda / y \in \lambda[\hat{y} - \bar{y}, \bar{y} - \hat{y}], \lambda > 0\}$. *If* $\succcurlyeq$ *is strictly monotone increasing at* $\bar{y}$ (*for any* $y \in F$, $y \preccurlyeq \bar{y}$, $y \neq \bar{y} \Rightarrow u(y - \hat{y}) < u(\bar{y} - \hat{y})$), *then* $\bar{y}$ *is a minimal point of* F *with* $\succcurlyeq$.

Two Ideas of Group Decision Making

Lemma 6. *Let* $S_i \subset Y, 0 \in \mathrm{cor}S_i (i = 1, \cdots, r)$. $S = \bigcap_{i=1}^{r} S_i$.

$u(y) = \inf\{\lambda / y \in \lambda S, \lambda > 0\}, u_i(y) = \inf\{\lambda / y \in \lambda S_i, \lambda > 0\}, i = 1, \cdots, r$. *Then*

(1) $u(y) \geqslant \max_{1 \leqslant i \leqslant r} u_i(y), \forall y \in Y$.

(2) *If* $S_i (i = 1, \cdots, r)$ *are convex, then*

$$u(y) = \max_{1 \leqslant i \leqslant r} u_i(y), \forall y \in Y.$$

Proof. (1) is obvious by the definitions of $u(y)$ and $u_i(y)$. For arbitrary definite $\varepsilon > 0$, there exist $\lambda_i > 0$, $y \in \lambda_i S(i = 1, \cdots, r)$ such that

$$\lambda_i < u_i(y) + \varepsilon \leqslant \max_{1 \leqslant i \leqslant r} u_i(y) + \varepsilon (i = 1, \cdots, r).$$

If S_i is convex, $0 \in S_i(i = 1, \cdots, r)$, then

$$y \in \lambda_i S_i = (\max_{1 \leqslant i \leqslant r} \lambda_i) \frac{\lambda_i}{\max\limits_{1 \leqslant i \leqslant r} \lambda_i} S_i \subset (\max_{1 \leqslant i \leqslant r} \lambda_i) S_i, i = 1, \cdots, r.$$

Hence

$$u(y) \leqslant \max_{1 \leqslant i \leqslant r} \lambda_i < \max_{1 \leqslant i \leqslant r} u_i(y) + \varepsilon.$$

Let $\varepsilon \to 0$. Then (2) holds.

When r decision makers (DM) have their own preferences $\succ_i (i=1, \cdots, r)$, what group preferences will be possible?

Suppose that $\bar{y}^i \in F \subset Y$ is a minimal point of F with $\succ_i$. There exists $\hat{y}^i \in Y$ such that $y \succ_i \hat{y}^i, \forall y \in F$. $0 \in \text{cor}[\hat{y}^i - \bar{y}^i, \bar{y}^i - \hat{y}^i]$. By Lemma 6 and Theorem 2, let $S_i = [\hat{y}^i - \bar{y}^i, \bar{y}^i - \hat{y}^i]$, $S = \bigcap_{i=1}^{r} S_i$. We may define a group preference:

$$y \prec z \Longleftrightarrow \max_{1 \leqslant i \leqslant r} u_i(y - \hat{y}^i) \leqslant \max_{1 \leqslant i \leqslant r} u_i(z - \hat{y}^i),$$

where $u_i(y) = \inf\{\lambda / y \in \lambda S_i, \lambda > 0\}$. The optimal solution $\bar{y}$ of

$$\min_{y \in F} \max_{1 \leqslant i \leqslant r} u_i(y - \hat{y}^i) = \max_{1 \leqslant i \leqslant r} u_i(\bar{y} - \hat{y}^i)$$

can be considered as a preference point of F with $\succ$. When $F = f(R)$, the optimal solution $\bar{x}$ of

$$\min_{x \in R} \max_{1 \leqslant i \leqslant r} u_i(f(x) - \hat{y}^i) = \max_{1 \leqslant i \leqslant r} u_i(f(\bar{x}) - \hat{y}^i)$$

is a preference solution of $f(x)$ on R with $\succ$.

Consider $V\text{-}\min_{x \in R}(u_1(f(x)), \cdots, u_r(f(x)))$. Each DM should give a weighted vector $w^i \in E_r$ that reflects the strength of his own veto right. For example (in[3]), $w^i = (1, \cdots, 1, 1 + \lambda_i, 1, \cdots, 1)^T$, where $\lambda_i > 0$ is the strength of the i-th DM's veto right. The group preference can be defined by

$$x^1, x^2 \in R, x^1 \prec x^2 \Longleftrightarrow w^{iT} u(f(x^1)) \leqslant w^{iT} u(f(x^2)), i = 1, \cdots, r,$$

where $u(f) = (u_1(f), \cdots, u_r(f))^T$. So the group domination structure is a convex cone $D = \{d / w^{iT} d \geqslant 0, i = 1, \cdots, r\}$. The polar cone of D is $D^* = \left\{\sum_{i=1}^{r} \alpha_i w^i / \forall \alpha_i \geqslant 0\right\}$. For any $\alpha_i > 0 (i = 1, \cdots, r)$, the optimal solution of

$$\min_{x \in R} \left(\sum_{i=1}^{r} \alpha_i w^i \right)^T u(f(x))$$

can be considered as a preference solution of $f(x)$ on R with $\succ$.

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标 量 化 与 群 决 策

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摘 要

多目标问题是否可以用单目标标量函数的极值问题来代替，是个基本理论问题．本文在对目标与约束不加任何凸性或光滑要求的前提下，对偏好关系为一般二元关系以及向量半序的情况，讨论了标量函数存在的条件，并由此导出一种群决策的偏好关系和一种控制结构．