A Relationship between the Bowen Ratio and Sea–Air Temperature Difference under Unstable Conditions at Sea

S. A. HSU

Coastal Studies Institute, Louisiana State University, Baton Rouge, Louisiana

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ABSTRACT

At the air–sea interface, estimates of evaporation or latent heat flux and the Monin–Obukhov stability parameter require the measurements of dewpoint (T_{dew}) or wet-bulb temperature, which are not routinely available as compared to those of air (T_{air}) and sea surface temperature (T_{sea}). On the basis of thermodynamic considerations, this paper first postulates that the quantity of ($q_{sea} - q_{air}$) for the difference in specific humidity between the sea surface and its overlying air is related to the quantity of ($T_{sea} - T_{air}$). Using hourly measurements of all three temperatures, that is, T_{sea}, T_{air}, and T_{dew} from a buoy in the Gulf of Mexico under a severe cold air outbreak, a linear correlation between ($q_{sea} - q_{air}$) and ($T_{sea} - T_{air}$) does exist with a compelling high correlation coefficient, r, of 0.98 between these two quantities. Second, based on this Clausius-Clapeyron effect, the Bowen ratio B is proposed to relate to the quantity of ($T_{sea} - T_{air}$) only such that $B = a(T_{sea} - T_{air})^b$. Using all data for these three temperatures available from four stations in the Gulf from 1993 through 1997 reveal that for deepwater a varies from 0.077 to 0.078, b from 0.67 to 0.71, and r from 0.85 to 0.89. Similar equations for the nearshore region are also provided. Limited datasets from the open ocean also support this generic relationship between B and the quantity of ($T_{sea} - T_{air}$).

1. Introduction

At the air–sea interface under unstable conditions when the sea is warmer than the air, the sensible heat flux H_s is defined as [see, e.g., Smith 1980, Eq. (4)]

$$H_s = \rho C_p C_T (T_{sea} - T_{air}) U_{10}, \tag{1}$$

where ρ is the air density and C_p the specific heat capacity at constant pressure, T_{sea} is the ‘‘bucket’’ seawater temperature in the wave-mixed layer, T_{air} is the mean air temperature at the 10-m reference height, C_T is the sensible heat flux coefficient, and U_{10} is the wind speed at the 10-m reference height.

The latent heat flux H_l is defined as (Roll 1965)

$$H_l = L_T E = L_T C_E \rho (q_{sea} - q_{air}) U_{10}, \tag{2}$$

where L_T is the latent heat of vaporization, E is the evaporation, C_E is the latent heat flux coefficient, and q_{sea} and q_{air} are the specific humidity for the sea and air, respectively.

Since the total heat flux requires the addition of H_s and H_l and since values of q need the input of vapor pressure, which is not always measured, the so-called Bowen ratio B has been suggested (see, e.g., Roll 1965) and from Eqs. (1) and (2), we have

$$B = \frac{H_s}{H_l} = \frac{C_p C_T (T_{sea} - T_{air})}{L_T C_E (q_{sea} - q_{air})}, \tag{3}$$

where B may vary from 0.1 to 0.3 in the Tropics (e.g., Pond et al. 1971) and from 0.61 to 0.78 during cold air outbreaks over a midlatitude coastal water (Chou et al. 1986).

The Bowen ratio in the marine environment has been studied by many investigators (see, e.g., Roll 1965; Pond et al. 1971; Kondo 1976; Hicks and Hess 1977; Rao et al. 1986; Liu and Niiler 1990; Konda and Imasato 1996). The spatial variation of B is large, particularly for shallow seas during winter when cold air outbreaks occur frequently. An example is shown in Fig. 1. During the Air Mass Transformation Experiments (AMTEX) from 14–28 February 1974, in the sea areas of the southwest islands of Japan, the value of B was about 0.7–0.8 over the Yellow Sea, whereas it was 0.3–0.4 over the AMTEX region, and 0.1–0.2 over the subtropical Pacific Ocean. Kondo (1976) speculated that the saturation vapor pressure (e_s) is very low at low temperatures, but as the temperature increases, e_s increases exponentially, so that the ratio of the sea–air vapor pressure difference to the sea–air temperature difference takes a large value for high temperatures. The main purpose of this research is to further substantiate this Clausius-Clapeyron effect physically and mathematically.

Corresponding author address: Dr. S. A. Hsu, Coastal Studies Institute, 308 Howe-Russell Geoscience Bldg., Louisiana State University, Baton Rouge, LA 70803.
E-mail: sahsu@antares.esl.lsu.edu

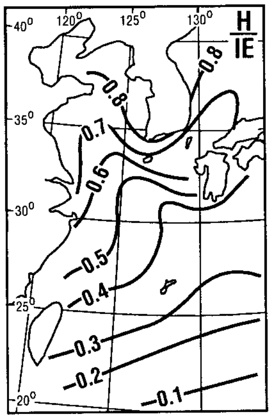

FIG. 1. The mean distribution of Bowen ratio for the entire period of AMTEX 1974 over the East China Sea region (after Kondo 1976).

The total heat flux H_{total} can be estimated from B and H_s and from Eq. (3),

$$H_{total} = H_s + H_l = H_s\left(1 + \frac{H_l}{H_s}\right) = \left(1 + \frac{1}{B}\right)H_s. \quad (4)$$

Also, in the atmospheric surface boundary layer the value of B is needed in order to compute the Monin–Obukhov stability length, which takes the form (see Panofsky and Dutton 1984, p. 132)

$$L = -\frac{u_*^3 C_p \rho T_{air}}{\kappa_a g H_s\left(1 + \frac{0.07}{B}\right)}, \quad (5)$$

where u_* is the friction velocity, κ_a is the von Kármán constant, and g is the gravitational acceleration.

The objectives of this paper are 1) to offer a possible explanation regarding the variation of B based on thermodynamic considerations, 2) to formulate a relationship between $(T_{sea} - T_{air})$ and $(q_{sea} - q_{air})$, and 3) to develop a formula that relates B to $(T_{sea} - T_{air})$ only so that one may ‘‘bypass’’ the requirement of $(q_{sea} - q_{air})$ measurements.

2. Thermodynamic considerations

At the sea surface, the specific humidity q_{sea} is related to the saturation vapor pressure e_{sea} through (see, e.g., Hsu 1988, pp. 20–21)

$$q_{sea} = 0.62\frac{e_{sea}}{P}, \quad (6)$$

where

$$e_{sea} = 6.1078 \times 10^{[7.5T_{sea}/(237.3+T_{sea})]}, \quad (7)$$

and p is the atmospheric pressure. Thus, q_{sea} is related nonlinearly to T_{sea}.

Similarly,

$$q_{air} = 0.62\frac{e_{air}}{P}, \quad (8)$$

where

$$e_{air} = 6.1078 \times 10^{[7.5T_{dew}/(237.3+T_{dew})]}, \quad (9)$$

in which T_{dew} is the dewpoint temperature in degrees Celsius. So, q_{air} is related to T_{dew} nonlinearly.

The problem now is to find a relationship between T_{dew} and T_{air} so that q_{air} can be related to T_{air}. This is accomplished as follows.

For the dry adiabatic lapse rate (see, e.g., Hsu 1988, pp. 23–24),

$$\frac{dT_{air}}{dz} = -\frac{g}{C_p} = -\frac{0.98\ \mathrm{K}}{100\ \mathrm{m}}, \quad (10)$$

where z is the altitude. In a well-mixed atmospheric boundary layer from surface to the lifting condensation level (LCL: Hsu 1988, pp. 26–27),

$$T_{air\,LCL} - T_{air\,sfc} = -\frac{0.98\ \mathrm{K}}{100\ \mathrm{m}}H_{LCL} \quad (11)$$

where H_{LCL} is the height of the LCL. For dewpoint lapse rate (see, e.g., McIlveen 1986, p. 151),

$$\frac{dT_{dew}}{dz} = -\frac{g}{L_T}\frac{R_v}{R}T_{dew}, \quad (12)$$

where R_v and R is the gas constant for water vapor and dry air, respectively. For typical low tropospheric T_{dew} between 283 K (or 10°C) and 293 K (or 20°C), Eq. (12) becomes approximately

$$\frac{dT_{dew}}{dz} = -\frac{0.18\ \mathrm{K}}{100\ \mathrm{m}}, \quad (13)$$

or

$$T_{dew\,LCL} - T_{dew\,sfc} = -\frac{0.18\ \mathrm{K}}{100\ \mathrm{m}}H_{LCL}. \quad (14)$$

At the LCL, $T_{dew\,LCL} = T_{LCL}$, and Eqs. (14)–(11) become

$$T_{air\,sfc} - T_{dew\,sfc} = \frac{0.80\ \mathrm{K}}{100\ \mathrm{m}}H_{LCL} \quad (15)$$

$$\therefore\ H_{LCL} = 125(T_{air\,sfc} - T_{dew\,sfc}), \quad (16)$$

where H_{LCL} is in meters and the dewpoint depression at the surface is in degrees Celsius.

Therefore, T_{dew} is linearly related to T_{air} near the sea surface through the parameterization of H_{LCL}. From Eqs. (8), (9), and (16), it is inferred that q_{air} is related nonlinearly to T_{air} so that the composite quantity of $(q_{sea} - q_{air})$ may also be related nonlinearly to that of $(T_{sea} - T_{air})$. A proof of this idea is done in the next section.

3. Field results

Before the field results shown in Fig. 2 are presented, a relationship between the quantity $(q_{sea} - q_{air})$ as a

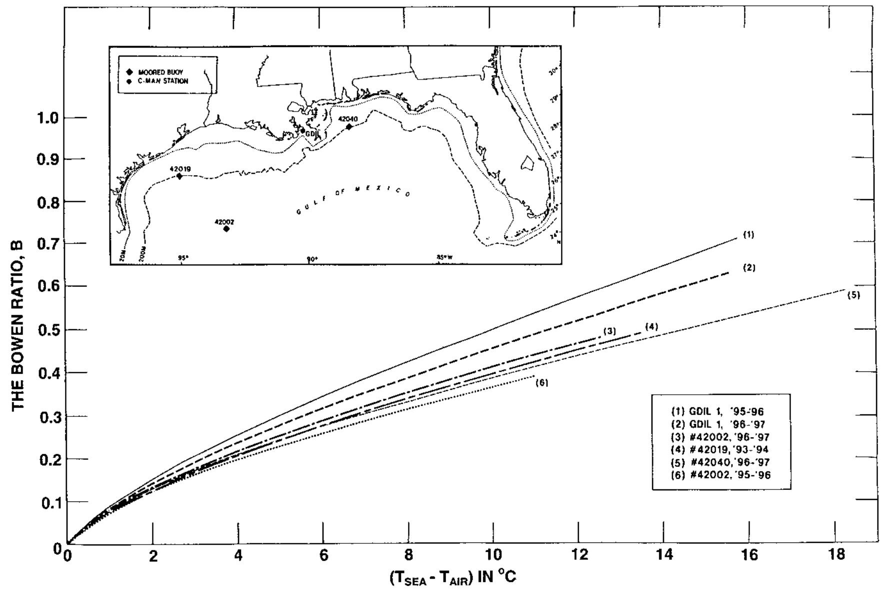

FIG. 2. Relationships between the Bowen ratio B and the quantity $(T_{sea} - T_{air})$ for the Gulf of Mexico (see text for explanation). The stations used are shown in the insert.

function of the quantity $(T_{sea} - T_{air})$ must be verified for the idea as discussed above. Between 18 and 22 December 1996, a severe cold air outbreak occurred over the northern Gulf of Mexico, producing a large range of both $(q_{sea} - q_{air})$ and $(T_{sea} - T_{air})$ as shown in Fig. 3. Hourly data for both T_{air} and T_{dew} were obtained for Buoy 42040. The results are plotted in Fig. 3. Two curves are obtained, one is linear and the other nonlinear. It can be seen that the linear correlation between $(q_{sea} - q_{air})$ and $(T_{sea} - T_{air})$, with a very high correlation coefficient ($r = 0.98$), is superior to the nonlinear one ($r = 0.96$). That is, the linear equation provided in Fig. 3 can directly account for $r^2 = (0.98)^2 = 96.0\%$ of the variability in $(q_{sea} - q_{air})$ using $(T_{sea} - T_{air})$ alone. This correlation is very compelling. If we accept that the linear correlation between $(q_{sea} - q_{air})$ and $(T_{sea} - T_{air})$ does exist, we may then infer from Eq. (3) that B is also related to $(T_{sea} - T_{air})$ only. In fact, this idea is verified in Fig. 4 using the same dataset and Eq. (3) for the same period shown in Fig. 3. In other words, we postulate that

$$B = a(T_{sea} - T_{air})^b, \tag{17a}$$

or

$$\ln B = \ln A + b \ln(T_{sea} - T_{air}), \tag{17b}$$

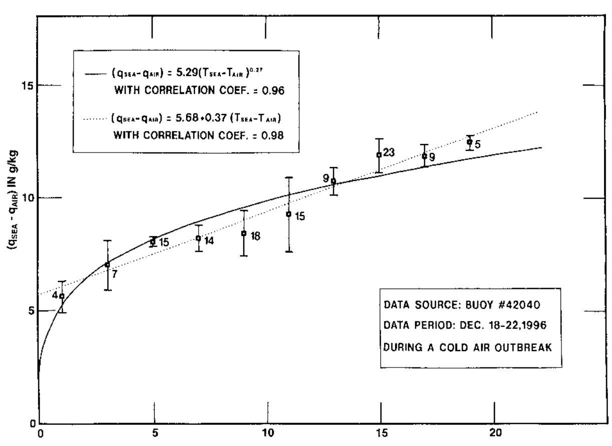

FIG. 3. Relationships between $(q_{sea} - q_{air})$ and $(T_{sea} - T_{air})$ obtained from buoy 42040 (see Fig. 2 for location) during a cold air outbreak 18–22 December 1996. The mean, standard deviation, and the number of measurements are also provided in the figure.

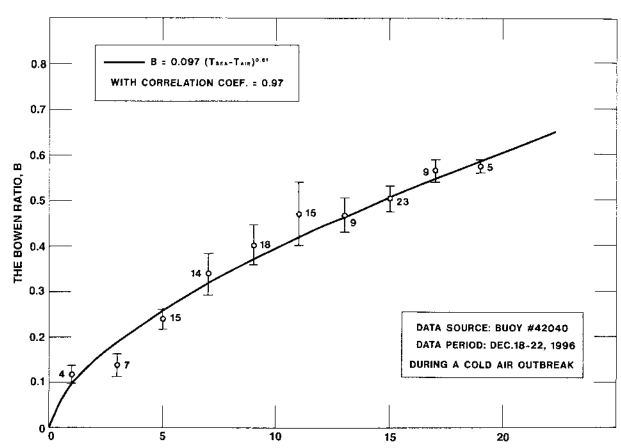

FIG. 4. The relationship between the Bowen ratio B and the quantity $(T_{sea} - T_{air})$ obtained from buoy 42040 (see Fig. 2 for location) during a cold air outbreak 18–22 December 1996. The mean, standard deviation, and number of measurements are also provided in the figure.

where a and b need to be determined from field data. The results from all available pairs of B as obtained from Eq. (3) with $C_T = 1.1 \times 10^{-3}$ (see Large and Pond 1982) and $C_E = 1.12 \times 10^{-3}$ (Smith et al. 1994) versus $(T_{\rm sea} - T_{\rm air})$ are shown in Fig. 2. Values of a and b along with the correlation coefficient r from the linear regression of Eq. (17b) between B and $(T_{\rm sea} - T_{\rm air})$ are obtained as follows according to the curve numbers for buoy/station

1) GDIL1, 1995–96:
   $B = 0.087(T_{\rm sea} - T_{\rm air})^{0.76}$ with $r = 0.79$
2) GDIL1, 1996–97:
   $B = 0.087(T_{\rm sea} - T_{\rm air})^{0.72}$ with $r = 0.77$
3) 42002, 1996–97:
   $B = 0.077(T_{\rm sea} - T_{\rm air})^{0.69}$ with $r = 0.85$
4) 42019, 1993–94:
   $B = 0.077(T_{\rm sea} - T_{\rm air})^{0.71}$ with $r = 0.89$
5) 42040, 1996–97:
   $B = 0.077(T_{\rm sea} - T_{\rm air})^{0.70}$ with $r = 0.89$
6) 42002, 1995–96:
   $B = 0.078(T_{\rm sea} - T_{\rm air})^{0.67}$ with $r = 0.87$.

If one accepts these high correlation coefficient values between B and $(T_{\rm sea} - T_{\rm air})$, one may say that Eq. (17) is verified. Note that hourly data for these four locations for the period given in Fig. 2 were based on measurements provided by the National Data Buoy Center, and for brevity and comparison purposes each individual point from each hour is not plotted in Fig. 2. According to Breaker et al. (1998), uncertainties in the calculated values of specific humidity from these buoys were estimated and ranged between 0.27% and 2.1% of the mean values as compared to nearby ship reports. Note that these hourly buoy data are also available on the World Wide Web. Note also that although the transfer coefficient of C_T and C_E may vary with stability (e.g., Konda 1996, Table 2), according to Garratt (1992, pp. 101–104), at present $C_E = C_H \simeq 1.1 \times 10^{-3}$ ($\pm$15%) under near neutral conditions. This is supported most recently by DeCosmo et al. (1996) that $C_{EN} = C_{HN} = 1.1 \times 10^{-3}$ and by Fairall et al. (1996) that $C_E = 1.11 \times 10^{-3}$. Futhermore, according to Smith (1988, pp. 15 470), Friehe and Schmitt (1976) summarized eddy correlation data then available to find $C_E = 1.32 \times 10^{-3}$, corresponding to $C_{EN} = 1.18 \times 10^{-3}$; the difference between C_E and C_{EN} was within the 15% range as suggested by Garratt (1992). Therefore, we use $C_T = 1.1 \times 10^{-3}$ based on Large and Pond (1982) and $C_E = 1.12 \times 10^{-3}$ from Smith et al. (1994) for our computations.

The results as assembled in Fig. 2 are interesting in that for different years at different locations from shelf break to the deeper Gulf of Mexico shown by curves 3 through 6, variations of a are only between 0.077 and 0.078 and b between 0.67 and 0.69 along with r between 0.85 and 0.89. In the nearshore area as represented by GDIL1 (for Grand Isle, Louisiana), however, both a and b values are higher as shown by curves 1 and 2 due possibly to land effects.

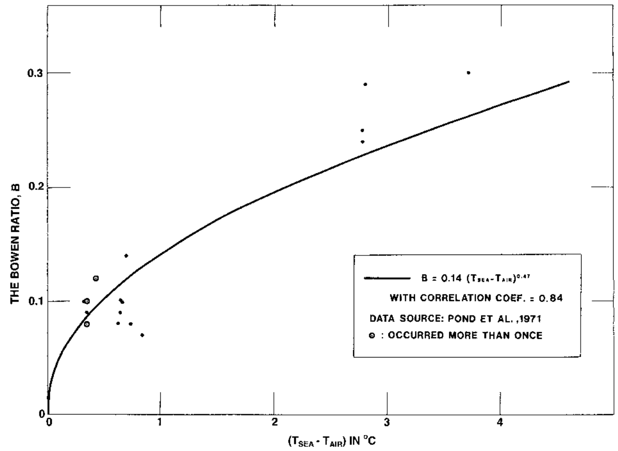

FIG. 5. The relationship between the Bowen ratio B and the quantity $(T_{\rm sea} - T_{\rm air})$ for open ocean.

In order to apply Eq. (17) to the deep ocean, Fig. 5 is provided. All data points from Pond et al. (1971) were incorporated except number 12 from the Oregon State University run, which was questionable, as noted by the authors. It is surprising that with only 19 data points the correlation coefficient reaches 0.84. If one accepts this high correlation between B and $(T_{\rm sea} - T_{\rm air})$, Eq. (17) is also applicable to the deep ocean; although coefficients a and b may vary, the generic relationship does exist. Certainly, more data are needed to further verify Eq. (17) to deep ocean conditions.

4. Conclusions

On the basis of thermodynamic considerations, it is postulated that the quantities of $(q_{\rm sea} - q_{\rm air})$ and $(T_{\rm sea} - T_{\rm air})$ are related. This postulation was verified during a severe cold air outbreak over the northern Gulf of Mexico under which large ranges of $(q_{\rm sea} - q_{\rm air})$ versus $(T_{\rm sea} - T_{\rm air})$ along with a very high-correlation coefficient of 0.98 were found. With this compelling correlation, we further postulate that the Bowen ratio B is also related to the quantity of $(T_{\rm sea} - T_{\rm air})$ such that $B = a(T_{\rm sea} - T_{\rm air})^b$. Based on simultaneous hourly measurements of all three temperatures, that is, of $T_{\rm air}$, $T_{\rm sea}$, and $T_{\rm dew}$ in the Gulf of Mexico at four stations from 1993 through 1997, we found that in the Gulf of Mexico, values of a varied from 0.077 to 0.078, b from 0.67 to 0.71, and correlation coefficient r from 0.85 to 0.89. Because curve 5 covers the range of $(T_{\rm sea} - T_{\rm air})$ from 0° to 18°C, it is recommended for operational use. Equations for the nearshore environment are also obtained. Limited data from the open ocean also support this generic relationship between B and $(T_{\rm sea} - T_{\rm air})$. Certainly, similar measurements are needed from other ocean basins to further substantiate our proposed formulas.

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REFERENCES

Breaker, L. C., D. B. Gilhousen, and L. D. Burroughs, 1998: Preliminary results from long-term measurements of atmospheric moisture in the marine boundary layer in the Gulf of Mexico. *J. Atmos. Oceanic Technol.,* **15,** 661–676.

Chou, S.-H., D. Atlas, and E.-N. Yeh, 1986: Turbulence in a convective marine atmospheric boundary layer. *J. Atmos. Sci.,* **43,** 547–564.

DeCosmo, J., K. B. Katsaros, S. D. Smith, R. J. Anderson, W. A. Oost, K. Bumke, and H. Chadwick, 1996: Air–sea exchange of water vapor and sensible heat: The Humidity Exchange Over the Sea (HEXOS) results. *J. Geophys. Res.,* **101** (C5), 12 001–12 016.

Fairall, C. W., E. F. Bradley, D. P. Rogers, J. B. Edson, and G. S. Young, 1996: Bulk parameterization of air–sea fluxes for Tropical Ocean Global Atmosphere Coupled Ocean–Atmosphere Response Experiment. *J. Geophys. Res.,* **101** (C2), 3727–3764.

Friehe, C. A., and K. F. Schmitt, 1976: Parameterization of air–sea interface fluxes of sensible heat and moisture by the bulk aerodynamic formulas. *J. Phys. Oceanogr.,* **6,** 801–809.

Garratt, J. R., 1992: *The Atmospheric Boundary Layer.* Cambridge University Press, 316 pp.

Hicks, B. B., and G. D. Hess, 1977: On the Bowen ratio and surface temperature at sea. *J. Phys. Oceanogr.,* **7,** 141–145.

Hsu, S. A., 1988: *Coastal Meteorology.* Academic Press, 260 pp.

Konda, M., and N. Imasato, 1996: A new method to determine near-sea surface air temperature by using satellite data. *J. Geophys. Res.,* **101** (C6), 14 349–14 360.

Kondo, J., 1976: Heat balance of the East China Sea during the Air Mass Transformation Experiment. *J. Meteor. Soc. Japan,* **54,** 382–398.

Large, W. G., and S. Pond, 1982: Sensible and latent heat flux measurements over the ocean. *J. Phys. Oceanogr.,* **12,** 464–482.

Liu, W. T., and P. P. Niiler, 1990: The sensitivity of latent heat flux to the air humidity approximations used in ocean circulation models. *J. Geophys. Res.,* **96** (C6), 9745–9753.

McIlveen, J. F. R., 1986: *Basic Meteorology.* Van Nostrand Reinhold, 457 pp.

Panofsky, H. A., and J. A. Dutton, 1984: *Atmospheric Turbulence.* John Wiley, 397 pp.

Pond, S., G. T. Phelps, J. E. Paquin, G. McBean, and R. W. Stewart, 1971: Measurements of the turbulent fluxes of momentum, moisture and sensible heat over the ocean. *J. Atmos. Sci.,* **28,** 901–917.

Rao, G. R. L., M. V. Rao, P. H. Prasad, and K. G. Reddy, 1986: Distribution of Bowen ratio over the Indian Ocean. *Mausam,* **37,** 71–72.

Roll, H. U., 1965: *Physics of the Marine Atmosphere.* Academic Press, 426 pp.

Smith, S. D., 1980: Wind stress and heat flux over the ocean in gale force winds. *J. Phys. Oceanogr.,* **10,** 709–726.

——, 1988: Coefficients for sea surface wind stress, heat flux, and wind profiles as a function of wind speed and temperature. *J. Geophys. Res.,* **93** (C12), 15 467–15 472.

——, K. B. Katsaros, W. A. Oost, and P. G. Mestayer, 1994: The impact of the HEXOS programme. Preprints, *Second Int. Conf. Air–Sea Interaction and on Meteorology and Oceanography of the Coastal Zone,* Lisbon, Portugal, Amer. Meteor. Soc., 226–227.