第27卷第1期 数 学 研 究 与 评 论 Vol.27, No.1
2007年2月 JOURNAL OF MATHEMATICAL RESEARCH AND EXPOSITION Feb., 2007

Article ID: 1000-341X(2007)01-0028-07 **Document code**: A

On Strongly Clean General Rings

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Abstract: In this paper, we introduce the concept of strongly clean general ring (with or without unity) and extend some of the basic results to a wider class. We prove that every strongly π-regular general ring is strongly clean and the corner of the strongly clean general ring is strongly clean. It is also proved that upper triangular matrix rings of the commutative clean general ring with $J(I) = Q(I)$ are strongly clean.

Key words: strongly clean general ring, strongly π-regular general ring, upper triangular matrix ring.
MSC(2000): 16U99; 16S50
CLC number: O153

1. Introduction

An element in a ring R is called strongly clean if it is the sum of an idempotent and a unit which commute, and R is called a strongly clean ring if every element of R is strongly clean. The notion was introduced by Nicholson in [5] where he proved that every strongly π-regular ring is strongly clean (A ring R is strongly π-regular if all chains of the forms $aR \supseteq a^2R \supseteq \cdots$ terminate, or equivalently, all chains $Ra \supseteq Ra^2 \supseteq \cdots$ terminate). In [7], Nicholson and Zhou defined the notion of a clean general ring, and some properties about clean rings[1,3,4] were extended. In this paper we extend the definition of a strongly clean ring to general rings (with or without unity), and extend some of the basic results.

Let I be a general ring. We call a general ring strongly clean if every element is the sum of an idempotent and an element $q \in Q(I)$ which commute ($q \in Q(I)$ means $q+p+qp = 0 = p+q+pq$ for some $p \in I$). In this paper, we prove that if I is strongly π-regular (that is, for every $a \in I$, there exist $n \in \mathbf{N}$ and $y \in I$ such that $a^n = a^{n+1}y$), then I is strongly clean. For $A \lhd I$, we prove that if I is a strongly clean general ring, then A and the corner of A are both strongly clean. Finally, we proved that upper triangular matrix rings of the commutative clean general ring with $J(I) = Q(I)$ are strongly clean.

By the term ring we mean an associative ring with unity and by a general ring we mean an associative general ring with or without unity. For clarity, R, S will always denote rings, and a general ring will be written as I. We denote the group of units of the ring R by $U(R)$, and

Received date: 2005-11-18; **Accepted date**: 2006-07-03
Foundation item: the Foundation for Excellent Doctoral Dissertation of Southeast University (YBJJ0507); the National Natural Science Foundation of China (10571026); the Natural Science Foundation of Jiangsu Province (Bk2005207).

the Jacobson radical of the general ring I by $J(I)$. We write $A \triangleleft I$ for an ideal of I and write $M_n(I)$ $(T_n(I))$ for the general ring of all (respectively all upper triangular) $n \times n$ matrices over the general ring I.

2. Strongly clean general rings

Let I be a general ring with $p,\ q \in I$, and write $p * q = p + q + pq$. Let

$$Q(I) = \{q \in I | p * q = 0 = q * p \text{ for some } p \in I\}.$$

Note that $J(I) \subseteq Q(I)$. Recall that R is strongly clean if every element $a \in R$ can be written as the sum of an idempotent and a unit which commute.

Lemma 2.1 *The following statements are equivalent for a ring R.*

(1) R is strongly clean.

(2) For each $a \in R$, $a = e + q$ and $eq = qe$ where $e^2 = e$ and $q \in Q(R)$.

Proof Let $a \in R$. If R is strongly clean, write $a + 1 = e + u$ and $eu = ue$ where $e^2 = e$ and $u \in U(R)$. Then $a = e + q,\ eq = qe$ where $q = u - 1$ and $q * p = 0 = p * q$ with $p = u^{-1} - 1$. Conversely, if $a - 1 = e + q$ and $eq = qe$ where $e^2 = e$ and $q \in Q(R)$, then we have $a = e + u$ where $u = q + 1 \in U(R)$ and $eu = ue$.

An element a in a general ring I is called a strongly clean element if $a = e + q$ and $eq = qe$ where $e^2 = e \in I$ and $q \in Q(I)$; and I is called a strongly clean general ring if every element of I is strongly clean. Hence idempotents, nilpotents and elements of $Q(I)$ are all strongly clean. Clearly, every homomorphic image of a strongly clean general ring is strongly clean, and the direct product $\Pi_i K_i$ and the direct sum $\oplus_i K_i$ of general rings are strongly clean if and only if each K_i is strongly clean.

Theorem 2.2 *Let I be a general ring. The following are equivalent for $a \in I$.*

(1) $a^n = a^n x a^n$ and $xa^n = a^n x$ for $n \in \mathbf{N}$ and $x \in I$.

(2) $a^k = a^{k+1}x$ and $a^l = ya^{l+1}$ for $k,\ l \in \mathbf{N}$ and $x,\ y \in I$.

(3) $a^n = a^{n+1}x$ and $xa = ax$ for $n \in \mathbf{N}$ and $x \in I$.

(4) $a^n = e + q$ where $e^2 = e \in I$ and $eq = qe = q \in Q(I)$ for some $n \in \mathbf{N}$.

Proof (1)$\Longrightarrow$ (2). Since $xa^n = a^n x$, $a^n = a^n x a^n = a^{2n} x = a^{n+1}(a^{n-1}x)$. Similarly, $a^n = (xa^{n-1})a^{n+1}$.

(2)$\Longrightarrow$ (3). By (2), we can assume that $a^n = a^{n+1}x = ya^{n+1}$. Then $a^n = ya^{n+1} = y(ya^{n+1})a = y^2 a^{n+2} = \cdots = y^{n+1}a^{2n+1} = y^n(ya^{n+1})a^n = y^n(a^{n+1}x)a^n = y^{n-1}(ya^{n+1})xa^n = y^{n-1}(a^{n+1}x)xa^n = \cdots = a^{n+1}x^{n+1}a^n = a^n x^n a^n$. Set $e = a^n x^n$, then $a^n = ea^n$ and $a^n e = a^n a^n x^n = a^{n-1}(a^{n+1}x)x^{n-1} = a^{n-1}a^n x^{n-1} = \cdots = a^{n+1}x = a^n$. Similarly, there exists $f = y^n a^n$, such that $a^n = fa^n = a^n f$. Therefore, $e = a^n x^n = fa^n x^n = fe = y^n a^n e = y^n a^n = f$. Set $b = ex^n e \in eIe$, and $c = ey^n e \in eIe$. Then $e = a^n b = ca^n$. In addition, $ba^n = eba^n = ca^n ba^n = cea^n = ca^n = a^n b = e$. Set $z = a^{n-1}b = a^{n-1}ebe = a^{2n-1}b^3 a^n \in eIe$, whence $az = a^n b = e$.

Then $z = ez = ze$ and $a^{n+1}z = a^{n+1}a^{n-1}b = a^{2n}b = a^n$. Set $g = za$. Then $g^2 = z(az)a = zea = za = g \in eI$, whence $eg = g$. Since $Ia^n = Iea^n = Iba^{2n} \subseteq Ia^{2n} \subseteq Ia^{n+1} \subseteq Ia^n$, we have $Ie = Ia^n = Ia^{n+1} = Ia^n a = Ia^{n+1}za \subseteq Iza = Ig$, whence $e = eg = g = za$. Therefore, $za = az$.

(3)$\Longrightarrow$ (4). $a^n = a^{n+1}x = a(a^{n+1}x)x = a^{n+2}x^2 = \cdots = a^{2n}x^n = a^n x^n a^n$. Let $b = x^n a^n x^n$. Then we have $a^n b a^n = a^n$ and $ba^n b = b$. Set $e = ba^n$ and $q = a^n - ba^n$. Note that $xa = ax$, so $e^2 = e$ and $q \in Q(I)$ with $p = b - ba^n$, whence $eq = qe = q$, as required.

(4)$\Longrightarrow$ (1). If $a^n = e + q$, $eq = qe = q$ where $e^2 = e$ and $q \in Q(I)$. Then there exists $p = ep = pe$ such that $p*q = 0 = q*p$. Thus we have $a^n(e+p)a^n = (e+q)(e+p)(e+q) = e+q = a^n$, and $a^n(e+p) = (e+p)a^n$.

Corollary 2.3 *Let R be a ring. Then R is strongly π-regular if and only if for every $a \in R$, there exist $n \in \mathbf{N}$, $e^2 = e \in R$ and $q \in Q(R)$ such that $a^n = e + q$ and $eq = qe = q$.*

From the proof of Theorem 2.2, the following result is immediate.

Corollary 2.4 *Let I be a general ring. The following are equivalent for $a \in I$.*

(1) $a^2x = a = ya^2$ for $x,\ y \in I$.

(2) $a = aba$ and $ab = ba$ for some $b \in I$.

(3) $a = e + q$ and $eq = qe = q$ where $e^2 = e \in I$ and $q \in Q(I)$.

Recall that a ring R is strongly regular if every element of R can be written as the product of an idempotent and a unit which commute (or equivalently, for each $a \in R$, there exists $b \in R$ such that $a = aba$ and $ab = ba$). By Corollary 2.4, we obtain easily a new characterization of strongly regular rings.

Corollary 2.5 *Let R be a ring. Then R is strongly regular if and only if for every $a \in R$, there exist $e^2 = e \in R$ and $q \in Q(R)$ such that $a = e + q$ and $eq = qe = q$.*

An element a in a general ring I is called strongly π-regular if it satisfies the condition of Theorem 2.2. In [2], H. Chen and M. Chen introduced a strongly π-regular ideal. We define analogously a strongly π-regular general ring in case for any $x \in I$ there exist $n \in \mathbf{N}$ and $y \in I$ such that $x^n = x^{n+1}y$. The proof of [2, Theorem 2.3] adapts to prove the next result.

Lemma 2.6 *The following statements are equivalent for a general ring I.*

(1) I is strongly π-regular.

(2) Every element in I is strongly π-regular.

Proof (2)$\Longrightarrow$ (1). It is clear.

(1)$\Longrightarrow$ (2). Suppose that I is strongly π-regular and let $x \in I$. Then we have $n \in \mathbf{N}$ and $y \in I$ such that $x^n = x^{n+1}y$. Also we have some $m \in \mathbf{N}$ and $z \in I$ such that $y^n = y^{n+1}z$. We may assume $m = n$. One easily checks that $x^n = x^{2n}y^n$ and $y^n = y^{2n}z^n$. Set $a = x^n$, $b = y^n$ and $c = z^n$. Then $a = a^2b$ and $b = b^2c$. Since I is strongly π-regular, there exist $k \in \mathbf{N}$ and $d \in I$ such that $(c-a)^k = (c-a)^{k+1}d$. Note that $ac = (a^2b)c = a(a^2b)bc = a^3(b^2c) = a^3b = a^2$ and $abc = a^2(b^2c) = a^2b = a$. Hence $(c-a)^2 = c^2 - ca - ac + a^2 = c(c-a)$, whence $ab(c-a)^2 = abc(c-a) = a(c-a) = 0$. Observe that $b(c-a) = b^2c(c-a) = b^2(c-a)^2 = b[b(c-a)](c-a) =$

$b[b^2(c-a)^2](c-a) = b^3(c-a)^3 = \cdots = b^k(c-a)^k$. Thus $b(c-a)^2d = b^k(c-a)^{k+1}d = b^k(c-a)^k = b(c-a)$. So $ab(c-a) = ab(c-a)^2d = 0$, whence $a = abc = aba$. Furthermore, $0 = ab(c-a)d = ab^2(c-a)^2d = ab[b(c-a)] = ab^2(c-a)$. It implies that $ab^2a = ab^2c$. Therefore, $a = aba = a(b^2c)a = ab^2a^2$. By Theorem 2.2, x is a strongly π-regular element in I.

Theorem 2.7 *Every strongly π-regular general ring is strongly clean.*

Proof Let I be a strongly π-regular general ring and let $a \in I$. By Lemma 2.6 there exist $n \in \mathbf{N}$ and $b \in I$ such that $a^n = a^{n+1}b$ and $ab = ba$. Then $a^n = a^{n+1}b = a(a^{n+1}b)b = a^{n+2}b^2 = \cdots = a^{2n}b^n = a^nb^na^n$. Set $e = a^nb^n = b^na^n$ is an idempotent in I. In fact, we may assume n is even and using a virtual 1 for clarity. We write $q = a-e$ and $p = (a-a^2+a^3-\cdots+a^{n-1})(e-1)+a^{n-1}b^ne-e$. Thus we have

$$\begin{aligned} q * p =& a - e + (a - a^2 + a^3 - \cdots + a^{n-1})(e-1) + a^{n-1}b^ne - e + \\ & (a^2 - a^3 + a^4 - \cdots + a^n)(e-1) + a^nb^ne - ae - a^{n-1}b^ne + e \\ =& a - ae + (a + a^n)(e-1) = 0. \end{aligned}$$

Similarly, $p * q = 0$. Therefore, $a = e + q,\ q \in Q(I)$ whence $eq = qe$.

3. Extensions of strongly clean general rings

If S is a ring and I is a general ring such that $I =_S I_S$ is a bimodule, the ideal extension of S by I is defined to be the additive group $E(S;I) = S \bigoplus I$ with multiplication $(r,a)(s,b) = (rs, rb+as+ab)$. This is an associative ring if and only if the conditions $s(ab) = (sa)b$, $a(sb) = (as)b$ and $(ab)s = a(bs)$ are satisfied for all $s \in S$ and $a,\ b \in I$. In particular, $I' = E(\mathbf{Z};\ I)$ is the standard unitization of the general ring I.

Proposition 3.1 *The following are equivalent for a general ring I.*

(1) I is strongly clean.

(2) Whenever $I \lhd R$ where R is a ring, each $a \in I$ is a strongly clean element of R.

(3) There exists a ring S such that $I =_S I_S$ and $(0,a)$ is strongly clean in $E(S;I)$ for all $a \in I$.

Proof (1)$\Longrightarrow$(2). Let $a \in R$. By (1) we have $-a = e + q$, $e^2 = e$, $q \in Q(I)$ and $eq = qe$. Then $a = (1-e) + (-1-q)$ where $(1-e)^2 = 1-e$ and $-1-q$ is a unit of R.

(2)$\Longrightarrow$(3). This is trivial with $S = \mathbf{Z}$.

(3)$\Longrightarrow$(1). Given S as in (3), write $R = E(S;I)$. Let $a \in I$, then $(0,-a) = \alpha + \beta$ in R where $\alpha^2 = \alpha,\ \beta \in U(R)$ and $\alpha\beta = \beta\alpha$. Hence $\alpha = (e,x)$ and $\beta = (-e,y)$ where $e \in S$ and $-a = x + y$ for $x,\ y \in I$. Since $\alpha^2 = \alpha$, we obtain $e^2 = e$ and $x = ex + xe + x^2$. We have $e = 1$ because β is a unit in R, so $(-x)^2 = -x$ is an idempotent in I. Note that $(-1,y) = \beta$ is a unit in R, then exists $\beta^{-1} = (-1,z)$ such that $(-y)*(-z) = 0 = (-z)*(-y)$. Hence we have $a = (-x) + (-y)$ and $(-x)(-y) = (-y)(-x)$ by $\alpha\beta = \beta\alpha$, as required.

Proposition 3.2 *Let I be a strongly clean general ring and $A \lhd I$. Then A is strongly clean.*

Proof Assume first that $I = R$ is a ring. Let $a \in A$, by Lemma 2.1 we have $-a = e - u$ where $e^2 = e \in R$, $u \in U(R)$ and $eu = ue$. Thus $1 - e = u^{-1}[u(1-e)] = u^{-1}[a(1-e)] \in A$. Write $u = 1 + q$ where $q \in Q(R)$. Then $a = -e + u = (1-e) + q$ whence $q \in A \cap Q(R) = Q(A)$, and $(1-e)q = q(1-e)$.

If I has no unity, write $R = E(\mathbf{Z};I)$. Then $A \cong (0, A) \lhd R$ so it suffices by (2) of Proposition 3.1 to show that $(0, a)$ is strongly clean in R for each $a \in A$. But if $-a = f + q$ is a strongly clean expression of $-a$ in I, then $(0, a) = (1, -f) + (-1, -q)$ where $(1, -f)^2 = (1, -f)$, $(-1, -q) \in U(R)$ (the inverse is $(-1, -p)$) where $q * p = 0 = p * q$, and $(1, -f)(-1, -q) = (-1, -q)(1, -f)$.

Theorem 3.3 *Let I be a strongly clean general ring and let $A \lhd I$ with $e^2 = e \in I$. Then eAe is also strongly clean.*

Proof Given $a \in A$ and $eae \in A$. A is a strongly clean general ring by Proposition 3.2, so there exist $f^2 = f \in A$ and $q \in Q(A)$ such that $eae = f + q$, $fq = qf$. Thus we have

$$feae = f + fq = f + qf = eaef,$$

whence

$$feae = ef + efq = fe + qfe = eaef. \tag{*}$$

Note that $q \in Q(A)$, there exists $p \in A$ such that $q * p = 0 = p * q$. Since $qp = pq$ and $fq = qf$, it is obtained that $fp = pf$ and $peae = eaep$. Thus we have $ef = ef + ef(q + p + qp) = (ef + efq) + (ef + efq)p = feae + feaep = feae + pfeae = fe + qfe + p(fe + qfe) = fe + (q + p + pq)fe = fe$ (by $(*)$). Therefore, $eae = efe + eqe$ where $(efe)^2 = efe$ and $eqe \in Q(A) \cap eAe = Q(eAe)$ commute.

In [8], we showed that semiperfect rings and matrix rings over strongly clean rings need not be strongly clean. However, we do not know whether the corner of the strongly clean ring is strongly clean. The following result answers the question.

Corollary 3.4 *Let R be a strongly clean ring with $e^2 = e \in R$. Then eRe is also strongly clean.*

Proof It follows from Theorem 3.3 and Lemma 2.1.

Lemma 3.5 *Let I be a general ring. Then* $\begin{pmatrix} q_1 & * & \cdots & * \\ & q_2 & \cdots & * \\ & & \ddots & \vdots \\ & & & q_n \end{pmatrix} \in Q(T_n(I))$ *if and only if each $q_i \in Q(I)$.*

Proof It is directly verified.

Theorem 3.6 *Let I be a commutative clean general ring with $J(I) = Q(I)$. Then $T_n(I)$ is strongly clean for all $n \geq 1$.*

Proof We prove the implication by induction on n. It is true if $n = 1$. Assume that $n > 1$ and

$T_{n-1}(I)$ is strongly clean. Let

$$A = \begin{pmatrix} A_1 & \alpha \\ 0 & a_{nn} \end{pmatrix} \in T_n(I),$$

where $A_1 \in T_{n-1}(I)$, $a_{nn} \in I$ and α is an $(n-1)\times 1$ matrix. By (1) and by induction hypothesis, A_1 and a_{nn} have strongly clean expressions in $T_{n-1}(I)$ and in I respectively,

$$A_1 = E_1 + Q_1,\ a_{nn} = e_{nn} + q_{nn}.$$

Therefore, it suffices to prove that there exists an $(n-1)\times 1$ matrix α_1 such that

$$A = \begin{pmatrix} A_1 & \alpha \\ 0 & a_{nn} \end{pmatrix} = \begin{pmatrix} E_1 & \alpha_1 \\ 0 & e_{nn} \end{pmatrix} + \begin{pmatrix} Q_1 & \alpha - \alpha_1 \\ 0 & q_{nn} \end{pmatrix}$$

is a strongly clean expression in $T_n(I)$. Let $E = \begin{pmatrix} E_1 & \alpha_1 \\ 0 & e_{nn} \end{pmatrix}$ and $Q = \begin{pmatrix} Q_1 & \alpha - \alpha_1 \\ 0 & q_{nn} \end{pmatrix}$. Clearly, $Q \in Q(T_n(I))$ by Lemma 3.5. We denote

$$\begin{pmatrix} a & & \\ & \ddots & \\ & & a \end{pmatrix}_{(n-1)\times(n-1)}$$

by aI_{n-1} for clarity. It is easy to verify that

$$E^2 = E \Longleftrightarrow E_1\alpha_1 + \alpha_1 e_{nn} = \alpha_1 \Longleftrightarrow (E_1 + e_{nn}I_{n-1})\alpha_1 = \alpha_1. \tag{1}$$

$$\begin{aligned} EQ = QE &\Longleftrightarrow E_1(\alpha - \alpha_1) + \alpha_1 q_{nn} = Q_1\alpha_1 + (\alpha - \alpha_1)e_{nn} \\ &\Longleftrightarrow (Q_1 - q_{nn}I_{n-1} - 2e_{nn}I_{n-1})\alpha_1 + (E_1 + e_{nn}I_{n-1})\alpha_1 = (E_1 - e_{nn}I_{n-1})\alpha. \end{aligned} \tag{2}$$

Combining (1) with (2) gives

$$(Q_1 - q_{nn}I_{n-1} - 2e_{nn}I_{n-1})\alpha_1 + \alpha_1 = (E_1 - e_{nn}I_{n-1})\alpha.$$

Because $(-2e_{nn}) * (-2e_{nn}) = 0,\ \ (-2e_{nn}) \in Q(I)$. Note that $q_{nn} \in Q(I),\ Q_1 \in Q(T_{n-1}(I))$ and $Q(I) = J(I) \lhd I$. Hence $Q_1 - q_{nn}I_{n-1} - 2e_{nn}I_{n-1} \in Q(T_{n-1}(I))$. By Lemma 3.5, there exists $P \in T_{n-1}(I)$ such that

$$P * (Q_1 - q_{nn}I_{n-1} - 2e_{nn}I_{n-1}) = (Q_1 - q_{nn}I_{n-1} - 2e_{nn}I_{n-1}) * P = 0. \tag{3}$$

So $\alpha_1 = (E_1 - e_{nn}I_{n-1})\alpha + P(E_1 - e_{nn}I_{n-1})\alpha$. Next we verity that $\alpha_1 = (E_1 - e_{nn}I_{n-1})\alpha + P(E_1 - e_{nn}I_{n-1})\alpha$ satisfies (1) and (2). Set $X = Q_1 - q_{nn}I_{n-1} - 2e_{nn}I_{n-1}$. Using (3), we get $PX = XP$. Note that $E_1Q_1 = Q_1E_1$, then $E_1X = XE_1$. Observe that $PE_1 = (PE_1 + E_1)(X + P + XP) + PE_1 = (PX + P + X)E_1 + (PX + P + X)E_1P + E_1P = E_1P$. Therefore, $E_1 + e_{nn}I_{n-1},\ Q_1 - q_{nn}I_{n-1} - 2e_{nn}I_{n-1}$ and P all commute. Thus

$$\begin{aligned} (E_1 + e_{nn}I_{n-1})\alpha_1 &= (E_1 + e_{nn}I_{n-1})[(E_1 - e_{nn}I_{n-1})\alpha + P(E_1 - e_{nn}I_{n-1})\alpha] \\ &= (E_1 - e_{nn}I_{n-1})\alpha + P(E_1 - e_{nn}I_{n-1})\alpha = \alpha_1. \end{aligned}$$

So α_1 satisfies (1). Similarly, α_1 satisfies (2). Therefore, the proof is completed.

In [7], Nicholson and Zhou proved that every uniquely clean general ring I satisfies $J(I) = Q(I)$. By Theorem 3.6, if I is a commutative uniquely clean general ring, then $T_n(I)$ is strongly clean. Thus, we obtain a new class of strongly clean general rings.

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关于强 Clean 一般环

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摘要: 本文介绍了强 clean 一般环的概念并将一些基本的结果推广到这个更广的环类. 证明了强 clean 一般环的角落环和强 π- 正则一般环都是强 clean 的，还讨论了强 clean 一般环的扩张并且证明了满足条件 $J(I) = Q(I)$ 的交换 clean 一般环的上三角矩阵环是强 clean 的.

关键词: 强 clean 一般环，强 π- 正则一般环，上三角矩阵环.