The Dissipation of Fluctuating Tracer Variances

ROLAND A. DE SZOEKE

College of Oceanic and Atmospheric Sciences, Oregon State University, Corvallis, Oregon

(Manuscript received 12 February 1997, in final form 23 January 1998)

ABSTRACT

The evolution of the covariance of two tracers involves a quantity called codissipation, proportional to the covariance of the gradients of the two tracers, and analogous to the dissipation of tracer variance. The evolution of the variance of a composite tracer—a linear combination of two simple, primary tracers—depends on the ''composite dissipation,'' a combination of the individual simple tracer dissipations and the codissipation. The composite dissipation can be negative (implying growth of the variance of the composite tracer) for structures in which the correlation of the simple tracer gradients are large enough (i.e., large codissipation). This situation occurs in the phenomena of double diffusion and salt fingering. A particular composite tracer called watermass variation, a measure of water-type scatter about the mean tracer versus tracer relationship, lacks production terms of the conventional form—tracer flux multiplying tracer gradient—in its variance evolution balance. Only codissipation can produce variance of watermass variation. The requirements that watermass variance production and dissipation be in equilibrium, and that no other composite tracer variance be tending to grow due to codissipation, lead to a particular relation among codissipation and the simple dissipations and between the simple dissipations themselves. The latter are proportional to one another, the proportionality factor being the square of the slope of the mean tracer versus tracer relation. The same results can be obtained by modifying Batchelor's argument to give the equilibrium cospectrum of two tracer gradients at high wavenumbers in a well-developed field of isotropic turbulence. As a consequence of these arguments, the turbulent eddy tracer fluxes are also proportional, with the mean tracer–tracer slope as proportionality factor. Further, the ratio of turbulent diffusivities of two tracers is unity. The dissipation of buoyancy, a composite tracer constructed from temperature and salinity, is proportional at equilibrium to thermal dissipation multiplied by a factor that depends on the stability ratio. This previously established result is obtained here under less restrictive conditions.

1. Introduction

This paper examines the links between variance dissipations of several tracers and the watermass relations among the mean tracers. Dissipation by molecular diffusive processes of the variance of the concentration of a tracer is related, under temporally stationary and spatially homogeneous conditions, to the turbulent transport of the tracer down its mean gradient (Osborn and Cox 1972). This principle has been most commonly applied to infer vertical heat flux from thermal variance dissipation because this is the only tracer dissipation that has been amenable to direct observation in the ocean. It should also be possible to calculate salinity flux from salinity variance dissipation, although the small scales on which the salinity variance dissipation presumably takes place have resisted measurement (Gregg 1987). A similar statement could be made about the variance dissipation of any arbitrary tracer. The ability to infer one tracer variance dissipation from another would be an important tool for estimating and modeling mixing of tracer concentrations in the ocean. A firm appreciation of the limitations and implications of such theoretical inferences is just as important. Stommel and Csanady (1980) showed how the large-scale heat and salt (or freshwater) transports that must be exchanged between watermasses determine the *T–S* relations of those watermasses. This poses a fascinating link, first considered by Stern (1968), between processes on the microscale (tracer variance dissipation and downgradient tracer fluxes) and the large scale (watermass relations and ocean circulation).

A quantity of considerable dynamical importance in the ocean is buoyancy, which is approximately a linear combination of temperature and salinity. It has been suggested that because of the stable temperature–salinity relation occurring in most parts of the ocean, the dissipation of salinity variance, and hence of buoyancy variance, can be related to temperature variance dissipation (Gregg 1987; Gargett and Moum 1995). If this can be done, diapycnal buoyancy flux can be inferred. Hence, the general relations among tracer variance dissipations, and mean watermass properties, are topics of considerable dynamical interest.

Buoyancy variance is composed of a linear combi-

Corresponding author address: Dr. Roland A. de Szoeke, College of Oceanic and Atmospheric Sciences, Oregon State University, Oceanography Adm Bldg 104, Corvallis, OR 97331-5503.
E-mail: szoeke@oce.orst.edu

nation of thermal variance $\overline{T'^2}$, salinity variance $\overline{S'^2}$, and the covariance $\overline{T'S'}$. An evolution equation can be obtained for the covariance of two tracers (Stern 1968). This contains a term analogous to the simple variance dissipations, which we call codissipation. The buoyancy variance evolution equation then contains a sum of terms proportional to the simple thermal and salinity dissipations and the codissipation. A similar statement can be made for the variance of any linear combination of temperature and salinity (or any other tracers). The appropriately weighted sums of simple dissipations and codissipation in the variance equations of such composite tracers appear to play a role analogous to dissipation for a simple tracer. The curious fact about composite-tracer "dissipation" is that its positive sign cannot be guaranteed, especially when there is a wide disparity in the molecular diffusivities of the simple tracers of which it is composed. This characteristic is evident in the phenomena of double diffusion and salt fingering, where structures in mixtures of several tracers of disparate molecular diffusivities can grow if their gradients are correlated above a certain threshold.

One particular composite tracer we find especially interesting is the watermass variation between two tracers. This tracer is constructed by subtracting the simple tracers, each scaled by its mean gradient. This combination measures the variation from the mean watermass relation on the tracer–tracer diagram (Stern 1968, 1975). Its variance equation contains no production terms of the standard form of a turbulent flux multiplying a mean gradient (because the latter is zero by construction). Only the composite dissipation term appears, made up of the individual dissipations and the codissipation. Since, for equilibrium, the composite dissipation of watermass variation must vanish, a particular relation is implied between the codissipation and the simple dissipations.

The dissipation of an arbitrary composite tracer is a specific function of the correlation between the gradients of the turbulent fluctuations of the simple tracers. If fluctuation–gradient correlation exceeds a certain level, a certain range of composite-tracer variances will tend to grow. Only a particular value of the fluctuation–gradient correlation will ensure nonnegative dissipation of all possible composite tracers. This value determines a unique relation between the simple tracer dissipations and among the codissipation and the simple dissipations. The same relation is obtained by generalizing Batchelor's (1959) argument for the form of the gradient spectrum of a passive tracer being deformed by a field of isotropic, homogeneous turbulence at statistical equilibrium. As all composite-tracer dissipations are then positive (except for watermass variation, whose variance dissipation is zero), the dissipation for buoyancy, in particular, is proportional to thermal dissipation. The proportionality factor depends on the stability ratio of the mean temperature and salinity gradients, according to a formula obtained by Gargett and Moum (1995) under more restrictive assumptions.

2. Dissipation, codissipation

In this section we derive evolution equations for tracer variance $\overline{C_j'^2}$ and for the covariance of two tracers $\overline{C_1'C_2'}$. The former is routine, while the latter, though derived in a similar way, was obtained by Stern (1968).

Consider a tracer whose concentration per unit volume is C_j in a turbulent solenoidal motion field. The tracer concentration is governed by

$$\frac{D}{Dt}C_j \equiv (\partial_t + \mathbf{u}\cdot\boldsymbol{\nabla})C_j = D_j\nabla^2 C_j, \tag{1a}$$

$$\boldsymbol{\nabla}\cdot\mathbf{u} = 0, \tag{1b}$$

where D_j is the molecular diffusivity of the substance C_j. Suppose the motion and tracer fields consist of an ensemble mean and fluctuations:

$$\mathbf{u} = \overline{\mathbf{u}} + \mathbf{u}', \qquad C_j = \overline{C}_j + C_j'. \tag{2}$$

Substitute in (1) and average over the ensemble

$$\frac{\overline{D}}{Dt}\overline{C}_j = -\boldsymbol{\nabla}\cdot\overline{\mathbf{u}'C_j'} + D_j\nabla^2\overline{C}_j, \tag{3a}$$

$$\boldsymbol{\nabla}\cdot\overline{\mathbf{u}} = 0, \tag{3b}$$

where $\overline{D}/Dt = \partial_t + \overline{\mathbf{u}}\cdot\boldsymbol{\nabla}$ is the substantial derivative following the mean motion. Subtract (3) from (1):

$$\frac{\overline{D}}{Dt}C_j' + \mathbf{u}'\cdot\boldsymbol{\nabla}\overline{C}_j + \boldsymbol{\nabla}\cdot\mathbf{u}'C_j' = \boldsymbol{\nabla}\cdot\overline{\mathbf{u}'C_j'} + D_j\nabla^2 C_j' \tag{4a}$$

$$\boldsymbol{\nabla}\cdot\mathbf{u}' = 0. \tag{4b}$$

Construct an equation for the mean square of fluctuations by multiplying (4a) by $2C_j'$ and averaging:

$$\frac{\overline{D}}{Dt}\overline{C_j'^2} + \boldsymbol{\nabla}\cdot\overline{\mathbf{u}'C_j'^2} = -2\overline{\mathbf{u}'C_j'}\cdot\boldsymbol{\nabla}\overline{C}_j + D_j\nabla^2\overline{C_j'^2} - \chi_j, \tag{5a}$$

(no summation over j) where

$$\chi_j \equiv 2D_j\overline{|\boldsymbol{\nabla}C_j'|^2} \tag{5b}$$

is the mean dissipation of variance of species j.

Construct an equation for the correlation of fluctuations of two tracers C_1, C_2 by writing (4a) for $j = 1$ and $j = 2$, multiplying by C_2' and C_1' respectively, adding and averaging. The result is

$$\frac{\overline{D}}{Dt}\overline{C_1'C_2'} + \boldsymbol{\nabla}\cdot\overline{\mathbf{u}'C_1'C_2'} = -\overline{\mathbf{u}'C_2'}\cdot\boldsymbol{\nabla}\overline{C}_1 - \overline{\mathbf{u}'C_1'}\cdot\boldsymbol{\nabla}\overline{C}_2 + \boldsymbol{\nabla}\cdot(D_1\overline{C_2'\boldsymbol{\nabla}C_1'} + D_2\overline{C_1'\boldsymbol{\nabla}C_2'}) - \chi_{12}, \tag{6a}$$

where

$$\chi_{12} = (D_1 + D_2)\overline{\nabla C_1' \cdot \nabla C_2'} \tag{6b}$$

is the codissipation. Wyngaard et al. (1978) obtained a similar equation in which they called χ_{12} ''molecular destruction.'' They displayed cospectra of temperature and humidity measured in an atmospheric boundary layer, showing clear evidence of a (−5/3)-power wavenumber dependence in an inertial subrange.

The structure of Eq. (5a) is familiar and has been often remarked on (Tennekes and Lumley 1972). The term $-2\overline{\mathbf{u}'C_j'} \cdot \nabla \overline{C}_j$ is called the turbulent variance production. Equation (6) is a little less familiar, though all its terms have counterparts in Eq. (5). The first two terms on the right may be called the covariance production; they tend to increase the covariance whenever the turbulent flux of one tracer is down the gradient of the other. Codissipation χ_{12} and covariance $\overline{C_1'C_2'}$ may have either sign, while χ_j, $\overline{C_j'^2}$ are of course positive. In what follows, we shall neglect the molecular diffusion of variances on the right of (5a) and (6a). We shall also neglect the divergence of triple correlations among velocity and tracers on the left of (5a) and (6a). A scaling argument for this neglect will be presented below.

3. Watermass variation

Under certain circumstances, a linear combination of the fluctuations of C_1' and C_2' may be constructed, for which the variance equation contains no production terms of the form described above. We will call this combination the watermass variation. Its variance can only be increased by the codissipation's assuming the appropriate sign.

Suppose that the mean fields in (5a) and (6a) vary in one dimension only, which we label the z axis. Then the production terms in (5a), (6a) assume the form

$$-\overline{\mathbf{u}'C_j'} \cdot \nabla \overline{C}_k = F_j \partial_z \overline{C}_k, \tag{7a}$$

where

$$F_j = -\overline{w'C_j'}. \tag{7b}$$

As the consequences of this simplification are very important in what follows, we shall examine the circumstances in which it is an adequate approximation. Suppose that the three-dimensional tracer flux can be related to tracer gradients by means of a diffusivity tensor K, that is,

$$-\overline{\mathbf{u}'C'} = \mathsf{K} : \nabla \overline{C},$$

and that the principal axes of K can be found with characteristic values K_1, K_2, K_3 such that $K_1 \sim K_2 \gg K_3$. [The principal axes are often held to be aligned with the isopycnal surfaces in the ocean (Redi 1982; McDougall and Church 1986), although this is not central to the present argument.] Take the z coordinate to be aligned with the third axis of the principal system. Then it is sufficient for the approximation (7a) to be accurate that the vectors $\nabla \overline{C}_1$ and $\nabla \overline{C}_2$ are aligned within order $(K_3/K_1)^{1/2}$ radians of each other and of the z axis. For $K_3 \sim 10^{-5}$–10^{-4} m^2 s^{-1} (Polzin et al. 1997) and $K_1 \sim 10^3$ m^2 s^{-1} (Freeland et al. 1975), this alignment threshold is 1–3 ($\times 10^{-4}$) radians. Inspection of oceanographic sections confirms that this requirement is often fulfilled in the ocean on the large scale.

Substituting (7) into (5a) and (6a) we see that the production terms become, respectively, $2F_j\partial_z\overline{C}_j$ and $F_1\partial_z\overline{C}_2 + F_2\partial_z\overline{C}_1$. If F_1, F_2 are eliminated among (5a) for $j = 1$, 2 and (6a), one obtains

$$2\frac{\overline{D}}{Dt}\overline{C_1'C_2'} - m^{-1}\frac{\overline{D}}{Dt}\overline{C_1'^2} - m\frac{\overline{D}}{Dt}\overline{C_2'^2} + 2\nabla \cdot \overline{\mathbf{u}'C_1'C_2'} - m^{-1}\nabla \cdot \overline{\mathbf{u}'C_1'^2} - m\nabla \cdot \overline{\mathbf{u}'C_2'^2} = \chi_2 m + \chi_1 m^{-1} - 2\chi_{12}, \tag{8}$$

where only the molecular transport divergence terms have been neglected and

$$m = \frac{\overline{C}_{1z}}{\overline{C}_{2z}} = \frac{\partial \overline{C}_1}{\partial \overline{C}_2} \tag{9}$$

is the slope of the mean property–property relation. If we assume that m varies only gradually in time and space, Eq. (8) may be written

$$\frac{\overline{D}}{Dt}\overline{C_\perp'^2} = -\chi_1 - m^2\chi_2 + 2m\chi_{12}, \tag{10a}$$

where

$$C_\perp' = C_1' - mC_2' \tag{10b}$$

is the watermass variation, the combination of the tracer fluctuations C_1', C_2' that measures their displacement from the mean $\overline{C}_1$ versus $\overline{C}_2$ property–property relation. Obtaining this equation entails commuting mean gradients $\overline{C}_{1z}$, $\overline{C}_{2z}$, as though they were constant, with time and space derivatives. The neglected terms can be shown to be of order K_3/L_z^2 times $\overline{C_1'^2}$ or $m^2\overline{C_2'^2}$, where K_3 is the ''vertical'' turbulent diffusivity and L_z is a vertical scale over which $\overline{C}_{1z}$ or $\overline{C}_{2z}$ varies. This estimate can be obtained by supposing that turbulent flux of variance can be represented, for scaling purposes, by

$$-\overline{w'C_1'^2} \sim K_3\partial_z\overline{C_1'^2},$$

with similar relations for other tracer-squared transports. An upper bound for K_3 from microstructure and dye-release experiments on turbulent vertical diffusion in the ocean is 10^{-4} m^2 s^{-1} (Polzin et al. 1997). Hence, by taking $L_z \gtrsim 100$ m, we obtain

$$K_3/L_z^2 \lesssim 10^{-8}\ \mathrm{s}^{-1} \doteqdot (3\ \mathrm{yr})^{-1}.$$

If we assume that microstructure turbulence adjusts to energy production sources and sinks on a much shorter timescale than this, then the neglected terms are indeed small. The divergence terms on the left of (8a) are also of order $(K_3/L_z^2)\overline{C_\perp'^2}$ and are negligible for the same reasons.

Equation (10a) is notable for lacking production terms of the form (7a), that appear in (5a) and (6a). The dissipations χ_1 and χ_2 certainly act to decrease $\overline{C_\perp'^2}$, while the codissipation χ_{12} can increase or decrease $\overline{C_\perp'^2}$, depending on whether it has the same or opposite sign as m. In any case, it follows from (10a), at least on the adjustment timescales of microstructure turbulence, that *only* χ_{12} can increase $\overline{C_\perp'^2}$.

4. Watermass equilibrium and evolution

Next we consider the conditions under which watermass variance $\overline{C_\perp'^2}$ can achieve equilibrium, $\overline{D}\,\overline{C_\perp'^2}/Dt = 0$. We introduce the correlation coefficient of gradients,

$$r = \frac{\overline{\nabla C_1' \cdot \nabla C_2'}}{[\overline{|\nabla C_1'|^2}\ \overline{|\nabla C_2'|^2}]^{1/2}} \tag{11}$$

(where, of course, $|r| \leq 1$), in terms of which the codissipation may be written

$$\chi_{12} = (\chi_1\chi_2)^{1/2}\frac{r}{r_0}, \tag{12a}$$

where

$$r_0 = \frac{2(D_1D_2)^{1/2}}{D_1 + D_2}. \tag{12b}$$

(Note that $r_0 \leq 1$) Hence the right side of (10a) may be written

$$-\chi_2\left[m - \left(\frac{\chi_1}{\chi_2}\right)^{1/2}\frac{r}{r_0}\right]^2 - \chi_1\left(1 - \frac{r^2}{r_0^2}\right). \tag{13}$$

Net production of $\overline{C_\perp'^2}$ is only possible for $|r| > r_0$; otherwise $\overline{C_\perp'^2}$ decays. Watermass variance $\overline{C_\perp'^2}$ is in steady state if the expression (13) is zero; that is,

$$m = \left(\frac{\chi_1}{\chi_2}\right)^{1/2}\left\{\frac{r}{r_0} \pm \left(\frac{r^2}{r_0^2} - 1\right)^{1/2}\right\}. \tag{14}$$

These expressions are only meaningful for $|r| \geq r_0$. We may call them the metastable limits for the property–property slope m. Stern (1968, 1975), considering temperature and salinity as the tracers and defining

$$q = m\left(\frac{\chi_S}{\chi_T}\right)^{1/2}\left(\frac{D_S}{D_T}\right)^{-1/2},$$

obtained (14) as a relation between q and r. For $r = \pm r_0$, the metastable limits coincide in what might be called the stable limit:

$$m = \pm\left(\frac{\chi_1}{\chi_2}\right)^{1/2}. \tag{15}$$

If the two tracers are indeed temperature and salinity, for which $D_1 = 1.4 \times 10^{-7}$ m^2 s^{-1}, $D_2 = 1.0 \times 10^{-9}$ m^2 s^{-1}, respectively, then $r_0 = 0.17$. This modest value does not seem an insuperable barrier for $|r|$ to attain or exceed. For more closely matched molecular diffusivities D_1 and D_2, r_0 is nearer to 1, so it seems conceivable that $|r| < r_0$. The complex value given by (14) in that case for the property–property relation slope simply means that equilibrium for $\overline{C_\perp'^2}$ is impossible while $|r| < r_0$. We shall return to this case in a moment.

If m attains a value given by (14) at which $\overline{C_\perp'^2}$ becomes stationary, it is plausible that $\overline{C_1'^2}$, $\overline{C_2'^2}$ also become stationary, so that from (5a) and (7), divergence of triple correlation terms being neglected,

$$2F_j\,\partial_z\overline{C}_j = \chi_j \tag{16}$$

for $j = 1, 2$. This situation depicts the well-known production–dissipation model proposed by Osborn and Cox (1972). Substituting (16) into (14), one obtains for the metastable property–property slope

$$m = \frac{F_1}{F_2}\left[\frac{r}{r_0} \pm \left(\frac{r^2}{r_0^2} - 1\right)^{1/2}\right]^2. \tag{17}$$

When $|r| = r_0$, the stable slope is

$$m = \frac{F_1}{F_2}. \tag{18}$$

Stommel and Csanady (1980) obtained a value for the *T–S* slope exactly like (18) from consideration of the steady throughput of large-scale heat and freshwater flux between two distinct watermasses.

If one supposes that the turbulent flux of a property is down its mean gradient,

$$F_j = K_j\,\partial_z\overline{C}_j, \tag{19}$$

with turbulent diffusivity coefficient K_j, then one obtains for the turbulent Schmidt number, the ratio of the two diffusivities

$$\frac{K_1}{K_2} = \frac{F_1}{F_2}m^{-1} = \left[\frac{r}{r_0} \pm \left(\frac{r^2}{r_0^2} - 1\right)^{1/2}\right]^{-2}. \tag{20}$$

The two possible values are reciprocals of each other. For $|r| = r_0$, the turbulent Schmidt number is 1.

In any case, nonzero r, and hence nonzero codissipation, is essential for the attainment of an equilibrium for the positive definite quantity $\overline{C_\perp'^2}$. Otherwise the right side of (10a) is purely negative. Evidently some minimal degree of organization of the fluctuating gradients of the random fields C_1', C_2' is necessary so that a correlation of at least $|r| = r_0$ [given by (12b)] is attained in order to achieve equilibrium. This conclusion is rather unexpected since it seems plausible in Eq. (6a) for the evolution of the covariance $\overline{C_1'C_2'}$ to neglect the codissipation term χ_{12}. It is not obviously positive as χ_1, χ_2 are, nor are the gradients $\nabla C_1'$ and $\nabla C_2'$ obviously correlated, and anyway they are multiplied by the sum of molecular diffusivities $D_1 + D_2$, which is small. Indeed, in writing down an evolution equation for buoyancy variance (see below), Gregg (1987) neglected the codissipation. On the contrary, however, the molecular

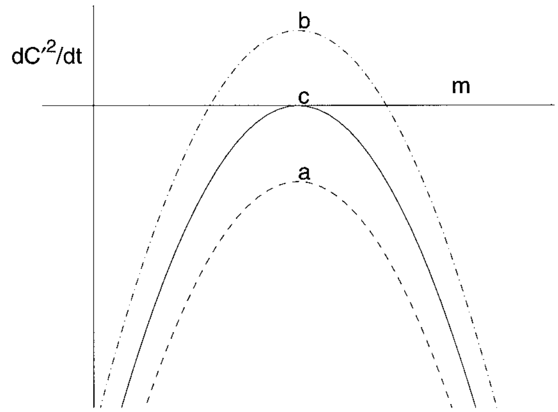

FIG. 1. Watermass variance tendency $\partial_t\overline{C'^2_\perp}$ as a function of mean tracer–tracer slope $m = \partial\overline{C_1}/\partial\overline{C_2}$ for (a) small codissipation, $|r| < r_0$; (b) large codissipation, $|r| > r_0$; and (c) marginal codissipation, $|r| = r_0$.

diffusivities determine the threshold value (12b) of the gradient correlation, which must be attained or exceeded for equilibrium.

Some qualitative remarks can be made about the evolution of $\overline{C'^2_\perp}$, the gradient-correlation coefficient *r*, and perhaps on a longer timescale, the property–property slope *m* from the form of Eq. (10a). The right side of (10a) must be negative if $|r| < r_0$ [case A]; it is positive for *m* between the bounds given by (14), negative outside those bounds as long as $|r| > r_0$ (case B). It is negative for $|r| = r_0$, except at the value given by (15) where it is zero (case C). These cases are shown schematically in Fig. 1.

Consider case A. It seems that $\overline{C'^2_\perp}$ must decrease forever at a finite rate, which is absurd. But *r* may evolve during this process. First, the fluxes F_1, F_2, linked to the dissipations χ_1, χ_2 approximately through Eq. (16), may adjust to bring

$$\left(\frac{\chi_1}{\chi_2}\right)^{1/2}\frac{r}{r_0}$$

close to *m*, which gives the minimum decay rate for $\overline{C'^2_\perp}$. Second, as $\overline{C'^2_\perp}$ decays, the correlation between C'_1 and C'_2 becomes stronger (with regression coefficient *m*). While this correlation is probably dominated by low wavenumbers and frequencies, it is plausible that it extends to the higher wavenumbers and frequencies that dominate the spectra of $\nabla C'_1$ and $\nabla C'_2$. Hence, the gradient-correlation coefficient $|r|$ may be expected to increase, bringing the minimum absolute value of the right side of (10a) closer to zero. These simultaneous adjustments of F_1, F_2, and *r* while $\overline{C'^2_\perp}$ decreases may be expected to continue until the value (15) or (18) at $|r| = r_0$ is achieved, upon which $\overline{C'^2_\perp}$ is stationary.

On a much longer timescale, the property–property slope *m* may also adjust. If the fluxes F_1, F_2, adjusted to local equilibrium as just described, become very dissimilar to nearby F_1, F_2, values or to externally imposed boundary conditions (e.g., surface heat and freshwater fluxes in the case where C_1, C_2 are temperature and salinity), then local convergences or divergences of tracer transports must result, slowly changing the mean gradients of $\overline{C_1}$, $\overline{C_2}$, and hence *m*. In any case, the adjustments must tend toward stationary $\overline{C'^2_\perp}$.

Case B presents two situations.

(i) *Decay of watermass variance.* Suppose that *m* is initially outside the bounds of the two metastable limits given by (14). Then $\overline{C'^2_\perp}$ decreases at a fixed rate, and to avert an absurdity ($\overline{C'^2_\perp}$ may not become negative) $|r|$ must increase, for fixed flux ratio F_1/F_2, until it reaches a value at which the local *m* coincides with one of the metastable limits (14) and $\overline{C'^2_\perp}$ becomes stationary.

(ii) *Double diffusion.* If *m* is initially between the metastable limits given by (14), $\overline{C'^2_\perp}$ tends to grow. This condition encompasses double-diffusive processes, both ''fingering'' and ''diffusion'' (Turner 1973). Double-diffusion is most effective when large disparities in diffusion coefficients D_1, D_2 give a low threshold correlation coefficient r_0. Then the degree of organization in the structures involved in these processes is evidently sufficient to lead to growth of $\overline{C'^2_\perp}$. The conventional schematic explanation for salt fingering is instructive for understanding Eq. (10a). Suppose two water types, A (cold, fresh) and B (warm, salty), are brought close together (Fig. 2) with the former lying below the latter. In any disturbance deforming the interface between the two types the faster diffusing ''substance'' (usually temperature) will tend to equalize across the interface much more than the slower (salinity). This means local evolution of the two waters to A′, B′ as shown in Fig. 2. Because A′ may be lighter than B′, since fresher, it may experience a buoyant upward acceleration causing the initial disturbance to grow. The dynamics and energetics of salt fingering are beyond the scope of this discussion. What is relevant in this context is that watermass variation tends to grow. The line joining A, B in the property–property diagram in Fig. 2b represents waters for which $C'_\perp = 0$; lines parallel to this but displaced from it represent nonzero isopleths of $C'_\perp$. So the generation of the waters A′, B′ implies creation of $\overline{C'^2_\perp}$. This can only come from the term $m\chi_{12}$ on the right side of (10a), which is positive because

$$\overline{\nabla T'\cdot\nabla S'} = \overline{\frac{\partial T'}{\partial x}\frac{\partial S'}{\partial x}} > 0$$

in the vicinity of the disturbance in Fig. 2 and is evidently large enough to overcome $-\chi_1 - m^2\chi_2$ for growing salt fingers. This illustrates graphically, and in a familiar context, that the generation term for $\overline{C'^2_\perp}$ is the codissipation. Again we stress that this must be so because Eq. (10a) lacks terms of the usual production form, such as $-\overline{\mathbf{u}'C'_j}\cdot\nabla\overline{C}_k$ in (5a) or (6a). In Eq. (6a), however, the codissipation still acts to reduce the covariance of *T* and *S.*

To prevent unlimited growth of $\overline{C'^2_\perp}$, *r* should adjust

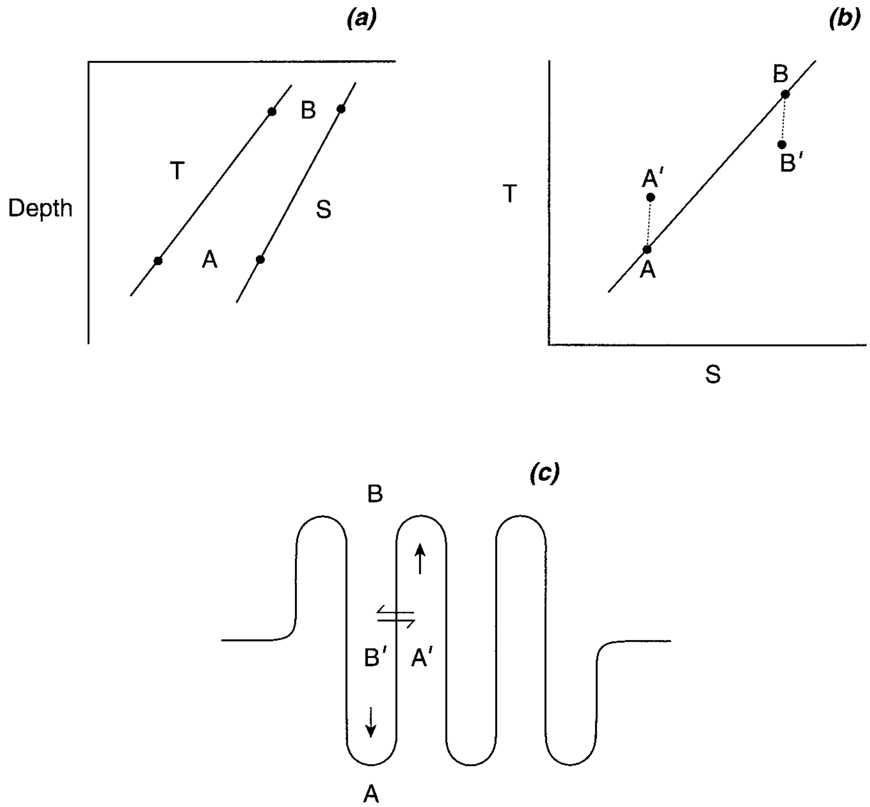

FIG. 2. Schematic of salt fingering. (a) Mean temperature and salinity profiles. (b) *T–S* diagram. (c) A disturbance on an interface between cold, fresh water (A) and warm, salty water (B) above. Interpenetrating fingers exchange heat readily, but very little salinity. Alternating fingers move to configurations A′, B′ on *T–S* diagram, tending to make watermass variation $\overline{C_\perp'^2}$ increase.

so that m assumes one of the metastable limits (14). For reasons opposite to those enunciated for case (a), the gradient-correlation coefficient $|r|$ should decrease, bringing the limits (14) closer to the given m. It is conceivable that this process may be arrested by attaining a stationary value of $\overline{C_\perp'^2}$ at some m given by (14) for $|r| > r_0$ strictly, as in (i). However, it is also possible that $|r| \to r_0$ from above so as to match the stable limit of m given by (15) or (18).

Watermass stability. Case C is like case A or case B(i): the right side of (10a) is negative. This again leads to the absurd situation of $\overline{C_\perp'^2}$ decaying at a finite rate forever unless $|r| = r_0$ is achieved, along with $\chi_{12}/\chi_2 = m$. At this point $\overline{C_\perp'^2}$ becomes stationary, and

$$\chi_{12} = \pm(\chi_1\chi_2)^{1/2}, \qquad m = \pm\left(\frac{\chi_1}{\chi_2}\right)^{1/2}. \qquad (21)$$

(The signs should be chosen so as to give χ_{12} and m the same sign as $\overline{C_1'C_2'}$.)

5. Composite tracers

Tracers whose molecular transport is governed by the usual downgradient molecular diffusion law we call *simple.* Any linear combination of simple tracers we may call a composite tracer: Its molecular diffusion is not given by the usual downgradient law if the molecular diffusivities of the individual tracers differ. The watermass variation that we considered above is an example of a composite tracer. Another example, which occurs in practical oceanographic applications, is conductivity, which may be approximately linear over a suitably restricted range of temperature and salinity. A third example of great dynamical importance is furnished by buoyancy, also to be considered an approximately linear function of temperature and salinity. We may obtain an evolution equation for composite tracer variance in which the combination of terms that seem analogous to dissipation cannot be guaranteed to be positive. If we assume that the codissipation of the component simple tracers has adjusted to a metastable value, given by (14), in order to ensure at least the stationarity of watermass variance, this is still not sufficient to guarantee the stationarity of all conceivable composite tracers. However, adopting the principle that all composite tracers possess positive dissipation rates leads to the conclusion that the codissipation must assume the stable value given by (21).

An equation for the variance of the composite tracer

$s' = a_1 C_1' + a_2 C_2'$ can be formed by taking the appropriate combination of (5a) (for $j = 1, 2$) and (6a). This gives

$$\frac{\overline{D}}{Dt}\overline{s'^2} = P_s - \chi_s, \tag{22a}$$

where

$$P_s = -2\overline{w's'}\,\overline{s}_z, \qquad \chi_s = a_1^2\chi_1 + a_2^2\chi_2 + 2a_1a_2\chi_{12}, \tag{22b, c}$$

and $\overline{s} = a_1\overline{C_1} + a_2\overline{C_2}$. (Divergences of molecular diffusive fluxes of variance, and of triple correlation fluxes, have been neglected.) Equation (10a) may be obtained as a special case of (22) by setting $a_1 = 1$, $a_2 = -m = -\overline{C_{1z}}/\overline{C_{2z}}$. In that case $\nabla\overline{s} = \hat{\mathbf{z}}\overline{s}_z = 0$. Equations (22b, c) define χ_s and P_s, the composite dissipation of $\overline{s'^2}$ and its production. For the stationarity of $\overline{C_\perp'^2}$, we concluded above that one of the metastable limits (14), which relate the property–property slope m to r/r_0 [or scaled codissipation $\chi_{12}/(\chi_1\chi_2)^{1/2}$; see Eq. (12)] and χ_1/χ_2, must pertain. By substituting (14) and (12) into the definition of χ_s, Eq. (22), the latter may be written

$$\frac{\chi_s}{a_1^2\chi_1} = 1 + R_s^{-2}\phi^2 - 2R_s^{-1}\frac{r}{r_0}\phi, \tag{23a}$$

where

$$R_s = -\frac{a_1}{a_2}m, \qquad \phi = \frac{r}{r_0} \pm \left(\frac{r^2}{r_0^2} - 1\right)^{1/2}. \tag{23b, c}$$

We may call R_s the mean tracer gradient ratio. There are two sign possibilities in the definition of ϕ; these give values that are the inverses of each other: $\phi_+\phi_- = 1$. The correlation coefficient r and ϕ_+, ϕ_- all have the same sign. Hence $r\phi/r_0$ is positive. Without loss of generality we can take $r > 0$. An alternative form of (23a) is

$$\frac{\chi_s}{a_1^2\chi_1} = (1 - R_s^{-1})R_s^{-1}\left\{R_s - \left[\frac{r}{r_0} + \left(\frac{r^2}{r_0^2} - 1\right)^{1/2}\right]^2\right\}, \tag{24}$$

where the positive branch of (23c) has been taken; a similar form can be obtained by taking the negative root. From (23a), $\chi_s > 0$ for $R_s < 0$. However, for $R_s > 0$, χ_s vanishes for

$$\frac{r}{r_0} = \frac{R_s^{1/2} + R_s^{-1/2}}{2}. \tag{25}$$

For r/r_0 greater than this value, one of the branches of (23c) gives negative values of χ_s, as Fig. 3, which shows $\chi_s/a_1^2\chi_1$ for two positive values of R_s, demonstrates. This implies a tendency for the growth of $\overline{s'^2}$. On Fig. 4, where the principal C_1 versus C_2 watermass relation is shown by the line with slope $\partial C_2/\partial C_1 = 1/m$, C_1 versus C_2 variability parallel to the horizon with slope

$$\partial C_2/\partial C_1 = -a_1/a_2 = R_s/m$$

will tend to grow if it contains structures with gradient correlation coefficients exceeding the threshold given by (25). Put the other way round, given structures with $r/r_0 > 1$, fluctuations about horizons with slope $\partial C_2/\partial C_1 = R_s/m$ will tend to grow, if

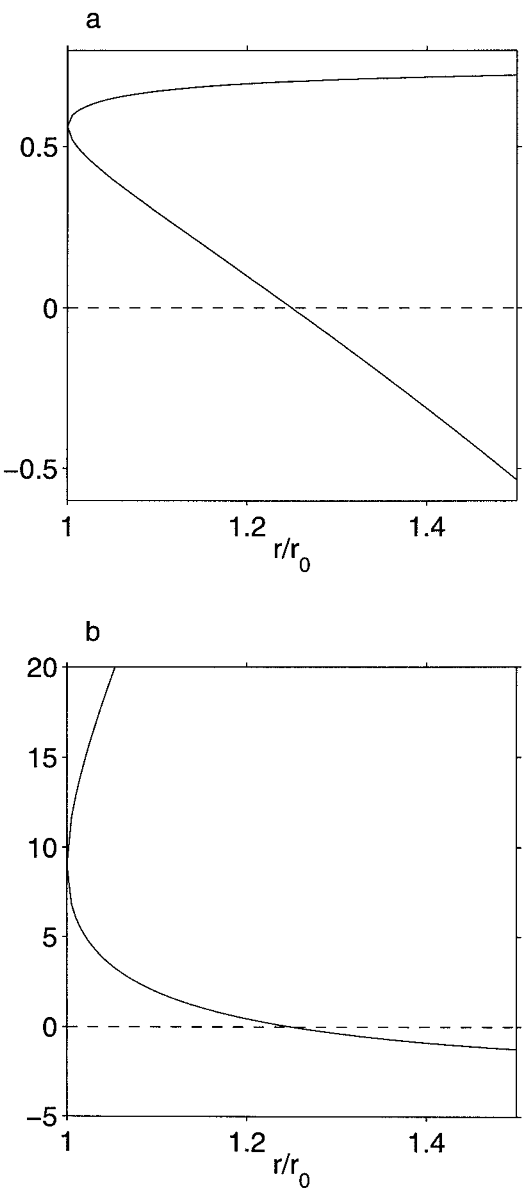

FIG. 3. Scaled composite dissipation $\chi_s/a_1^2\chi_1$ as a function of scaled codissipation gradient $\chi_{12}/(\chi_1\chi_2)^{1/2} = r/r_0$ for mean tracer gradient ratio $R_s = 4$ (a), and $R_s = 0.25$ (b).

$$1 < R_s < R_M, \tag{26a}$$

or

$$R_M^{-1} < R_s < 1, \tag{26b}$$

where

$$R_M = \left[\frac{r}{r_0} + \left(\frac{r^2}{r_0^2} - 1\right)^{1/2}\right]^2. \tag{26c}$$

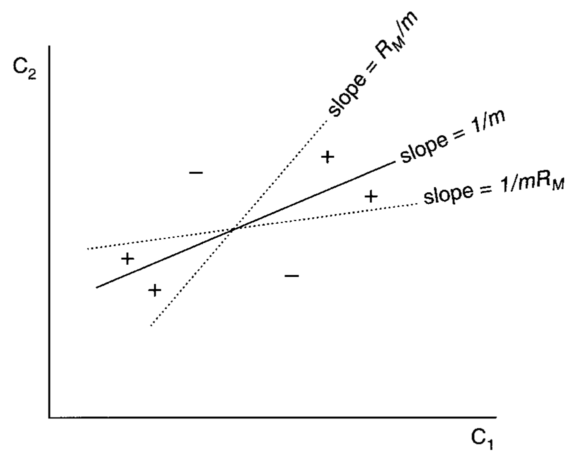

FIG. 4. The schematic C_1 vs C_2 watermass diagram showing the mean watermass relation with slope $\partial C_2/\partial C_1 = 1/m$, and the horizon limits given by (26) between which composite tracers, constant along lines of slope $\partial C_2/\partial C_1 = R_s/m$, tend to grow, if structures having gradient correlation $r > r_0$ are present.

Because $\chi_s/a_1^2\chi_1 = 0$ for $R_s = 1$, this case is explicitly excluded from these ranges. To ensure that no composite tracer variance grows as a consequence of χ_s being negative for some R_s, it is necessary to require that

$$r = r_0. \tag{27}$$

We may call the idea that leads to this requirement the principle of nonnegative dissipation of composite tracers because, as well as ensuring equilibrium of watermass variance $\overline{C_\perp'^2}$ (i.e., zero dissipation), it guarantees that all other composite tracers have positive dissipation χ_s. This dissipation can presumably be supplied at equilibrium by the production term P_s in (22a). It is conceivable that this principle is violated in the ocean by structures whose gradient-correlation coefficient exceeds r_0. But then either any member of the ranges of composite tracers given by (26) grows, or the turbulent flux of that tracer is against its mean gradient, that is,

$$P_s < 0.$$

The latter implies a reversal of the usual intuitive interpretation of the variance growth balance (22a)! From (27) it follows that codissipation is

$$\chi_{12} = \text{sgn}m(\chi_1\chi_2)^{1/2}, \tag{28}$$

and

$$\chi_1^{1/2} = |m|\chi_2^{1/2} \tag{29}$$

[cf. (15)]. Remarkably, these results coincide with a calculation of χ_{12} from application of the arguments of Batchelor (1959) to the joint straining of the scalar fields C_1, C_2 by the motion field of a three-dimensional, isotropic, homogeneous, inertial subrange of turbulence. (This argument is given in the appendix.) But (28) followed merely from the principle that all composite tracers possess nonnegative dissipation rates. This principle says nothing about the spectral constituents of χ_{12}, whether they are homogeneous or isotropic, or about the existence of an inertial subrange of turbulent motion. However, the Batchelor (1959) argument does suppose turbulent spectra to be at equilibrium, and this appears to be the reason for the concurrence of the two lines of argument. From (28) and (29), it follows that

$$\chi_s = a_1^2\chi_1(1 - R_s^{-1})^2. \tag{30}$$

The result (28) is like Hill's (1989a) deduction from consideration of the similarity of scalar dissipation and codissipation profiles for temperature and water vapor in an atmospheric boundary layer.

It has been suggested that differential turbulent vertical diffusion of temperature and salinity, perhaps by double-diffusive or salt-fingering processes, is an important mechanism for determining and altering the thermohaline circulation (Gargett and Ferron 1996). We saw above that the ratio of the turbulent diffusivities K_1/K_2 of two tracers is related to r/r_0 by (20). For K_1/K_2 different from 1, r/r_0 must exceed 1. Then it follows from the remarks above, and the inequalities given by (26), that the variance of composite tracers corresponding to horizons with slopes between K_1/mK_2 and K_2/mK_1 (but excluding the watermass horizon of slope $1/m$) will grow or experience countergradient fluxes. This situation certainly does not satisfy the principle of nonnegative dissipation of composite tracer variances. But the deduction of growth, or countergradient flux, of a range of composite tracer variances constructed from T and S poses a test of the Gargett–Ferron suggestion.

6. Buoyancy dissipation and transport

For the buoyancy anomaly $b' = g\alpha T' - g\beta S'$ (where $C_1' = T'$, $C_2' = S'$, $a_1 = g\alpha$, $a_2 = -g\beta$), Eq. (22) is

$$\frac{\overline{D}}{Dt}\overline{b'^2} = P_b - \chi_b, \tag{31a}$$

where

$$P_b = -2\overline{w'b'}\,\overline{b}_z, \tag{31b}$$

and from (30),

$$\chi_b = g^2\alpha^2\chi_T(1 - R_b^{-1})^2, \quad \text{where} \quad R_b = \frac{\alpha\overline{T}_z}{\beta\overline{S}_z}. \tag{32}$$

The parameter R_b is called the stability ratio; $\chi_T = 2D_T\overline{|\nabla T'|^2}$ is dissipation of thermal variance. Buoyancy dissipation is given by the thermal dissipation, multiplied by a factor depending on the stability ratio. Gargett and Moum (1995) proposed the relation (32) from an argument based on the assertion that $T' \simeq (\overline{T}_z/\overline{S}_z)S'$ at variance-dominating scales. This is a far stronger assertion than requiring that the variance of all linear combinations of T' and S' should be stationary. From stationarity and the neglect of transport divergence terms in (31), the component of buoyancy transport down the mean gradient $\overline{b}_z$ can be inferred:

$$-2\overline{w'b'}\,\overline{b}_z = \chi_b \quad (33)$$

(Osborn and Cox 1972), where of course χ_b is given by (32). This hypothesis has been questioned by Davis (1994), essentially on the grounds of the redness of the spectra of temperature and salinity variability. It may be argued, on the other hand, that Davis' objection can be met by large-scale averaging on isopycnals, themselves defined by prefiltering over microstructure scales (de Szoeke and Bennett 1993). This procedure serves to truncate the low-frequency contributions to variance and the effect of their secular variation in (31).

7. Summary

The main result of this paper is the deduction that the correlation coefficient of tracer gradients should be at the value $r_0 = 2(D_1D_2)^{1/2}/(D_1 + D_2)$. This follows from the principle that the dissipation of all composite tracers should be nonnegative. An alternative statement of the result is that the codissipation of the tracer covariance is given by the geometric mean of the individual tracer variance dissipations. This result coincides with the conclusions based on extensions and modifications of Batchelor's (1959) arguments for scalar spectra in the dissipation range to give the form of the cospectrum of two passive tracers being deformed by an equilibrium field of isotropic, homogeneous, turbulent motion. A further consequence of the result is that the mean slope of the tracer–tracer watermass relation should coincide with the square root of the ratio of dissipations of the tracers and with the ratio of turbulent tracer fluxes down their respective gradients. The latter concurs with an exactly similar relation obtained by Stommel and Csanady (1980) from consideration of how large-scale heat and freshwater transports, imposed at the ocean surface, determine watermass relations through the necessity of closing heat and salt budgets.

It also follows that the ratio of turbulent diffusivities of two tracers is unity. This is at variance with the suggestion that this ratio is not 1, made by Gargett and Ferron (1996), who examined the consequences of this proposition for the thermohaline circulation.

A further consequence is that the buoyancy variance dissipation is proportional to temperature variance dissipation, with a proportionality factor of $g^2\alpha^2(1 - R_b^{-1})^2$, R_b being the density ratio. (A similar relation can be obtained for an arbitrary composite tracer.) This result accords with Gargett and Moum's (1995) proposed formula for the buoyancy variance dissipation. We must stress that this result goes hand in hand with the other consequences, summarized here, from the principle of nonnegative composite-tracer dissipations.

Relaxing the principle of nonnegative dissipation for all composite tracers, except for the so-called watermass variation whose dissipation must be zero, leads to a less restrictive relation among tracer gradient correlation (or codissipation), watermass-diagram slope, and the ratio of tracer dissipations. It also leads to a specific relation between the ratio of turbulent diffusivities of the tracers and the tracer gradient correlation; this ratio may differ from 1 if the latter is greater than r_0. A consequence of this, it must be stressed, is that a range of composite tracers contained in horizons surrounding the mean watermass relation will experience negative dissipation. Hence the variances of these tracers must grow or their turbulent fluxes must be against their mean gradients (negative production).

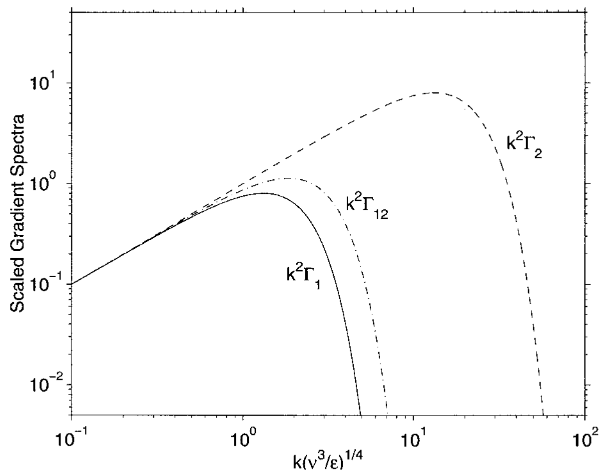

FIG. 5. Schematic showing the scaled scalar gradient spectra $k^2\Gamma_1, k^2\Gamma_2$ and cospectrum $k^2\Gamma_{12}$ for $D_1/D_2 = 100$.

These conclusions are drawn for circumstances in which large-scale mean gradients of tracers are collinear, although it seems sufficient that the mean gradients should be aligned to within order $(K_D/K_H)^{1/2}$, where K_D, K_H are the turbulent diapycnal (vertical) and horizontal diffusion coefficients. This criterion covers a wide range of circumstances in the ocean, although it may not pertain in regions of strong currents, such as the western boundary currents, or in regions where strong intrusive mixing occurs so that $\nabla\overline{T}$ and $\nabla\overline{S}$ are significantly misaligned.

Temperature variance dissipation χ_T is frequently directly measured in the ocean (Dillon and Caldwell 1980; Gregg 1987). Variance dissipations of other tracers, for example, salinity, have not been directly measured because of stringent instrumental resolution requirements. The gradient spectrum of a tracer is peaked at a wavenumber that scales with $(\text{diffusivity})^{-1/2}$. The band around this wavenumber contributes dominantly to the tracer dissipation. For salinity, with $D_S \sim 10^{-2}D_T$, this band occurs around a wavenumber a decade higher than for temperature dissipation, itself peaked in the band 1–10 cm (Fig. 5). Measurements at such small scales have been beyond the limits of practicability. Measurement of codissipation χ_{TS} entails similar difficulties. However, conductivity sensors with very fine resolution capability are beginning to be used to probe into the dissipation range of salinity (Nash and Moum 1998; Nash et al.

1998, manuscript submitted to *J. Atmos. Oceanic Technol.*). The results of this paper pose relationships among tracer variance dissipations that can be used to infer one tracer dissipation from another, for example, and that invite experimental validation, for their own sake and for their wider implications for ocean mixing and circulation.

Acknowledgments. It is a pleasure to acknowledge useful discussions with Clayton Paulson. This research was supported by the National Science Foundation under Grant numbers 9319892 and 9402891 and by the Jet Propulsion Laboratory under the TOPEX/Poseidon Extended Mission Announcement of Opportunity Contract 958127.

APPENDIX

The Cospectrum of Diffusive Scalars

An expression is derived for the homogeneous, isotropic cospectrum of two diffusive scalars at wavenumbers higher than the Kolmogorov wavenumber $(\epsilon/\nu^3)^{1/4}$.

Batchelor (1959) showed that the isotropic autospectrum of a scalar C_1 in the convective–diffusive range of wavenumber $k \gg (\epsilon/\nu^3)^{1/4}$, where ϵ is kinetic energy dissipation and ν is kinematic viscosity, is

$$\Gamma_1(k) = \frac{\chi_1}{\gamma' k} \exp\left(-\frac{D_1 k^2}{\gamma'}\right). \tag{A1}$$

In this expression

$$\gamma' = \frac{1}{q}\left(\frac{\epsilon}{\nu}\right)^{1/2}$$

is an estimate of the most negative principal strain-rate magnitude [Batchelor (1959) chose $q = 2.0$ for the parameter in this formula; others (Gibson 1968; Grant et al. 1968; Williams and Paulson 1977; Dillon and Caldwell 1980) argued for larger values]; D_1 is the molecular diffusivity of the scalar; and $\chi_1 = 2D_1\overline{|\nabla C_1'|^2}$ is the dissipation rate of the scalar variance $\overline{C_1'^2}$. It may easily be verified that

$$\int_0^\infty k^2\Gamma_1(k)\, dk = \frac{\chi_1}{2D_1}. \tag{A2}$$

The purpose of this appendix is to derive the isotropic cospectrum of two scalars C_1 and C_2. In terms of the lagged covariance

$$S_{12}(s) = \overline{C_1'(\mathbf{x})C_2'(\mathbf{x} + \mathbf{r})} \tag{A3}$$

which, being homogeneous and isotropic, is held to depend only on the displacement magnitude $s = |\mathbf{r}|$, the cospectrum is given by

$$\Gamma_{12}(k) = \frac{2}{\pi}\int_0^\infty S_{12}(s) ks \sin ks\, ds, \tag{A4}$$

where $k = |\mathbf{k}|$ is wavenumber magnitude. The quadrature spectrum is zero because of isotropy. By arguments similar to those given by Batchelor (1959), the form of the cospectrum is, in the wavenumber range stated above,

$$\Gamma_{12}(k) = \frac{\chi_{12}}{\gamma' k} \exp\left\{-\frac{(D_1 + D_2)k^2}{2\gamma'}\right\}, \tag{A5}$$

where D_2 is the diffusivity of scalar C_2, and $\chi_{12} = (D_1 + D_2)\overline{\nabla C_1' \cdot \nabla C_2'}$ is the codissipation. The gradient covariance, dominated by wavenumbers from the diffusive wavenumber range, is

$$\overline{\nabla C_1' \cdot \nabla C_2'} = \int_0^\infty k^2\Gamma_{12}(k)\, dk = \frac{\chi_{12}}{D_1 + D_2}. \tag{A6}$$

Hill (1978) reviewed arguments and evidence for several semiempirical models of the shapes of scalar spectra and cospectra in the inertial, viscous–convective, and diffusive wavenumber ranges. For scalars and fluids with large Prandtl numbers ν/D_k, such as temperature or salinity in water, the spectral shape used in (A1) is well supported (Dillon and Caldwell 1980). The scaled forms of gradient spectra $k^2\Gamma_1(k)$, $k^2\Gamma_2(k)$, $k^2\Gamma_{12}(k)$ are shown in Fig. 5 for $D_1/D_2 = 100$, appropriate for temperature and salinity in water.

In the viscous–convective wavenumber band $(\epsilon/\nu^3)^{1/4} < k \ll (\epsilon/\nu D_1^2)^{1/4}$ (assuming $\nu \gg D_1 > D_2$) both C_1 and C_2 are being strained by the turbulent motion field, and neither is much attenuated by its respective diffusion. Hence it seems plausible that the correlation of the contributions of the gradients of C_1 and C_2 from this band should be high. That is, the band-limited correlation coefficient, defined by

$$r_{BL} = \frac{|\overline{\nabla C_1' \cdot \nabla C_2'}^{BL}|}{\left(\overline{|\nabla C_1'|^2}^{BL}\,\overline{|\nabla C_2'|^2}^{BL}\right)^{1/2}} = \frac{\left|\int_{k_\nu}^{k_1} k^2\Gamma_{12}(k)\, dk\right|}{\left(\int_{k_\nu}^{k_1} k^2\Gamma_1(k)\, dk \cdot \int_{k_\nu}^{k_1} k^2\Gamma_2(k)\, dk\right)^{1/2}}, \tag{A7}$$

where $k_\nu = (\epsilon/\nu^3)^{1/4}$, $k_1 \ll (\epsilon/\nu D_1^2)^{1/4}$, should be nearly 1. Substitution from (A5) shows that

$$\int_{k_\nu}^{k_1} k^2\Gamma_{12}(k)\, dk \simeq \frac{1}{2}\chi_{12}\gamma'^{-1}k_1^2,$$

with similar forms for Γ_1, Γ_2. Hence, from (A7),

$$r_{BL} \simeq \frac{|\chi_{12}|}{(\chi_1\chi_2)^{1/2}} \simeq 1. \tag{A8}$$

It is interesting to observe that, from (A2), (A6), and (A8), the whole-band correlation coefficient is

$$r = \frac{|\overline{\nabla C_1' \cdot \nabla C_2'}|}{\left(\overline{|\nabla C_1'|^2}\,\overline{|\nabla C_2'|^2}\right)^{1/2}} \simeq \frac{2(D_1 D_2)^{1/2}}{D_1 + D_2} \leq 1. \quad \text{(A9)}$$

Equation (A8) shows that the codissipation is

$$\chi_{12} = \pm(\chi_1 \chi_2)^{1/2}. \quad \text{(A10)}$$

Its tendency should be to reduce the magnitude of covariance $\overline{C_1' C_2'}$ in Eq. (6a) in the main text. Hence, we should choose the sign of χ_{12} to be that of $\overline{C_1' C_2'}$. [Where the correlation of C_1 and C_2 is zero, or statistically insignificant, so too will be the correlation of the gradients $\overline{\nabla C_1' \cdot \nabla C_2'}$, and the argument that follows from setting Eq. (A7) to 1 will fail.] If C_1, C_2 are seawater properties (e.g., temperature, salinity, etc.), then the sign of the correlation $\overline{C_1' C_2'}$ is very likely the same as the sign of the slope $\partial\overline{C}_1/\partial\overline{C}_2$ of the mean property–property relation that pertains in the vicinity of the point of interest. (Note that this is a far weaker assertion than maintaining that $C_1' \simeq (\partial\overline{C}_1/\partial\overline{C}_2)C_2'$.) The relation (A10) is often asserted for scalars in the atmosphere, such as temperature and humidity for which the Prandtl number is not large (Antonia et al. 1978; Hill 1989b), so that the principle enunciated in (A7) and (A8) can scarcely apply. Even so, some indirect evidence for (A10), for $\nu/D_k \sim 1$, is available (Andreas 1987).

It may not be necessary to the argument for a three-dimensional, homogeneous, isotropic scalar cospectrum of the form (A5) that there be an isotropic, homogeneous spectrum of turbulent velocity with an inertial subrange at scales larger than $(\nu^3/\epsilon)^{1/4}$ to provide the strain rate.

REFERENCES

Andreas, E. L., 1987: On the Kolmogorov constants for the temperature–humidity cospectrum and the refractive index spectrum. *J. Atmos. Sci.,* **44,** 2399–2406.

Antonia, R. A., A. J. Chambers, and C. A. Friehe, 1978: Statistical properties of optical refractive index fluctuations in the marine boundary layer. *Bound.-Layer Meteor.,* **15,** 243–253.

Batchelor, G. K., 1959: Small-scale variation of convected quantities like temperature in turbulent fluid. Part 1. *J. Fluid Mech.,* **5,** 113–133.

Davis, R. E., 1994: Diapycnal mixing in the ocean: The Osborn–Cox model. *J. Phys. Oceanogr.,* **24,** 2560–2576.

de Szoeke, R. A., and A. F. Bennett, 1993: Microstructure fluxes across density surfaces. *J. Phys. Oceanogr.,* **23**, 2254–2264.

Dillon, T. M., and D. R. Caldwell, 1980a: The Batchelor spectrum and dissipation in the upper ocean. *J. Geophys. Res.,* **85,** 1910–1916.

Freeland, H. J., P. B. Rhines, and T. Rossby, 1975: Statistical observations of trajectories of neutrally buoyant floats in the North Atlantic. *J. Mar. Res.,* **33,** 383–404.

Gargett, A. E., and J. N. Moum, 1995: Mixing efficiencies in turbulent tidal fronts: Results from direct and indirect measurements of density flux. *J. Phys. Oceanogr.,* **25,** 2583–2608.

——, and B. Ferron, 1996: The effects of differential vertical diffusion of T and S in a box model of thermohaline circulation. *J. Mar. Res.,* **54,** 827–866.

Gibson, C. H., 1968: Finestructure of scalar fields mixed by turbulence. *Phys. Fluids,* **11,** 2305–2327.

Grant, H. L., B. A. Hughes, N. M. Vogel, and A. Moilliet, 1968: The spectrum of temperature fluctuations in turbulent flow. *J. Fluid Mech.,* **34,** 423–442.

Gregg, M. C., 1987: Diapycnal mixing in the thermocline: A review. *J. Geophys. Res.,* **92,** 5249–5286.

Hill, R. J., 1978: Models of the scalar spectrum for turbulent advection. *J. Fluid Mech.,* **88,** 541–562.

——, 1989a: Implications of Monin–Obukhov similarity theory for scalar quantities. *J. Atmos. Sci.,* **46,** 2236–2244.

——, 1989b: Structure functions and spectra of scalar quantities in the inertial-convective and viscous–convective ranges of turbulence. *J. Atmos. Sci.,* **46,** 2245–2251.

McDougall, T. J., and J. A. Church, 1986: Pitfalls with the numerical representation of isopycnal and diapynal mixing. *J. Phys. Oceanogr.,* **16**, 196–199.

Nash, J. D., and J. N. Moum, 1998: Estimating salinity variance dissipation rate from conductivity microstructure measurements. *J. Atmos. Oceanic Technol.,* in press.

——, D. R. Caldwell, M. J. Zelman, and J. N. Moum, 1998: A new thermocouple sensor for resolving small-scale temperature fluctuations. *J. Atmos. Oceanic Technol.,* in press.

Osborn, T. R., and C. S. Cox, 1972: Oceanic fine structure. *Geophys. Fluid Dyn.,* **3,** 321–345.

Polzin, K. L., J. M. Toole, J. R. Ledwell, and R. W. Schmitt, 1997: Spatial variability of turbulent mixing in the abyssal ocean. *Science,* **276,** 93–96.

Redi, M. H., 1982: Oceanic isopycnal mixing by coordinate rotation. *J. Phys. Oceanogr.,* **12**, 1154–1158.

Stern, M. E., 1968: T-S gradients on the micro-scale. *Deep-Sea Res.,* **15,** 245–250.

——, 1975: *Ocean Circulation Physics.* Academic Press, 246 pp.

Stommel, H. M., and G. T. Csanady, 1980: A relation between the T–S curve and global heat and atmospheric water transports. *J. Geophys. Res.,* **85**(C1), 495–501.

Tennekes, H., and J. L. Lumley, 1972: *A First Course in Turbulence.* The MIT Press, 300 pp.

Turner, J. S., 1973: *Buoyancy Effects in Fluids.* Cambridge University Press, 367 pp.

Wyngaard, J. C., W. T. Pennell, D. H. Lenschow, and M. A. LeMone, 1978: The temperature–humidity covariance budget in the convective boundary layer. *J. Atmos. Sci.,* **35,** 47–58.

Williams, R. M., and C. A. Paulson, 1977: Microscale temperature and velocity spectra in the atmospheric boundary layer. *J. Fluid Mech.,* **83,** 547–567.