Comments on ''On the Obscurantist Physics of 'Form Drag' in Theorizing about the Circumpolar Current*

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1. Introduction

In recent years the interest in the dynamical balance of the Antarctic Circumpolar Current (ACC) has seen a renaissance in oceanographic research. Various papers deal with the barotropic circulation in channels with simple topography—these are largely analytical solutions (e.g., Johnson and Bryden 1989; Krupitsky and Cane 1994; Wang and Huang 1995; Olbers and Völker 1996)—and a few numerical solutions have appeared, noteworthy eddy-resolving quasigeostrophic channel circulations (McWilliams et al. 1978; Wolff et al. 1991; Olbers 1993; Marshall et al. 1993) and the FRAM experiment (FRAM Group 1991). Apart from subtleties due to the different ingredients in these models the investigations have verified the early proposal of Munk and Palmén (1951) on the importance of the pressure force on the submarine topography—the bottom form stress—in the balance of the zonal momentum. In fact, the concept of a ''canonical'' balance may be formulated: The flux of momentum imparted by the surface wind stress is carried—via the interfacial form stress mechanism—by standing and transient eddies to the depth where the flow is blocked by topography and the bottom form stress acts to transfer the momentum to the earth.

To many oceanographers this dynamical concept of the ACC, however, still appears mysterious since the conventional tools of oceanographic science—for example, the Ekman and geostrophic transports and the Sverdrup balance—seem not to pertain to this canonical concept, which, moreover, seems to be insuffient to determine the zonal transport of the circumpolar flow. Warren et al. (1996, hereafter WLR) express their indisposition with the form stress concept and the implied balance of the ACC. In their view, the form drag mechanism appears to be irrelevant in the balance of the ACC, and they propose that ''Sverdrup dynamics seem to offer a more promising analysis of the real Circumpolar Current.''

Let me briefly repeat the basic line of arguments by which WLR degrade the importance of the form stress mechanism in particular and the '' 'form drag' force balance'' in general in the physics of the ACC. In their view, the momentum balance {Eq. [3][1] in WLR, (6) below} between the wind stress and the bottom form stress ''merely states that the [meridional Ekman flux] must balance the [net meridional geostrophic flow that can exist below the ridge crests] if the total meridional flow is zero.'' Of course, it is trivial, and maybe semantic at this stage, to state that the mass balance cares for the balance of mass fluxes and the balance of momentum has its own right (see section 2).

WLR continue to degrade the importance of the momentum balance [3] because it ''includes no attribute (e.g., volume transport) of the Circumpolar Current . . . in fact, according to the resolution [4] and [5]'' [the balances for the Ekman layer and the bottom layer, (2) and (4) below], ''the Circumpolar Current is irrelevant to the integrated momentum balance and could actually be absent without affecting [3], [4], and [5].'' However, a flow in response to passing over topographic features must set up a pressure field that correlates with the topographic elevation to produce a bottom form stress. This stress definitely ''knows'' ''that the current should flow eastward or westward,'' but the process of its generation cannot be inferred from the zonally averaged momentum balance [3] alone (see section 3).

In response to Munk and Palmén (1951), ''who sought an agency to oppose the eastward wind stress they propose as a more conventional, straightforward answer to their request . . . the Coriolis acceleration (not discussed by Munk and Palmén) associated with the

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[1] Numbers in square brackets refer to equations in WLR.

northward Ekman transport.'' Munk and Palmén do not need any defense from my side but apparently WLP take the momentum balance in the Ekman layer as the balance of the whole system. This is certainly appropriate for an unbounded domain with a motionless abyss but not for a bounded domain with a meridional circulation and deep currents. There must indeed be an agency that transmits the wind stress to greater depths (see sections 4 and 5) where it can be taken up by bottom friction or form stress (see section 6).

After all these arguments, the form drag—as a physical process—and the momentum balance [3]—as an independent statement about conservation of momentum in the system—seems to have disappeared from the WLR scenario: ''the proportionality'' of the pressure difference across the deep ridges and the wind stress ''is simply required for meridional mass balance and form drag talk is obscurantist in relation to the Circumpolar Current: the physics that it really describes seems to have nothing to do with that great current, but instead with mass conservation in the quite independent meridional circulation.'' The Sverdrup balance model of Stommel (1957) and the ''Sverdrupian'' estimate of the ACC transport by Baker (1982) and others are reconsidered. But apart from the fact that such a scenario is not dynamically closed, the ingredients of Sverdrup dynamics and more general, the barotropic vorticity dynamics, are not seen in its correct context: WLR try to convince us that the zonal wind stress cannot have a responsible role in the balance of the ACC since vorticity is implemented to the flow via the curl: ''adding a constant to the field of zonal wind stress, for example, would alter the meridional circulation, but, leaving the curl unchanged, would not affect the transport of the Circumpolar Current.'' We discuss the Sverdrup balance and the equivalence of the vorticity dynamics and the momentum dynamics in sections 7 and 8.

2. The interrelation of the momentum and mass balance and the meridional circulation

Starting from the simplified zonal momentum balance (absorbing the density into the pressure and stress)

$$-f v = -\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial z} \tag{1}$$

the relations [2] to [5] in WLR are obtained by vertical and zonal integration. Repeating here these equations, we have

$$-f \left\langle \int_{-E}^{0} v \, dz \right\rangle = \langle \tau_0 \rangle \tag{2}$$

$$-f \left\langle \int_{-H}^{-E} v \, dz \right\rangle = 0 \tag{3}$$

$$-f \left\langle \int_{-D(x)}^{-H} v \, dz \right\rangle = \left\langle p_D \frac{\partial D}{\partial x} - \tau_D \right\rangle, \tag{4}$$

where $p_D(x) = p(x, z = -D(x))$ is the bottom pressure. The zonal integral is indicated by angle brackets,

$$\langle \cdots \rangle := \oint_C dx \cdots,$$

and we are considering, as WLR, an Ekman layer $0 > z > -E$, an intermediate layer $-E > z > -H$ above the depth $z = -H$ of the highest ridge, and the bottom layer between $z = -H$ and the bottom at $z = -D(x)$. Furthermore, τ_0 is the zonal wind stress, and τ_D is the frictional stress at the bottom. As WLR, we assume that the wind stress is absorbed in the Ekman layer, meaning that $\tau \equiv 0$ beneath $z = -E$, and that τ_D may be neglected [as other small terms have been neglected in (1)]. We include τ_D for later reference.

WLR interpret (2)–(4) basically in the framework of mass balance, of course ''(2) specifies the meridional Ekman flux, (4) identifies the net meridional geostrophic flow that can exist below the ridge crests.'' The balance of these mass fluxes, however, is not derived from these equations nor from the total balance of momentum [Eq. (6) below] but from the balance of the total mass south of the path C,

$$\left\langle \int_{-D(x)}^{0} v \, dz \right\rangle = 0, \tag{5}$$

and the vanishing of the geostrophic transport and the neglection of any ageostrophic terms in the intermediate layer. Using (5) the integral over the entire depth yields the total balance of momentum [the sum of (2) to (4)] in the form

$$\left\langle \tau_0 - \tau_D + p_D \frac{\partial D}{\partial x} \right\rangle = 0, \tag{6}$$

which of course is completely independent of any specific form of mass exchange in the vertical.

Hence, Eq. (6) has its own right in the dynamics of the system. After all, it is one of the most fundamental constraints of the system, it is the balance of momentum in the whole water column: in the absence of body forces the fluxes of momentum into the system must balance.[2] WLR complain that Munk and Palmén have ignored ''the Coriolis acceleration associated with the northward Ekman transport'' in their seminal discussion of the momentum balance. The Coriolis acceleration only appears in the balance of the individual (nonmaterial) layers but has—as a consequence of mass conservation—no effect on the mass of the total water column, that is, there is no net Coriolis acceleration or force on the whole sys-

[2] It should be mentioned that the balance as presented in the form (6) is actually the budget of vorticity of the volume south of the latitude considered. To obtain a budget of momentum we must integrate (6) over a finite latitudinal strip.

tem. As WLR, I believe indeed that ''Ekman layer dynamics applies to the Drake Passage zone as much as is thought to the rest of the world ocean,'' but if ''the surface stress'' is ''absorbed in a thin boundary layer,'' this does not mean that the total system is in balance: there is still the question of balance below the Ekman layer and of the total water column. Here, one cannot evade the ''obscure form drag force'' unless one is willing to accept a large frictional bottom stress τ_D or a large lateral Reynolds stress, which transports momentum out of the current system. Both assumptions are contrary to observations (Morrow et al. 1992) and results from high-resolution models (see below).

3. The determination of the zonal transport

WLR dismiss the relevance of (6) to the ACC since it ''includes no attribute (e.g. volume transport) of the Circumpolar Current, . . . [it] could actually be absent,'' meaning presumably that the total momentum balance is of no use to determine the zonal transport. Of course, it is only under rare conditions (as, e.g., for a linear decay as in the case of radioactive substances, or the physically questionable Rayleigh damping of momentum) that a steady integral balance of a quantity contains, besides the fluxes across the boundaries, also a term involving the content of the quantity in the compartment.

But how is the transport of the zonal flow determined? One cannot expect that this can be done from just one integral balance. The total momentum balance (6) contains the part of the bottom pressure that is out of phase with variations of the topography along the zonal path of integration. This part, as the entire pressure field, is shaped by many different processes. For quantitative answers a full model including external forces by the wind stress and the surface fluxes of heat and freshwater as well as the advection of mass, heat, and salt must be solved. Qualitative answers and a deeper insight into the dynamics of the ACC can be obtained by cheaper methods, which may reveal mechanisms in trade for completeness.

The cheapest approach ignores the baroclinic state, considers quasigeostrophic dynamics, and derives the total momentum balance [i.e., Eq. (6), with linear bottom friction included] and the balances of the in-phase and the out-of-phase (the form stress) components of the pressure field involving the advection by the zonal mean flow. Evidently, this heavily truncated image of the full dynamics is the Charney–DeVore model (Charney and DeVore 1979): the pressure components are established by a standing barotropic Rossby wave generated by the mean flow U (the vertically integrated zonal velocity) over the topography. The implied form stress F becomes a function of the zonal velocity,

$$\left\langle p_D \frac{\partial D}{\partial x} \right\rangle = F[U] = -\frac{1}{2}\left(f \frac{D-H}{D} \right)^2 \frac{\epsilon U}{\epsilon^2 + k^2 (U - c_R)^2}. \quad (7)$$

Here, k is the zonal wavenumber of the topography, c_R the Rossby wave speed, and ϵ the parameter of linear bottom friction. The wave may get locked in resonance with the mean flow at the Rossby wave speed c_R and produces a large form stress. The total momentum balance

$$\tau_0 - \epsilon U + F[U] = 0 \quad (8)$$

determines the zonal transport U. The form stress definitely ''knows'' about the presence and direction of the flow; a westward wind and flow would generate a drag of different sign. For the two solutions of (7) and (8) in the resonant range the friction in the momentum balance (8) is negligible; these solutions are balanced by form stress. The off-resonant solution is controlled by friction. It is remarkable that friction is, in any case, essential to bring the pressure field out of phase with respect to the topography. Notice further that a westward wind would not be able to lock the flow in resonance. For reasonable oceanic parameters (reasonable values for the wind stress and the bottom friction) only the frictionally controlled solution exists.

This simple model—as any barotropic model—is incapable of correctly describing the observed transport of the ACC, not even the order of magnitude, but it captures an important mechanism by which the flow can set its bottom form stress. Extension to baroclinic conditions are tedious but still analytically managable (Olbers and Völker 1996). The physics are essentially the same as in the barotropic model. The form stress consists of a barotropic and a baroclinic pressure contribution, the latter is shaped by a baroclinic Rossby wave in resonance with the zonal flow. Again, the form stress is set by friction: The barotropic form stress vanishes for zero bottom friction, and the baroclinic form stress vanishes if bottom friction and vertical momentum transport in the fluid (by eddies, see below) vanish. This model yields reasonable transport magnitudes in the oceanic parameter range.

It is worth mentioning that this simple baroclinic model—as others (see Olbers et al. 1992) predicts that the barotropic part of the bottom form stress (the stress due to the surface pressure) acts as an extremely powerful drag. Whereas in barotropic conditions the form stress almost compensates the wind stress and the transport becomes a small fraction of the transport over flat bottom, it is found in baroclinic conditions that the barotropic form stress overcompensates the wind stress and the baroclinic part (due to tilting of the isopycnals) in fact drives the eastward flow. This mechanism has been observed in the most complete numerical model of the

ACC, the FRAM experiment (see, e.g., Stevens and Ivchenko 1997).

4. The physics of form stress

Why does the bottom form drag appear so obscure? After all, it is only a mathematical representation of the action of pressure forces on the topography and as such it is merely a zonal flux of zonal momentum out of the fluid. We are adapted to see the pressure gradient as force acting on fluid parcels but it should be remembered that pressure is part of the stress tensor, describing the isotropic part of the normal flux of momentum: p is the flux of any component of momentum in the same corresponding coordinate direction. If there is a ‘‘barrier’’ in the form of topography or a material interface (in a layer model), the pressure fluxes horizontal momentum in the horizontal direction across this barrier, establishing then either bottom form stress[3] or interfacial form stress. The interfacial form stress does not appear in balances evaluated for compartments bounded by level coordinates (the divergence of the form stress—the pressure gradient force—then propels the fluid in the same layer). Integrated vertically between two interfaces $z = -h_1(x)$ and $z = -h_2(x)$ along any contour with coordinate x and bounded by x_l, x_r, the integrated pressure ‘‘divergence’’ appears as

$$\int_{x_l}^{x_r} \frac{\partial p}{\partial x} \int_{-h_2(x)}^{-h_1(x)} dz\, dx = \int_{-h_2(x)}^{-h_1(x)} p\, dz \bigg|_{x_l}^{x_r} + \int_{x_l}^{x_r} \left[p(x, -h_1(x)) \frac{\partial h_1}{\partial x} - p(x, -h_2(x)) \frac{\partial h_2}{\partial x} \right] dx. \quad (9)$$

The first term on the rhs is the net lateral flux of horizontal momentum into the area (thus vanishing for closed contours), the second is the flux through the interfaces, that is, the form stress (in case of $h_2 = D$ this term is the bottom form stress). When we consider the wind-driven circulation in a basin with a flat bottom, bounded zonally by x_l and x_r, the net pressure difference across the basin (the first term on the rhs) opposes the external stress to achieve the balance of momentum. In a zonally unbounded domain this term is absent, but the bottom form stress is just the extension of this pressure difference to the submarine barriers.

Notice that the effect of form stress on a layer bounded by two surfaces is described by the difference of the interfacial form stresses acting at the surfaces: it takes the form of a vertical divergence of a vertical flux of horizontal momentum. Notice further that, in the case of material surfaces, the net meridional mass flux vanishes in each layer. Then there is no net Coriolis force acting on the layer and, thus, the vertical divergence of the interfacial form stress must balance all applied frictional stresses, and—in the deepest layer—the bottom form stress. The balance of zonal momentum in such an isopycnal model has thus the same general form as (2) to (4) with the Coriolis force replaced by the corresponding divergence of the interfacial form stress (see, e.g., Marshall et al. 1993). Notice that here other terms of the momentum equation (1) should be included; the divergence of the lateral Reynolds stress is generally small compared to the local Coriolis term, but not if integrated over the layer where the latter term becomes zero.

It is evident from the expression (9) that the interfacial stress contains a contribution from stationary deformations of the interface (the standing eddy part) and a contribution from transient eddies. Any flow over topography will generate the standing eddy part, whereas there are flow structures and, more important, low-resolution models that do not allow for transient eddies.

Evidently, the interfacial form stress, as defined above, vanishes for level surfaces. Below I will show that the continuous vertical transfer of momentum by a nonfrictional process—the interfacial form stress—can be generalized to level models with full thermodynamics.

5. The vertical transport mechanism in level models

A model of the ACC based on material layers may be suspicious in conditions where watermass conversion may occur. It is indeed of little help for quantitative prediction. But to say it again, such models are cheaper to run and analyze and may elucidate basic mechanisms such as the role of the eddies in the transfer of momentum in the water column. But where are the eddies—standing or transient—in the level counterpart considered in the section 2? In fact, they can be found in the Coriolis acceleration. The zonally averaged meridional velocity is related to the lateral transport of heat (or potential density) by the eddies and enters as a vertical divergence of a vertical flux of momentum in the balance of momentum in the same way as the interfacial form stress in layer models.

The mathematical details of this statement are easily outlined. Consider the piece of ocean bounded to the south by the Antarctic continent and to the north by a circumpolar path C at constant latitude through Drake Passage. Furthermore, let $A(z)$ denote the area on the level z from C to the south and bounded there by the continent. There are gaps in C [thus C is a $C(z)$] and ‘‘outbreaks’’ of $A(z)$ where topography stands above the level z, they are taken care of in the equations that

[3] It was first pointed out and exemplified by Holloway (1987) that the bottom form drag could act as well to accelerate the fluid. This actually is found in many cases of QG flow over topography, so it is more appropriate to call it bottom form stress.

follow. We discuss relations between line integrals along C and area integrals over $A(z)$ that are derived from the balances of mass and heat, which we write in the form

$$\nabla \cdot \mathbf{u} + \frac{\partial w}{\partial z} = 0 \tag{10}$$

$$\nabla \cdot \mathbf{J} + \frac{\partial J^{(z)}}{\partial z} + w\Theta_z = 0. \tag{11}$$

Here $\mathbf{u}$ is the horizontal velocity and w the vertical velocity, θ the perturbation of temperature about a horizontally averaged state $\Theta(z)$, and $\mathbf{J} = (J^{(x)}, J^{(y)})$ and $J^{(z)}$ are the horizontal and vertical flux components of heat, given by the advective and turbulent (sub-eddy-scale) parts $(\mathbf{I}, I^{(z)})$,

$$\mathbf{J} = \mathbf{u}\theta + \mathbf{I} \qquad J^{(z)} = w\ \theta + I^{(z)}. \tag{12}$$

Integration of the mass balance equation (10) over the area $A(z)$ yields after some mathematical substitutions

$$\langle v \rangle + \frac{\partial}{\partial z}\int_{A(z)} w\ dA = 0. \tag{13}$$

The area integral of w, which appears here, may also be evaluated using the thermohaline balance (11), which leads to

$$-\Theta_z \int_{A(z)} w\ dA = \langle J^{(y)} \rangle + \frac{dQ}{dz}, \tag{14}$$

where

$$Q(z) = \int_{A(z)} J^{(z)}\ dA \tag{15}$$

is the vertical heat flux across $A(z)$. Combining (13) and (14) we find a relation between the meridional circulation and heat transport in the form

$$\langle v \rangle = \frac{\partial}{\partial z}\left[(\Theta_z)^{-1}\left(\langle v\theta + I^{(y)} \rangle + \frac{dQ}{dz}\right)\right], \tag{16}$$

which is familiar in quasigeostrophic settings (e.g., Green 1970). Here, however, it is generalized for diabatic conditions. Notice that the integral of v in (13) and (16) has a geostrophic term at depths where C is blocked by topography, for unblocked contours it is entirely ageostrophic. The essence of (16) is that, in the presence of lateral boundaries, the Coriolis force can be rewritten as a divergence of a vertical transport of momentum—an ''interlevel form stress''—that is set by the lateral heat flux across C and the vertical heat flux south of C. We also learn from this relation how the watermass conversion rate Q enters the momentum balance. More important is, however, that the role of the eddies in the circumpolar belt is revealed: They are responsible for transferring horizontal momentum in the water column and heat laterally. These processes are interlinked; downward momentum transport is equivalent to southward heat transport.

6. The zonal momentum balance of the ACC

Quasigeostrophic models (McWilliams et al. 1978; Wolff et al. 1991; Marshall et al. 1993; Olbers 1993) and PE models (Killworth and Nanneh 1994; Stevens and Ivchenko 1997) have determined the magnitudes of the different contributions to the momentum balance in the zonal average. We summarize the results of these numerical investigations in the light of the above framework. Taking a time mean, the complete zonally integrated balance of zonal momentum becomes

$$-f\langle \overline{v} \rangle = \frac{\partial \mathcal{F}}{\partial z} = \left\langle \frac{\partial \overline{\tau}}{\partial z} \right\rangle - \left\langle \frac{\partial}{\partial y}\overline{uv} \right\rangle + \sum_{\text{ridges}} \delta\overline{p}, \tag{17}$$

where we have introduced the level expression of the interfacial form stress

$$\mathcal{F} = -f(\Theta_z)^{-1}\left(\langle \overline{v\theta} + \overline{I}^{(y)} \rangle + \frac{d\overline{Q}}{dz}\right). \tag{18}$$

The advective fluxes contained in $\mathcal{F}$ and $\overline{uv}$ can be split into mean and eddy terms, for example, $\overline{v\theta} = \overline{v}\,\overline{\theta} + \overline{v'\theta'}$. The sum of the pressure differences is extended over all ridges interrupting the path C; each ridge contributes the difference between the values on the eastern side and the western side, that is, $\delta p = p(x_{\text{east}}, z) - p(x_{\text{west}}, z)$. The vertical integral of this term yields the bottom form stress.

Summarizing the aforementioned investigations, the ''canonical'' balance of zonal momentum is then characterized as follows.

Ekman layer ($0 \geq z \geq -E$): At depths directly influenced by the wind stress the dominant balance is between the divergence of the frictional stress $\langle \overline{\tau} \rangle$ and the Coriolis force $f\ \langle \overline{v} \rangle$ (due to ageostrophic motion) or—as evident from the analysis above—the divergence of the interlevel (or facial) form stress,

$$-f\langle \overline{v} \rangle = \frac{\partial \mathcal{F}}{\partial z} \approx \left\langle \frac{\partial \overline{\tau}}{\partial z} \right\rangle. \tag{19}$$

By use of (14) at $z = 0$ and integration over the Ekman layer, we find

$$-f\int_{-E}^{0} \langle \overline{v} \rangle\ dz = -\mathcal{F}(z = -E) \approx \langle \overline{\tau}_0 \rangle \tag{20}$$

stating that the momentum input by wind stress at the sea surface is balanced by the interlevel/facial form stress at the bottom of the Ekman layer.

Intermediate layer ($-E \geq z \geq -H$): Below the Ekman layer but above the minimum depth H of topography along the path of consideration, the balance is between the lateral Reynolds stress divergence and the Coriolis force. All other terms are smaller by almost an order of magnitude. The dominant terms—the Reynolds stress divergence and the Coriolis force—are, however, more than two orders of magnitude smaller than the dominant terms in the Ekman layer. In other words: the

Coriolis force acting on the fluid of the layer is small (only the ageostrophic part enters the balance):

$$f\,\langle\overline{v}\rangle = f\,\langle\overline{v}_{\rm ag}\rangle \approx 0, \qquad (21)$$

but it is the difference of the two big interlevel/facial stresses at the top and bottom of the layer. In the framework of interlevel/facial form stress the balance is thus expressed by the approximate constancy of this stress. The constant is determined by (20), and we find

$$\mathcal{F} \approx -\langle\overline{\tau}_0\rangle. \qquad (22)$$

In this intermediate layer the stress associated with the heat flux is thus transmitted almost unchanged, it has the size of the wind stress but the process is neither frictional nor due to a vertical Reynolds stress.

Deep layer ($-H \geq z \geq -D_{\max}$): At depths where the topography interrupts the circumpolar path the bottom form stress comes into action. The balance occurs between this stress and the Coriolis force, which now has a large geostrophic component—the two terms are of the same magnitude as the terms in the Ekman layer. Other terms are negligible. We thus have

$$-f\langle\overline{v}\rangle = \frac{\partial\mathcal{F}}{\partial z} \approx \sum_{\rm ridges} \delta\overline{p}, \qquad (23)$$

and integration over the deep layer yields

$$-f\int_{-D_{\max}}^{-H}\langle\overline{v}\rangle\,dz = \mathcal{F}(z=-H) \approx -\left\langle\overline{p}_D\frac{\partial D}{\partial x}\right\rangle.$$

The balance thus occurs between the flux of momentum by interlevel/facial form stress at the top of the blocked layer and the pressure force on the topography, the bottom form stress.

Total balance ($0 \geq z \geq -D$): It is quite obvious from the above considerations, but also confirmed in the numerical experiments of the QG and FRAM models, that the vertically integrated balance occurs predominantly between the wind stress and the bottom form stress; that is,

$$\langle\overline{\tau_0}\rangle \approx -\left\langle\overline{p}_D\frac{\partial D}{\partial x}\right\rangle. \qquad (24)$$

For an eastward wind we must obviously have a southward geostrophic flow ($\delta\overline{p}$ negative, f negative) in the valleys between the blocking topography.

Primitive equation and QG models indicate that the standing eddy contribution dominates the heat flux $\langle\overline{J}^{(y)}\rangle$. If the intermediate layer outcrops at the surface, as in the real circumpolar flow and the FRAM experiment, the above canonical framework must be modified to include the direct acceleration by the wind (see Killworth and Nanneh 1994). For QG models the above balances hold with $Q \equiv 0$ and $(\mathbf{I}, I^{(z)}) \equiv 0$; that is, only the advective part of the interfacial form stress is present.

7. The Sverdrup balance

Despite these solid results on the dynamics of the ACC, WLR reconsider Sverdrup dynamics (following Stommel 1957) to put the problem of momentum balance aside. The transport estimates along 55°S (Baker 1982) or 54°S (Godfrey 1989) obtained from the simple flat-bottom Sverdrup balance are indeed intriguing, but it should be clear that the Sverdrup balance does not pose a dynamically closed problem: even if it could possibly be used to estimate the transport of the ACC, in the same way as the Sverdrup balance determines the transport of the Gulf Stream by a simple mass conservation argument, this approach can certainly not explain the dynamical balance of the current. The use of another quantity—the barotropic vorticity—and its balance cannot circumvent the fullfilment of the momentum balance. It can indeed be shown (see next section) that a properly posed vorticity problem must consider the balance (6).

The Sverdrup balance follows from the planetary vorticity balance derived from (1). Vertical integration from the surface to the bottom $z = -D(x)$ results in

$$\begin{aligned}\beta V &= -fw(-D) + {\rm curl}\,\boldsymbol{\tau}\Big|_{z=-D}^{z=0}\\ &= -fw_g(-D) + {\rm curl}(\boldsymbol{\tau}_0 - \boldsymbol{\tau}_D). \qquad (25)\end{aligned}$$

The term involving the geostrophic vertical velocity $w_g(-D) = -(1/f)\mathbf{J}(p_D, D)$ at the bottom generates barotropic vorticity by stretching the water column when it has to pass a topographic barrier. WLR propose to neglect this term, they argue that "the vertical velocity must be zero by regional averaging and that the Sverdrup balance should hold in that regional sense." As WLR also mention and as is evident from observations, "the current reaches the bottom . . . where it experiences non-zero vertical velocities." These are, however, not just local features, as WLR assume, since the current has to cross three major midocean ridges of large zonal extent and has to pass the sill in Drake Passge and numerous smaller barriers. Simple scaling reveals the danger to miss just one (half) of these features in this regional average: for a ridge with slope of 10^{-3} and a horizontal velocity of 0.01 m s^{-1} at the bottom we get $fw_g \sim 10^{-9}$ m s^{-2} (a fairly conservative estimate on the low side), whereas a meridional transport of 100 Sv (Sv $\equiv 10^6$ m^3 s^{-1}) implies $\beta V \sim 10^{-10}$ m s^{-2}, an order of magnitude smaller. In correspondence, scaling of the so-called bottom torque $\mathbf{J}(p_D, D)$ shows that it is not small at all when compared with the input of vorticity by the wind stress. In fact, the dominance of the stretching term can only be avoided if the bottom current very closely follows the contours of bathymetry.

Integrating (25) zonally and using as before the balance of mass (5) we find

$$\frac{\partial}{\partial y}\left\langle \tau_0 - \tau_D + p_D\frac{\partial D}{\partial x}\right\rangle = 0, \qquad (26)$$

which is the vorticity counterpart of the *form drag force balance* [as mentioned above, the zonally integrated momentum balance (6) is the vorticity balance for the piece of ocean south of the latitude given by the contour C]. So we are again stuck to the physics of form stress: either we accept a large friction—at least in some area as Drake Passage—or we have to face the importance of form drag.

But is the flat-bottom Sverdrup balance—in the local or the zonally accumulated form—applicable to the ACC, or is the rough agreement of transport estimates derived from it with observed values just a coincidence? Wells and DeCuevas (1995) considered the budget of vorticity over areas bounded by transport streamlines in the FRAM experiment. There is no evidence of a local flat-bottom Sverdrup balance anywhere in the ACC; instead, a local balance occurs between the planetary advection (the β term), the bottom torque (the stretching term), and the advective term of relative vorticity [so this should be included in (25)]. Integrating in the ''Sverdrupian range'' outside Drake Passage (40°W eastward to 70°W along 55°S), bottom torque and advection of relative vorticity integrate to large but almost compensating values, whereas the β term and the wind stress curl get values that also compensate but are a factor of 5 smaller. Should we regard this as a Sverdrup regime? Integrating further through the passage the β term must tend to zero and a balance between the bottom torque and the wind stress curl emerges in the complete zonal average (in accordance with the momentum balance). Of course, this is a model but it is the most complete one we currently have.

8. The curl problem

Finally I comment on WLR's suggestion that the transport of the ACC may entirely be determined by the curl of the wind stress. In the vorticity balance, indeed, only the curl of the wind stress appears, so it is tempting to speculate that the solution only depends on the curl and not the wind stress itself. Though contradicting our basic understanding of the behavior of stress-driven zonal flows, this fallacy is not easy to unravel (it is, of course, trivial that a constant wind stress does generate transport in a channel). As a matter of fact, there is an important example where transports only depend on the curl: the barotropic wind-driven circulation in a basin (the baroclinic state of this flow depends on the stress). In a periodic geometry, however, even the simplest transport model contains an important difference in the boundary conditions, which at first sight may be a purely mathematical problem but has a deeper physical meaning.

Consider the vorticity balance in the form

$$\epsilon\nabla^2\psi + \beta\frac{\partial\psi}{\partial x} = \mathbf{J}(p_D, D) + \mathrm{curl}\boldsymbol{\tau}_0, \qquad (27)$$

where our concern is to determine the transport pattern $U = -\psi_y$, $V = \psi_x$, which can be represented by a streamfunction ψ because of mass conservation. We have, for simplicity, represented the friction in (25) as linear bottom friction of the barotropic flow: curl $\boldsymbol{\tau}_D = \epsilon\nabla^2\psi$ (the conclusions do not depend on this specific choice). Integration of the resulting familiar barotropic vorticity equation (with prescribed forcing by the wind curl and the bottom torque) needs the specification of boundary conditions, which naturally are the kinematic conditions of zero normal flow. In a basin this is achieved by ψ = const on the rim and, without restriction, the constant can be set to zero. In multiply connected domains—as the Southern Ocean, or in simpler geometry, as a channel—zero normal flow requires ψ to be constant on each boundary but the constants are not equal; in fact, their difference is the unknown net transport value. The reason for this difficulty can be traced back to the elimination of a part of the baroclinic pressure field when stepping to the vorticity framework; here the information on the net transport has been lost. A complete solution must of course include a balanced pressure field satisfying the momentum equation corresponding to (27),

$$\epsilon\mathbf{U} + f\mathbf{k}\times\mathbf{U} = -h\boldsymbol{\nabla}p_D - \boldsymbol{\nabla}E + \boldsymbol{\tau}_0. \qquad (28)$$

Here $\mathbf{U} = (U, V)$, and E derives—together with the baroclinic part in p_D—from the vertical integral of the baroclinic pressure. It turns out that E is the total baroclinic potential energy of the in situ density, referred to the surface. This field does not impart any vorticity into the system; however, (27) must be solved such that the resulting $\mathbf{U}$ renders $\boldsymbol{\nabla}E$ as the gradient of a scalar. In a basin, given the transport $\mathbf{U}$, this pressure can be constructed uniquely by contour integration, starting with an arbitrary value at an arbitrary point on the boundary. In channel geometry, however, the unique reconstruction of the pressure from the gradient is possible if, and only if, integration around each continent vanishes:

$$\begin{aligned}\oint_L d\mathbf{s}\cdot\boldsymbol{\nabla}E &= \oint_L d\mathbf{s}\cdot[-\epsilon\mathbf{U} - f\mathbf{k}\times\mathbf{U} - h\boldsymbol{\nabla}p_D + \boldsymbol{\tau}_0] \\ &= \oint_L d\mathbf{s}\cdot[\tau_0 - \tau_D + p_D\boldsymbol{\nabla}h] = 0. \qquad (29)\end{aligned}$$

From (27) and Stokes theorem we notice that it is sufficient to consider (29) for one arbitrary contour L for each continent (there are thus as many constraints as unknown streamfunction differences). Only if complemented by these integral contraints, the vorticity problem (27) is physically and mathematically well posed. Of course, in this form the problem (27) and (29) is equivalent to (28), supplemented by the mass conservation $\nabla \cdot \mathbf{U} = 0$. Thus, finally, we are back to the necessity that the momentum balance equation (6) must be satisfied to obtain a consistent solution of the vorticity balance. And, it is apparent from (29) that the transport must indeed depend on the stress itself and not only on the curl.

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