Tidal Rectification: Friction or Not Friction?

ROBERT MAZÉ

Laboratoire de Physique des Océans, Université de Bretagne Occidentale, Brest, France

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ABSTRACT

A survey of the theoretical aspects of tidal rectification over a continental slope in the inviscid fluid approximation is proposed. In particular, the geostrophic degeneracy problem, evoked by A. W. Visser to justify the need for friction in the momentum equations, is examined. It appears that geostrophic degeneracy is only due to excessive approximations. In this process friction has only its classic effect, which leads to a slight modification of the current obtained in the inviscid theory.

The residual current due to nonlinear interactions between barotropic tidal currents, sloping topography, and earth rotation can be obtained by linearizing the equations describing the dynamics above the continental slope. The long period dynamics as thus defined are compared with oceanic data acquired by the Centre Militaire d'Oceanographie Service Hydrographique et Océanographique de la Marine in the continental slope area of the Bay of Biscay. Subject to the assumptions adopted for describing the spatial evolution of the semidiurnal current, these comparisons are quite satisfactory.

1. Intoduction

Above a continental slope, the tidal current is altered by both sloping topography and earth rotation. This process is named ''tidal rectification.'' It partly explains the long timescale alongslope current.

Evidence of such currents is generally observed in field data acquired over continental slope areas (Pingree and Le Cann 1989; Garreau and Mazé 1992, hereafter GM92). The tidal rectification current is generally recognizable by its periodic component close to 14 days. This timescale results from the interaction of the M_2–S_2 tidal components. As this current is rather large (typically 5–10 cm s^{-1} in the Bay of Biscay), understanding the generation process is an interesting challenge.

Qualitative theoretical explanations based on the conservation of potential vorticity or quantitative ones resulting from rotation of the major axis of the ellipse without tidal stress have been proposed. A summary of these works is given in an earlier paper (Mazé et al. 1998). A different approach to the quantitative aspect of this process has been suggested (GM92). The proposed solution has been critically examined (Visser 1994, hereafter V94). V94's arguments are based on a Lagrangian point of view and refer to geostrophic degeneracy to justify the use of friction.

The object of the present study is to prove that on one hand geostrophic degeneracy only results from excessive approximations and on the other hand that the friction parameterization used in the ''tidal stress'' theory is irrelevant and appears as an unnecessary complication of the problem. Field data acquired by the French Centre Militaire d'Océanographie Service Hydrographique et Océanographique de la Marine (CMO-SHOM) (MINT-94 experiment) in the continental slope area of the Bay of Biscay allow specification, at least partially, of the influence on tidal rectification of the development of the observed long-period current.

Corresponding author address: Dr. Robert Mazé, Laboratoire de Physique des Océans, Université de Bretagne Occidentale, B.P. 809, 29285 Brest, Cedex, France.

2. Eulerian and Lagrangian motions study

Consider the case of a shelf break parallel to the y axis with a constant slope a in the x axis: $h(x) = h_0 - ax$. In an inviscid, unstratified fluid, the momemtum equations may be written as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - fv = -g\frac{\partial \eta}{\partial x} \qquad (1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + fu = 0, \qquad (2)$$

and the continuity equation is

$$\frac{\partial}{\partial x}[(h + \eta)u] = -\frac{\partial \eta}{\partial t}, \qquad (3)$$

where u and v are the velocity components perpendicular and parallel to the isobaths, respectively; η is the free surface elevation.

a. Integration of Laplace equations

As Eqs. (1), (2), and (3) are nonlinear, it is impossible to compute the exact analytical solution for the description of a particular mechanism. The tidal oscillation description thus requires a linearization. For the study of a monochromatic oscillation of frequency ω, which does not require a high degree of precision, the Laplace equations corresponding to Eqs. (1), (2), and (3) are used:

$$\frac{\partial u_L}{\partial t} - f v_L = -g \frac{\partial \eta_L}{\partial x} \quad (4)$$

$$\frac{\partial v_L}{\partial t} + f u_L = 0 \quad (5)$$

$$\frac{\partial}{\partial x}(h u_L) = -\frac{\partial \eta_L}{\partial t}. \quad (6)$$

By introducing $\eta = \eta_0[F(x) - jG(x)] \exp j\omega t$, where $j = \sqrt{-1}$, it is found that

$$u_L = \frac{jg\omega}{(\omega^2 - f^2)} \frac{\partial \eta_L}{\partial x} \quad (7)$$

$$v_L = -\frac{fg}{(\omega^2 - f^2)} \frac{\partial \eta_L}{\partial x}, \quad (8)$$

with

$$\frac{\partial^2 \eta_L}{\partial x^2} - \frac{a}{h} \frac{\partial \eta_L}{\partial x} + \frac{(\omega^2 - f^2)}{gh} \eta_L = 0. \quad (9)$$

This well-known solution is clearly determined by imposing harmonic motion, which eliminates the following spurious solution:

$$u_1 = 0; \qquad v_1 = v_1(x); \qquad \frac{\partial \eta_1}{\partial x} = \frac{f v_1(x)}{g}. \quad (10)$$

In other words, the v_L expression is obtained by integrating Eq. (5) from t_0 to t:

$$v_L(x, t) - v_L(x, t_0) = -\frac{fg\eta_0}{(\omega^2 - f^2)} \left[\frac{\partial F}{\partial x} - j \frac{\partial G}{\partial x}\right] (\exp j\omega t - \exp j\omega t_0);$$

thus, $v_L(x, t_0)$ is defined by

$$v_L(x, t_0) = -\frac{fg\eta_0}{(\omega^2 - f^2)} \left[\frac{\partial F}{\partial x} - j \frac{\partial G}{\partial x}\right] \exp j\omega t_0$$

and $v_L(x, t)$ by

$$v_L(x, t) = -\frac{fg\eta_0}{(\omega^2 - f^2)} \left[\frac{\partial F}{\partial x} - j \frac{\partial G}{\partial x}\right] \exp j\omega t.$$

In this integration, t_0 is simply chosen so that $v_L(x, t_0) = 0$ in order to eliminate the spurious solution (10).

GM92 have then linearized Eqs. (1), (2), and (3) with the approximation $u\partial/\partial x = u_L\partial/\partial x$. The current components are then advected by a known current. We have found that

$$u = u_L(x^*, t); \qquad v = v_L(x^*, t);$$

$$\frac{\partial \eta}{\partial x} = -\frac{(\omega^2 - f^2)}{fg} v_L(x^*, t)$$

constitute an exact solution for Eqs. (1) and (2), and an approximate solution for Eq. (3) if

$$\frac{dx^*}{dt} = \frac{\partial x^*}{\partial t} + u_L \frac{\partial x^*}{\partial x} = 0 \quad (11)$$

from which

$$x^* = x - \int_{t_0}^{t} u_L \, dt + \sum_i^{\infty} f_i, \quad (12)$$

where $\Sigma_i f_i$ is a convergent series if $|\partial u_L/\partial x| < \omega$, which is generally verified over a continental shelf break. The f_i functions are easily determined by Eq. (11). To test the precision of this approximation, we have used the potential vorticity conservation principle (Q) on the trajectory $dQ/dt = 0$ with $Q = (\partial v/\partial x + f)/h$, with $\eta \ll h$. This verification is also a continuity accuracy test. It is easily shown that, in the case where $\partial/\partial y = 0$, if Eq. (2) and Eq. (3) are verified, $dQ/dt = 0$ is obtained. If $dQ/dt = 0$ and Eq. (2) are verified, then Eq. (3) is verified too.

b. Eulerian and Lagrangian motions study

In the dynamical context defined above, near the shelf break where $\partial u/\partial x$ is not negligible with respect to ω, then $u_L = D/h \cos\omega t$ with $D = h_0 u_0$ is a good approximation of u_L obtained with Eq. (7), which permits simple and locally correct analytical expressions for u_L. This formulation, used by both GM92 and V94, is equivalent considering the rigid-lid hypothesis $\partial(hu_L)/\partial x = 0$. In addition, for $\eta \ll h$ in the mass conservation equation (V94), Eqs. (1), (2), and (3) become

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} - f v = -g \frac{\partial \eta}{\partial x} \quad (13)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + f u = 0 \quad (14)$$

$$\frac{\partial (hu)}{\partial x} = 0. \quad (15)$$

In Lagrangian coordinates, this system becomes

$$\frac{du^\bullet}{dt} - f v^\bullet = -g \frac{\partial \eta^\bullet}{\partial x^\bullet}, \quad (16)$$

$$\frac{dv^\bullet}{dt} + f u^\bullet = 0, \quad \text{and} \quad (17)$$

$$\frac{\partial (h^\bullet u^\bullet)}{\partial x^\bullet} = 0. \quad (18)$$

The relations between the coordinates and solutions of Eqs. (13), (14), and (15) and (16), (17), and (18) are the following:

1) the Eulerian coordinates and solutions of Eqs. (13), (14), (15): x, h, $u(x, t)$, $v(x, t)$, $\eta(x, t)$, correspond to the coordinates and solutions of Eqs. (16), (17), (18): $x^\bullet$, $h^\bullet$, $u^\bullet$, $v^\bullet$, $\eta^\bullet$ so that

$$u^\bullet = u(x^\bullet, t); \qquad x^\bullet = x + \int_{t_0}^{t} u(x^\bullet, t)\, dt;$$

$$h^\bullet = h - a(x^\bullet - x); \qquad \eta^\bullet = \eta(x^\bullet, t), \tag{19}$$

where $x^\bullet$ ($u^\bullet$, $v^\bullet$) defines at instant t the position (the velocity components) of the particle that was at x at instant t_0 with the velocity components $u(x, t_0)$ and $v(x, t_0)$. Here $h^\bullet$ and $\eta^\bullet$ are the depth and the free surface elevation at instant t at $x^\bullet$;

2) the Lagrangian coordinates and solutions of Eqs. (16), (17), (18): $x^\bullet$, $h^\bullet$, $u^\bullet$, $v^\bullet$, $\eta^\bullet$ correspond to the coordinates and solutions of Eqs. (13), (14), (15): x, h, $u(x, t)$, $v(x, t)$, $\eta(x, t)$ so that

$$x = x^{*\bullet}; \qquad h = h^{*\bullet};$$

$$u(x, t) = u^{*\bullet} = u(x^{*\bullet}, t);$$

$$v(x, t) = v^{*\bullet} = v(x^{*\bullet}, t), \tag{20}$$

where x^* [$u(x^*, t_0')$, $v(x^*, t_0')$] defines at instant t_0' the position (the velocity components of the particle that was at x at instant t with the velocity components $u(x, t)$ and $v(x, t)$. We have

$$x = x^* + \int_{t_0'}^{t} u(x^*, t)\, dt. \tag{21}$$

As for the solutions of Eqs. (4), (5), (6), the instants t_0 and t_0' must be chosen so as to eliminate spurious solutions. As for the v_L determination by the integration of Eq. (5), $v^\bullet$ is obtained by integrating (17) from t_0 to t:

$$v^\bullet = v(x^\bullet, t) = v(x, t_0) - f\int_{t_0}^{t} u^\bullet\, dt, \quad \text{and}$$

with $u^\bullet = dx^\bullet/dt$, we have

$$v^\bullet = v(x^\bullet, t) = v(x, t_0) - f(x^\bullet - x). \tag{22}$$

The Eulerian component solution of Eq. (14) is, then,

$$v(x, t) = v(x^*, t_0') - f(x - x^*). \tag{23}$$

Equations (22) and (23) are verified for any motion studied with Eqs. (16), (17), (18) or Eqs. (13), (14), (15). For example,

$$u = \frac{D}{H}\cos\omega t; \qquad v = -fx;$$

$$g\frac{\partial\eta}{\partial x} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + f^2 x. \tag{24}$$

Obviously, the motion described by these expressions has no interest in the case of tidally induced motion. This solution corresponds to $\omega \ll \partial u/\partial x$; $\forall t$. These spurious solutions appear because the phenomenon is expressed by partial derivative equations. When integrating these equations, the constants that appear are defined by the boundary conditions and/or the initial conditions. The spurious solutions are eliminated due to these complementary conditions.

As far as we are concerned, the mechanism is characterized by the fact that $|\partial u/\partial x| < \omega$. If $|\partial u/\partial x| \ll \omega$, the solution must converge toward the solution of the Laplace equations. In the integration of Eq. (14), which leads to expression (24), the initial instant must be chosen as in the integration of Eq. (5) where $v(x,^* t_0') = 0$. The solution is, therefore,

$$v(x, t) = -f(x - x^*). \tag{25}$$

The introduction of this expression in Eq.(14) shows that this equation is verified, whatever the $u(x, t)$ and $v(x, t)$ expressions, under the following condition:

$$\frac{\partial x^*}{\partial t} + u\frac{\partial x^*}{\partial x} = \frac{dx^*}{dt} = 0. \tag{26}$$

The following solution suggested by GM92,

$$u(x, t) = \frac{D\cos\omega t}{h\sqrt{1 + \dfrac{2aD}{\omega h^2}\sin\omega t}};$$

$$v(x, t) = -\frac{f}{\omega}\frac{D\sin\omega t}{h\sqrt{1 + \dfrac{2aD}{\omega h^2}\sin\omega t}};$$

$$\frac{\partial\eta}{\partial x} = -\frac{(\omega^2 - f^2)}{fg}v, \tag{27}$$

is therefore totally defined and complete. This approximate solution of Eqs. (1), (2), (3) converges to the linear solution if $|u\partial/\partial x| \ll \omega$ (or $aD/h^2 \ll \omega$). V94 criticizes this solution based on on the solution of the Lagrangian equations (4), (5), (6) with the approximation $\partial(h^\bullet u^\bullet)/\partial x = 0$. Let $h^\bullet u^\bullet = D\cos\omega t$; thus,

$$u^\bullet = \frac{D\cos\omega t}{h\sqrt{1 - \dfrac{2aD}{\omega h^2}\sin\omega t}}, \tag{28}$$

and integrating (17), we obtain

$$v^\bullet - v_0 = \frac{fh}{a}\left(\sqrt{1 - \frac{2aD}{\omega h^2}\sin\omega t} - 1\right) = \frac{f}{a}(h^\bullet - h). \tag{29}$$

V94 affirms that $v^\bullet$ is determined only within the constant v_0. As seen above, this constant is perfectly determined by the fact that $v^\bullet$ must be harmonic [$v_0 = fh/a$ corresponds to the unacceptable solution (24)], and must converge toward the value $-fD\sin\omega t/\omega H$ if $|\partial u/\partial x| \ll \omega$.

Considering the approximation $h^\bullet u^\bullet = hu = D\cos\omega t$ used by V94, the Lagrangian component is, therefore,

$$v^\bullet = \frac{f}{a}(h^\bullet - h) \tag{30}$$

to which corresponds the Eulerian component:

$$v(x, t) = \frac{f}{a}[h - h(x^*)] \tag{31}$$

with

$$h(x^*) = h\sqrt{1 + \frac{2aD}{\omega h^2}\sin\omega t}. \tag{32}$$

The reasoning of V94 (i.e., the approximation: $h^\bullet u^\bullet = hu = D\cos\omega t$) leads to a Lagrangian circulation defined by

$$u^\bullet = \frac{D\cos\omega t}{h\sqrt{1 - \frac{2aD}{\omega h^2}\sin\omega t}}; \quad \text{thus, } \overline{u^\bullet} = 0 \quad \text{and} \tag{33}$$

$$v^\bullet = \frac{fh}{a}\left(\sqrt{1 - \frac{2aD}{\omega h^2}\sin\omega t} - 1\right); \quad \text{thus,}$$
$$\overline{v^\bullet} \simeq -\frac{1}{4}\frac{fa}{\omega^2}\frac{D^2}{h^3} \tag{34}$$

and an Eulerian circulation

$$u(x, t) = \frac{D}{h}\cos\omega t; \quad \text{thus, } \overline{u} = 0 \tag{35}$$

$$v(x, t) = \frac{fh}{a}\left(1 - \sqrt{1 + \frac{2aD}{\omega h^2}\sin\omega t}\right); \quad \text{thus,}$$
$$\overline{v} \simeq \frac{1}{4}\frac{fa}{\omega^2}\frac{D^2}{h^3}, \tag{36}$$

whereas with GM92's solution, the Eulerian velocities are

$$u(x, t) = \frac{D\cos\omega t}{h\sqrt{1 + \frac{2aD}{\omega h^2}\sin\omega t}}; \quad \text{thus, } \overline{u} = 0 \tag{37}$$

and

$$v(x, t) = -\frac{f}{\omega}\frac{D\sin\omega t}{h\sqrt{1 + \frac{2aD}{\omega h^2}\sin\omega t}}; \quad \text{thus,}$$
$$\overline{v} \simeq \frac{1}{2}\frac{fa}{\omega^2}\frac{D^2}{h^3} \tag{38}$$

and the Lagrangian velocities are

$$u^\bullet = u(x^\bullet, t) = \frac{D\cos\omega t}{h}; \quad \text{thus, } \overline{u^\bullet} = 0 \tag{39}$$

and

$$v^\bullet = v(x^\bullet, t) = -\frac{f}{\omega}\frac{D\sin\omega t}{h}; \quad \text{thus, } \overline{v^\bullet} = 0. \tag{40}$$

Hence, GM92's residual circulation balances the linear approximation of the Lagrangian drift, and when completing the V94 solutions, the residual circulations compensate only half of this drift.

Beyond the fact that it is not necessary to use the friction ''approach,'' this study shows that the approximations used have an important effect on the result. For this reason, it is necessary to verify them with an accurate numerical simulation of the complete system of Eqs. (1), (2), (3) as in a related study (Mazé et al. 1998).

3. The friction effect

The most important friction effect is the creation of a boundary layer above the bottom. This mechanism is expressed by the linearized Navier–Stokes equations:

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta_0}{\partial x} + \mu\frac{\partial^2 u}{\partial h^2},$$
$$\frac{\partial v}{\partial t} + fu = \mu\frac{\partial^2 v}{\partial h^2},$$
$$\frac{\partial}{\partial x}\int_0^H u\, dh = -\frac{\partial \eta_0}{\partial t}, \tag{41}$$

where h represents the depth and μ, the eddy viscosity coefficient, is assumed to be independent of h. (As a matter of fact, the momentum conservation equations over the continental shelf break must be written in the x, z axis system, where the x and z axes are perpendicular to the slope. The vertical diffusion term is actually $\mu\partial^2 u/\partial z^2$. The form used here is however sufficient because it respects the main friction characteristic, that is to say the creation of a boundary layer above the bottom.)

With the boundary conditions $\mathbf{u}(u, v) = 0$ for $h = H$ and $\partial\mathbf{u}/\partial h = 0$ for $h = 0$ (no surface friction), a classic calculation (Prandle 1982) leads to

$$u = u_0(x, t)\left[1 - \frac{\mathrm{Ch}(K_1 h)}{\mathrm{Ch}(K_1 H)} - (1 - b)\frac{\mathrm{Ch}(K_2 h)}{\mathrm{Ch}(K_2 H)}\right]$$

and

$$v = v_0(x, t)\left[1 + \frac{b\omega}{f}\frac{\mathrm{Ch}(K_1 h)}{\mathrm{Ch}(K_1 H)} - \left(1 + \frac{b\omega}{f}\right)\frac{\mathrm{Ch}(K_2 h)}{\mathrm{Ch}(K_2 H)}\right], \tag{42}$$

where $u_0(x, t)$ and $v_0(x, t)$ are defined by the relations:

$$j\omega u_0(x, t) - f v_0(x, t) = -g\frac{\partial \eta_0}{\partial x}$$

$$j\omega v_0(x, t) + f u_0(x, t) = 0.$$

Here b is a constant, $b = \omega(\omega - f)/(\omega^2 + f^2)$; K_1 and K_2 are complex numbers: $K_1 = k_1(1 + j)$, $K_2 = k_2(1 + j)$ with $k_1 = [(\omega + f)/2\mu]^{1/2}$ and $k_2 = [(\omega - f)/2\mu]^{1/2}$. The expressions in square brackets are complex numbers:

$$\mathrm{Ch}(K_1 h) = \cosh k_1 h \cos k_1 h + j \sinh k_1 h \sin k_1 h,$$

and, as $j\omega v_0(x, t) = -f u_0(x, t)$ and $jfu_0(x, t) = \omega v_0(x, t)$, u and v can be written as a linear combination of $u_0(x, t)$ and $v_0(x, t)$.

If $|\partial u/\partial x|$ is not negligible with respect to ω, the motion is defined by the equations:

$$\frac{\partial u^*}{\partial t} + u\frac{\partial u^*}{\partial x} - f v^* = -g\frac{\partial \eta^*}{\partial x} + \mu\frac{\partial^2 u^*}{\partial h^2}$$

$$\frac{\partial v^*}{\partial t} + u\frac{\partial v^*}{\partial x} + f u^* = \mu\frac{\partial^2 v^*}{\partial h^2}$$

$$\frac{\partial}{\partial x}\int_{-\eta_0}^{H} u^*\, dh = -\frac{\partial \eta_0^*}{\partial t}, \tag{43}$$

which are the Navier–Stokes equations written with the hydrostatic approximation and linearized with the approximation (GM92):

$$u^*\frac{\partial u^*}{\partial x} = u\frac{\partial u^*}{\partial x}. \tag{44}$$

We observe that for $x = x^*$

$$u^* = u(t, h, x^*) = u(t, h, x)$$

$$v^* = v(t, h, x^*) = v(t, h, x).$$

With (26) and for $x = x^*$, the expressions

$$\frac{\partial \eta_0^*}{\partial x} = \frac{\partial \eta_0}{\partial x}(t, x^*) = \frac{\partial \eta_0}{\partial x}$$

$$\frac{\partial^2 u^*}{\partial h^2} = \frac{\partial^2 u}{\partial h^2}(t, h, x^*) = \frac{\partial^2 u}{\partial h^2}(t, h, x)$$

constitute an exact solution of the momentum conservation equations and an approximate solution of the continuity equation.

Thus, the residual circulation is defined by

$$\overline{u}(x, h) = \frac{1}{T}\int_0^T u^*\, dt; \qquad \overline{v}(x, h) = \frac{1}{T}\int_0^T v^*\, dt. \tag{45}$$

The expression of x^* as a function of x, t, h is the following:

$$x^* = x - \int_{t_0}^{t} u\, dt + \sum_{i=1}^{\infty} f_i(x, t, h), \tag{46}$$

which is a convergent series if $|\partial u/\partial x| < \omega$. The functions $f_i(x, t, h)$ are defined by (26). Let $t_0 = t_0(x, h)$ be such as $v(t_0, x, h) = 0$. For example,

$$f_1(x, h, t) = \int_{t_0}^{t} u \int_{t_0}^{t} \frac{\partial u}{\partial x}\, dt \simeq \frac{1}{2}\frac{a}{H}\left(\int_{t_0}^{t} u\, dt\right)^2.$$

The exact analytical calculation of $\overline{u}(x, h)$ and $\overline{v}(x, h)$ is very long except when $h = H$, where $u = 0$; $u^* = u = 0$ and therefore $\overline{u}(x, H) = 0$.

If $h = 0$, when limiting x^* to $x - \int_{t_0}^{t} u\, dt$, we can obtain the "approximate" residual circulation, which remains a good approximation for fairly large depth.

For $\cos^2 k_1 H \ll \sinh^2 k_1 H$, $\cos^2 k_2 H \ll \sinh^2 k_2 H$, $\tanh^2 k_1 H \simeq 1$, and $\tanh^2 k_2 H \simeq 1$, which is the case for commonly used μ values, the residual current thus obtained close to the surface is, with $u_0(x, t) \simeq (D/H) \cos\omega t$:

$$\overline{u}(x, 0) = -\frac{1}{2}\frac{a}{\omega}\frac{D^2}{H^3}\Bigg[\frac{bk_1H}{\sinh^2 k_1H}(\cosh k_1H \cos k_1H - \sin k_1H \sin k_1H) + \frac{(1-b)k_2H}{\sinh^2 k_2H}(\cosh k_2H \cos k_2H - \sinh k_2H \sin k_2H)$$
$$+ \left(\frac{b \cosh k_1H \cos k_1H}{\sinh^2 k_1H} + (1-b)\frac{\cosh k_2H \cos k_2H}{\sinh^2 k_2H}\right)\left(\frac{b \sin k_1H}{\sinh k_1H} + (1-b)\frac{\sin k_2H}{\sinh k_2H}\right)\Bigg]$$

$$\overline{v}(x, 0) = \frac{1}{2}\frac{af}{\omega^2}\frac{D^2}{H^3}\Bigg[\left(1 + \frac{b\omega}{f}\frac{\cosh k_1H \cos k_1H}{\sinh^2 k_1H} - (1 + b\omega/f)\frac{\cosh k_2H \cos k_2H}{\sinh^2 k_2H}\right)^2 - \left(\frac{b\omega}{f}\frac{\sin k_1H}{\sinh k_1H} - (1 + b\omega/f)\frac{\sin k_2H}{\sinh k_2H}\right)^2$$
$$- \frac{b\omega}{f}\frac{k_1H}{\sinh^2 k_1H}(\cosh k_1H \cos k_1H - \sinh k_1H \sin k_1H)$$
$$- (1 + b\omega/f)\frac{k_2H}{\sinh^2 k_2H}(\cosh k_2H \cos k_2H - \sinh k_2H \sin k_2H)\Bigg].$$

If we suppress the friction by putting μ close to 0, k_1 and k_2 converge toward infinity, but $k_1H/\sinh k_1H$ converges toward zero. We then find

$$\overline{u}(x, 0) \to 0 \quad \text{and} \quad \overline{v}(x, 0) \to \frac{1}{2}\frac{af}{\omega^2}\frac{D^2}{H^3}, \tag{47}$$

which is the first term of the $\overline{v}$ development obtained in GM92. Consequently, friction modifies the current existing in an inviscid flow and creates of a bottom boundary layer for both the residual current and the harmonic current.

4. Comparison with the tidal stress method

The tidal stress method ‘‘requires’’ friction, which is parameterized by

$$\frac{1}{H}\frac{\boldsymbol{\tau}(H)}{\rho} = -k(x)\mathbf{U}, \tag{48}$$

where $\boldsymbol{\tau}(H)$ is the stress and ρ is the mass density.

With the current components u^* and v^* (independent of h), the problem is defined by the following system:

$$\frac{\partial u^*}{\partial t} + u^*\frac{\partial u^*}{\partial x} - fv^* = -g\frac{\partial \eta}{\partial x} - k(x)u^* \tag{49}$$

$$\frac{\partial v^*}{\partial t} + u^*\frac{\partial v^*}{\partial x} + fu^* = -k(x)v^* \tag{50}$$

$$\frac{\partial}{\partial x}[(H + \eta)u^*] = -\frac{\partial \eta}{\partial t}. \tag{51}$$

The free surface elevation η and the current components u^* and v^* are developed in a series:

$$v^* = \overline{v} + \sum_{n=1}^{\infty} v_n \exp jn\omega t. \tag{52}$$

These series are limited to the first two terms: $v^* = \overline{v} + v_1 \exp j\omega t$.

The harmonic motion is defined by

$$j\omega u_1 - fv_1 = -g\frac{\partial \eta_1}{\partial x} - ku_1 \tag{53}$$

$$j\omega v_1 + fu_1 = -kv_1 \tag{54}$$

$$\frac{\partial}{\partial x}(Hu_1) = -\frac{\partial \eta_1}{\partial t}, \tag{55}$$

and the residual motion by

$$\overline{u_1\frac{\partial u_1}{\partial x}} - f\overline{v} = -g\frac{\partial \overline{\eta}}{\partial x} - k\overline{u} \tag{56}$$

$$\overline{u_1\frac{\partial v_1}{\partial x}} + f\overline{u} = -k\overline{v} \tag{57}$$

$$\frac{\partial}{\partial x}(H\overline{u} + \overline{\eta_1 u_1}) = 0. \tag{58}$$

With the approximations $\eta_1 \simeq \eta_0$ and $k\mathbf{u} \simeq k\mathbf{u}_0$, where η_0 and $\mathbf{u}_0$ remain the solutions of Laplace equations, the following is obtained:

$$u_1 = u_0 + \frac{k}{f}\frac{(\omega^2 + f^2)}{(\omega^2 - f^2)}v_0 \quad \text{and}$$
$$v_1 = v_0 - \frac{2kf}{(\omega^2 - f^2)}u_0. \tag{59}$$

In addition, with

$$u_0 \simeq \frac{D}{H}\exp j\omega t, \qquad v_0 \simeq j\frac{f}{\omega}u_0, \quad \text{and} \quad k(x) = k = C^{\text{ste}}, \tag{60}$$

the residual components are

$$\overline{u} = 0 \quad \text{and} \quad \overline{v} = \frac{1}{2}\frac{af}{\omega^2}\frac{D^2}{H^3} \quad \text{if } k \to 0. \tag{61}$$

Here $\overline{v}$ is then the residual component parallel to the continental shelf break in an inviscid fluid, but it depends upon the friction parameterization:

$$k \propto \frac{1}{H} \Rightarrow \overline{v} = \frac{af}{\omega^2}\frac{D^2}{H^3}$$

or

$$k \propto \frac{1}{H^2} \Rightarrow \overline{v} = \frac{3}{2}\frac{af}{\omega^2}\frac{D^2}{H^3}.$$

Using the eddy viscosity defined above, the same reasoning and corresponding approximations lead to a determination of $\overline{v}$ by

$$\mu\frac{\partial^2 \overline{v}}{\partial h^2} = \overline{u\frac{\partial v}{\partial x}}, \tag{62}$$

where u and v are the current components at the depth h. With the boundary conditions,

$$\overline{v}|_{h=H} = 0 \quad \text{and} \quad \left.\frac{\partial \overline{v}}{\partial h}\right|_{h=0} = 0, \tag{63}$$

the integration of Eq. (62) leads to

$$\overline{v} = \lim_{\mu\to 0}\left(-\frac{1}{\mu}\int_h^H \int_0^h \overline{u\frac{\partial v}{\partial x}}\, dh\, dh'\right)$$
$$= \frac{af}{\omega(\omega - f)}\frac{D^2}{H^3}\left(1 + \frac{1}{4}\frac{(\omega^2 - f^2)}{(\omega^2 + f^2)}\right) \quad \text{for } h = 0.$$

It appears clear that such a variety of results obtained depending upon the chosen friction form raises a problem that needs to be resolved.

To explain the ‘‘geostrophic degeneracy’’ notion invoked by V94 to justify the arbitrary use of friction, the problem is addressed by splitting u^*, v^*, η^* in series in Eqs. (49), (50), and (51):

$$u^* = \overline{u} + \sum_{k=1}^{\infty} u_k \exp jk\omega t$$

$$v^* = \overline{v} + \sum_{k=1}^{\infty} v_k \exp jk\omega t$$

$$\eta^* = \overline{\eta} + \sum_{k=1}^{\infty} \eta_k \exp jk\omega t,$$

where $\overline{u}$, u_k, $\overline{v}$, v_k, $\overline{\eta}$, and η_k are complex numbers. Noting that

$$z_k \exp jk\omega t \cdot z_l \exp jl\omega t$$

$$= \frac{1}{2}(z_k^* z_l) \exp j(l-k)\omega t + 1/2(z_k z_l) \exp j(l+k)\omega t,$$

the Eqs. (49), (50), (51) can then be separated as

$$jk\omega u_k + \overline{u}\frac{\partial u_k}{\partial x} + u_k\frac{\partial \overline{u}}{\partial x} + \frac{1}{2}\sum_{k=1}^{\infty}\left(u_k^*\frac{\partial u_{2k}}{\partial x}\right) - f v_k$$
$$= -g\frac{\partial \eta_k}{\partial x}[-k(x)u_k] \tag{64}$$

$$jk\omega v_k + \overline{u}\frac{\partial v_k}{\partial x} + u_k\frac{\partial \overline{v}}{\partial x} + \frac{1}{2}\sum_{k=1}^{\infty}\left(u_k^*\frac{\partial v_{2k}}{\partial x}\right) + f u_k$$
$$= 0[-k(x)u_k] \tag{65}$$

$$\frac{\partial}{\partial x}\left[Hu_k + \overline{u}\,\eta_k + \frac{1}{2}\sum_{k=1}^{\infty}(u_k^*\eta_{2k})\right] = jk\omega\eta_k \tag{66}$$

with $\overline{\eta} \ll H$. The residual motion is defined by

$$\overline{u}\frac{\partial \overline{u}}{\partial x} + \frac{1}{2}\sum_{k=1}^{\infty}\left(u_k^*\frac{\partial u_k}{\partial x}\right) - f\overline{v} = -g\frac{\partial \overline{\eta}}{\partial x}[-k(x)\overline{u}] \tag{67}$$

$$\overline{u}\frac{\partial \overline{v}}{\partial x} + \frac{1}{2}\sum_{k=1}^{\infty}\left(u_k^*\frac{\partial v_k}{\partial x}\right) + f\overline{u} = 0[-k(x)\overline{v}] \tag{68}$$

$$\frac{\partial}{\partial x}\left[H\overline{u} + \frac{1}{2}\sum_{k=1}^{\infty}(u_k^*\eta_k)\right] = 0. \tag{69}$$

For $k(x) = 0$, we find that $\overline{u}$ is defined by

$$\overline{u} = -\frac{1}{2H}\sum_{k=1}^{\infty}(u_k^*\eta_k) \tag{70}$$

and

$$\frac{\partial \overline{v}}{\partial x} = -\left[\frac{1}{2}\sum_{k=1}^{\infty}\left(u_i^*\frac{\partial v_k}{\partial x}\right) + f\overline{u}\right]\Big/\overline{u} \tag{71}$$

with the boundary condition $\overline{v} = 0$ both on the abyssal plain and the continental shelf. For $k(x) \neq 0$,

$$\frac{\partial \overline{v}}{\partial x} + k(x)\overline{v}/\overline{u} = -\left[\frac{1}{2}\sum_{k=1}^{\infty}\left(u_k^*\frac{\partial v_k}{\partial x}\right) + f\overline{u}\right]\Big/\overline{u}. \tag{72}$$

It appears in this last expression that, if $k = 0$, then $\overline{u}$, u_k, and v_k take the values of the previous case, and $\overline{v}_x$ takes the value defined for $k(x) = 0$. It is also clear that the external force parameterization $-k\mathbf{u}$ has no other effect than to complicate the problem. The introduction of these $\overline{u}$ and $\overline{v}$ expressions in Eqs. (64), (65), (66) leads to a nonlinear system of $3N$ equations, with $N \to \infty$, which is impossible to solve exactly if $k(x) = 0$ and even less so if $k(x) \neq 0$. It is due to the nonlinearity of the equations, and definitely has nothing to do with the geostrophic degeneracy. The calculation of the exact residual current is impossible; thus, linearization of the equations leads to an approximation that will be stated precisely by a rigorous numerical simulation [setting of course $k(x) = 0$].

Two types of linearization can be considered:

1) The first one consists in simplifying equations (64), (65), (66) and (67), (68), (69): first, by truncating the series at first order, and, then, by neglecting some terms.
2) The second one consists of linearizing Eqs. (49), (50), and (51).

Let us first examine the first linearization type: The first-order truncation eliminates all the terms in the sum $\Sigma_{k=1}^{\infty}$ in Eqs. (64), (65), and (66). In Eqs. (67), (68), and (69) the sum is limited to the first term ($k = 1$). In the low "nonlinearity" cases (Huthnance 1973), the interaction term averaged current-harmonic current in (64) and (65) plus the terms $\overline{u}\partial\overline{u}/\partial x$ and $\overline{u}\partial\overline{v}/\partial x$ in (67) and (68) are also neglected. These approximations lead to systems (53), (54), (55) and (56), (57), (58). The problem is "degenerate" for $k(x) = 0$—that is, the residual component $\overline{v}$ cannot be determined without friction because all the terms allowing this calculation have been eliminated. Then, neglecting $u_1\partial\overline{v}/\partial x$ in (65) signifies that the relation $|u_1|\partial\overline{v}/\partial x \ll k\overline{v} \ll f|u_1|$ is admitted. Now $\overline{v}$ is of the order of $af|u_1|^2/(\omega^2 H)$. With the observed values for the Bay of Biscay: $a \simeq 10^{-1}$, $f \simeq 10^{-4}$ s^{-1}, $\omega \simeq 1.4 \times 10^{-4}$ s^{-1}, $|u_1| \simeq 0.5$ m s^{-1}, for $H = 300$ m, then $\partial\overline{v}/\partial x = 1.5 \times 10^{-4}$ s^{-1}; thus $\partial\overline{v}/\partial x > f$. This approximation is valid only in the case where H is large, that is, up to the continental shelf break. To solve the highest nonlinearity cases, Loder (1980) retains the term $u_1\partial\overline{v}/\partial x$ in (65) but supposes $u_1 = u_0 = D \exp j\omega t/H$. In this case, $\overline{u}$ (complex number) is equal to zero, and Eqs. (65) and (68) become

$$j\omega v_1 + u_0\frac{\partial \overline{v}}{\partial x} + f u_0 = 0(-kv_1) \tag{73}$$

$$\frac{1}{2}u_0^*\frac{\partial v_1}{\partial x} = 0(-k\overline{v}). \tag{74}$$

Thus, for $k(x) = 0$, (73) yields

$$v_1 = j\frac{f}{\omega}u_0 + \frac{j}{\omega}u_0\frac{\partial \overline{v}}{\partial x}. \tag{75}$$

Introducing (75) into (74) leads to

$$\frac{\partial^2 \overline{v}}{\partial x^2} + \frac{a}{H}\overline{v}_x + \frac{a}{H}f = 0. \tag{76}$$

This equation, therefore, defines the residual component $\overline{v}_{(0)}$ for $k(x) = 0$. With the boundary conditions $\overline{v}_{(0)}$ and $\overline{v}_{(0)x}$ everywhere on the abyssal plain ($H = H_0$) and, thus up to the continental shelf break in $x = 0$, it follows that

$$\overline{v}_{R(0)} = \mathrm{Re}(\overline{v}_{(0)}) \Rightarrow \overline{v}_{R(0)} = -\frac{1}{2}\frac{a}{H_0}fx^2 \quad \text{and}$$

$$\overline{v}_{I(0)} = \mathrm{Im}(\overline{v}_{(0)}) = 0. \tag{77}$$

If the external force ($-k\mathbf{u}$) is used with, for example, $k(x) = C^{\mathrm{ste}}$, solution of Eqs. (73) and (74) becomes more complex. The imaginary component $\overline{v}_I$ of $\overline{v}$ is necessary and leads to the resolution of two nonlinear equations. The unknown quantities are the real and imaginary parts of $\overline{v}$, $\overline{v}_R$, and $\overline{v}_I$. Obviously, this system of equations can be solved using the Runge–Kutta method, but Eqs. (73), (74) show clearly that, if $k(x) \to 0$, the solution must absolutely converge to $\overline{v}_{(0)}$. The approximations used remain excessive; neither the direction nor the intensity of $\overline{v}_{(0)}$ correspond to the observations. On the eastern side of the ocean and with a continental shelf oriented southeast to northwest as in the Bay of Biscay, the observations reveal a current orientation toward the northwest of 5–10 cm s^{-1} at a depth of 200 m on a continental shelf break slope of $a \simeq 0.1$. The obtained $\overline{v}_{(0)}$ is oriented southeastward. Its strength is too great, and, in addition, this current does not depend on the tidal current.

To study the influence of truncation examine now the direct linearization of (73) by the substitution $u^*\partial/\partial x = u_0\partial/\partial x$. Thus the motion is defined by

$$\frac{\partial u^*}{\partial t} + u_0\frac{\partial u^*}{\partial x} - fv^* = -g\frac{\partial \eta^*}{\partial x}, \tag{78}$$

$$\frac{\partial v^*}{\partial t} + u_0\frac{\partial v^*}{\partial x} + fu^* = 0, \tag{79}$$

$$\frac{\partial}{\partial x}(Hu^* + u_0\eta^*) = -\frac{\partial \eta^*}{\partial t}. \tag{80}$$

As has been demonstrated, the ‘‘external force’’ $-k\mathbf{u}$ is of no interest and is no longer included. This system is linear and the component u_0 provides the advection of u^*, v^*, η^*. It is true that $u^* = u_0(t, x^*)$; $v^* = v_0(t, x^*)$, and $\partial\eta^*/\partial x = \partial\eta_0/\partial x$ (in $x = x^*$) $= \partial\eta_0/\partial x^*(t, x^*)$ constitute an exact solution for (78) and (79) and an approximate solution for (80) if $dx^*/dt = \partial x^*/\partial t + u_0\partial x^*/\partial x = 0$. Thus,

$$\frac{\partial u^*}{\partial t} + u_0\frac{\partial u^*}{\partial x} = j\omega u^*;$$

$$\frac{\partial v^*}{\partial t} + u_0\frac{\partial v^*}{\partial x} = j\omega v^* = -fu^*. \tag{81}$$

When u^* and v^* are developed in a series, it follows that

$$\overline{u} = -\frac{1}{2\omega}ju_0^*\frac{\partial u_1}{\partial x} \quad \text{and} \quad v_1 = j\frac{f}{\omega}u_1, \tag{82}$$

which can be used in the previous method to determine $\overline{v}$ [which is directly defined by $\overline{v} - (½)ju_0^*\partial v_1/\partial x/\omega$]. The equations become

$$u_0\frac{\partial \overline{v}}{\partial x} + \frac{1}{2}u_0^*\frac{\partial v_2}{\partial x} = 0 \tag{83}$$

and

$$\frac{1}{2}u_0^*\frac{\partial v_1}{\partial x} + f\overline{u} = 0. \tag{84}$$

Thus, Eq. (83) demonstrates that $\overline{v}_x$ is defined by the second harmonic component v^*. In the case where u_0 can be represented by $u_0 = D\exp j\omega t/H$, it follows that

$$\overline{v}_{0_x} = \mathrm{Re}\left(\frac{\partial v}{\partial x}\right) = -\frac{1}{2}\mathrm{Re}\left(\frac{\partial v_2}{\partial x}\right). \tag{85}$$

Thus, with $\overline{v} = 0$ and $v_2 = 0$ on the abyssal plain:

$$\overline{v}_0 = -\frac{1}{2}\mathrm{Re}(v_2). \tag{86}$$

The unacceptable result (77) has vanished; it was due only to the approximation $\overline{u} = 0$ instead of Re ($\overline{u} = 0$). Besides, it appears clearly that this second harmonic v^* component defines $\overline{v}$.

5. Field data analysis

The data used in the present study were acquired during the MINT-94 experiment in the Bay of Biscay (18 May 1994–17 June 1994) carried out by the French Navy's CMO-SHOM (Outré and Pichon 1995; Pichon 1997). Doppler current meter measurements were acquired at a location on the 300-m isobath above the continental slope (Figs. 1 and 2). From an acoustic Doppler current profiler (ADCP) fixed on the sea bottom, data were obtained within layers of thickness $\delta h = 4$ m between 48 m and 280 m. At the beginning of the experiment, the water column was weakly stratified. A thermocline 5 m thick and characterized by $g' = 1.2 \times 10^{-3}$ m s^{-2} (reduced gravity) was located at 80-m depth. The amplitude and phase of the tidal components have been deduced from harmonic analysis of the data. In the study area, diurnal tides are weak. The semidiurnal tide includes fundamentally the M_2, S_2, K_2, and N_2, components. The tidal current, which is then reconstructed from these four components will be called hereafter ‘‘linear semidiurnal.’’ Therefore, at each depth, the cross-slope and the alongslope components of the above linear semidiurnal currents can be compared with the measured current (Figs. 3 and 4) for the two examples, the cross-slope component is in the

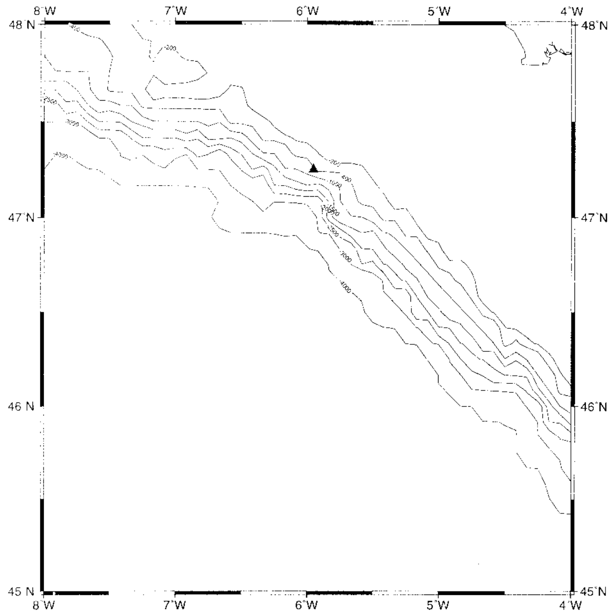

FIG. 1. Location of the ADCP mooring.

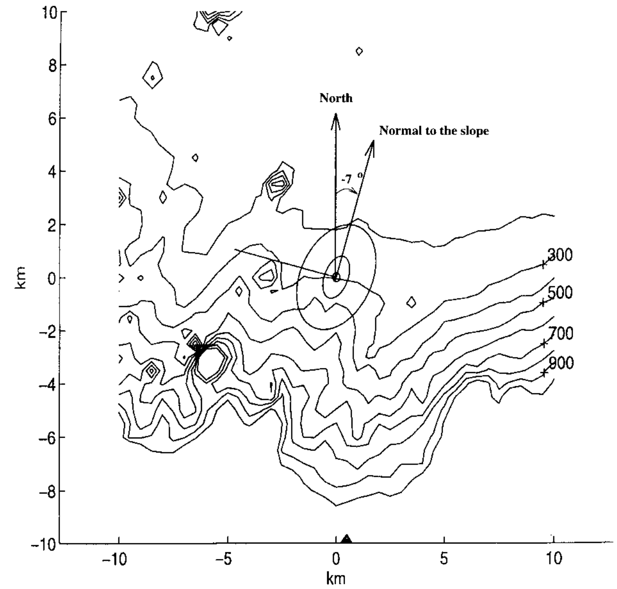

FIG. 2. Slope topography in the vicinity of the ADCP mooring.

south–north direction and the alongslope component is in the west–east direction. In many cases, the comparison between data and results given by theoretical models is a rather difficult challenge. Thus, the examples presented here are chosen to be more explicit and to show limiting cases: a ''good'' example and a ''less favorable'' one.

These observations show different characteristics of the cross-slope and the alongslope components. For the cross-slope component, differences between in situ measurements and linear semidiurnal values tend to balance. Thus, the residual component, that is, the low-frequency cross-slope component, practically vanishes. For the alongslope component, variations are not symmetrical. The maximum amplitude of the measured current in the east direction is weaker than that given by linear semidiurnal reconstruction. This feature appears clearly in the data at 184-m depth (Fig. 4) but much less in the data at 88 m (Fig. 3). In the west direction, the maximum measured value is strongly marked, leading to a low-frequency residual current (Fig. 6). This feature can be observed at 184 m, 88 m, and also at other depths. This unsymmetrical feature is, of course, related to the presence of a mean semidiurnal current. The magnitude of this current increases as the linear semidiurnal current increases. Consequently, the alongslope residual current contains a fortnightly component due to the linear semidiurnal current. It is possible that the aforementioned feature can be explained in different ways. However, it is clear that reference to frictional influence here is a misinterpretation.

To understand these observations in terms of the theoretical approach proposed in GM92, the standard approximation used previously for a monochromatic current can be adopted:

$$u_L = \frac{D}{H}\cos\omega t; \qquad v_L = -\frac{f}{\omega}\frac{D}{H}\sin\omega t$$

$$u^* = \frac{D}{H'}\cos\omega t; \qquad v^* = -\frac{1}{2}\frac{D}{H'}\sin\omega t$$

with

$$H' = H\left(1 + \frac{2a}{\omega}\frac{D}{H^2}\sin\omega t\right)^{1/2},$$

where

$$\frac{aD}{\omega H} = 0.4 \quad \text{and} \quad \frac{D}{H} = 0.4\ \text{m s}^{-1}.$$

Comparisons between u^* and u_L on one hand and between v^* and v_L on the other (Fig. 5) reveal evolutions quite similar to those suggested by observations (Fig. 4).

To understand the importance of tidal rectification in the development of the observed low-frequency current (Fig. 6), some assumptions must be made:

1) The linear current is the sum of four semidiurnal components; that is, at 88 m:

$$u_L = (u_{11} + u_{12}\sin\Omega t)\cos\omega t \quad (S \rightarrow N)$$

$$v_L = (v_{11} + v_{12}\sin\Omega t)\sin\omega t \quad (W \rightarrow E)$$

at 184 m:

$$u_L = (u_{21} + u_{22}\sin\Omega t)\cos\omega t$$

$$v_L = (v_{21} + v_{22}\sin\Omega t)\sin\omega t$$

for $t_0 < t < t_0 + 25$ days, where $t_0 = 8$ h, $\omega = 1.4$

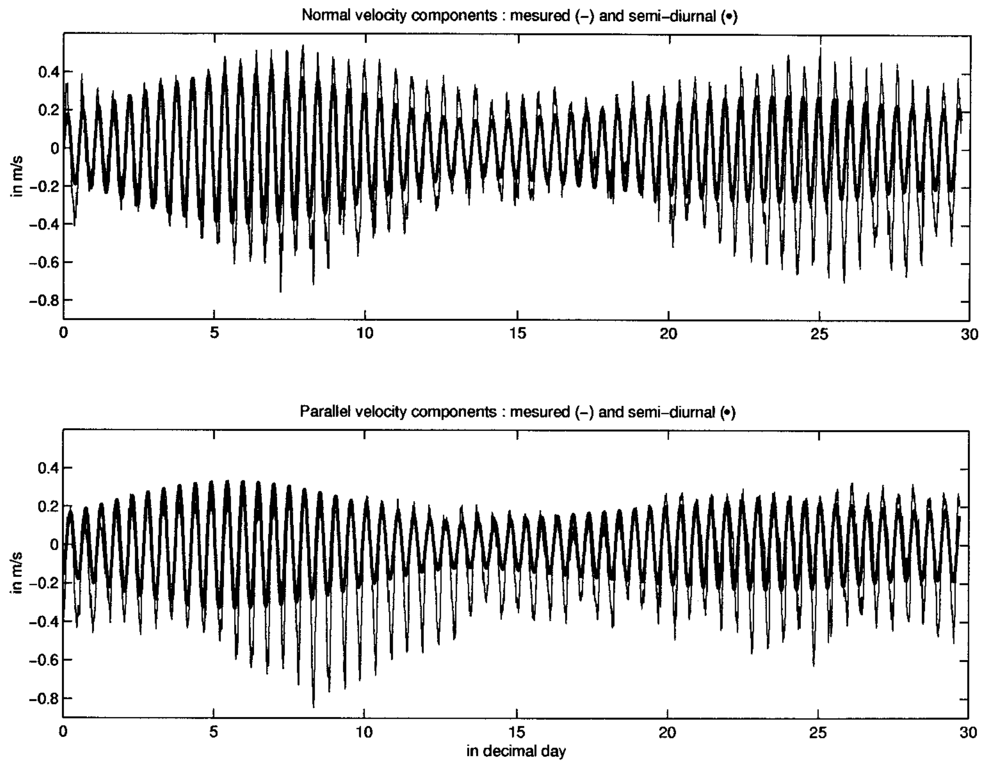

FIG. 3. Comparison between measured currents and calculated currents from M_2, S_2, N_2, and K_2 tidal components at depth h = 88 m (MINT-94 EPSHOM/CMO): (upper curves) cross-slope and (lower curves) alongslope components.

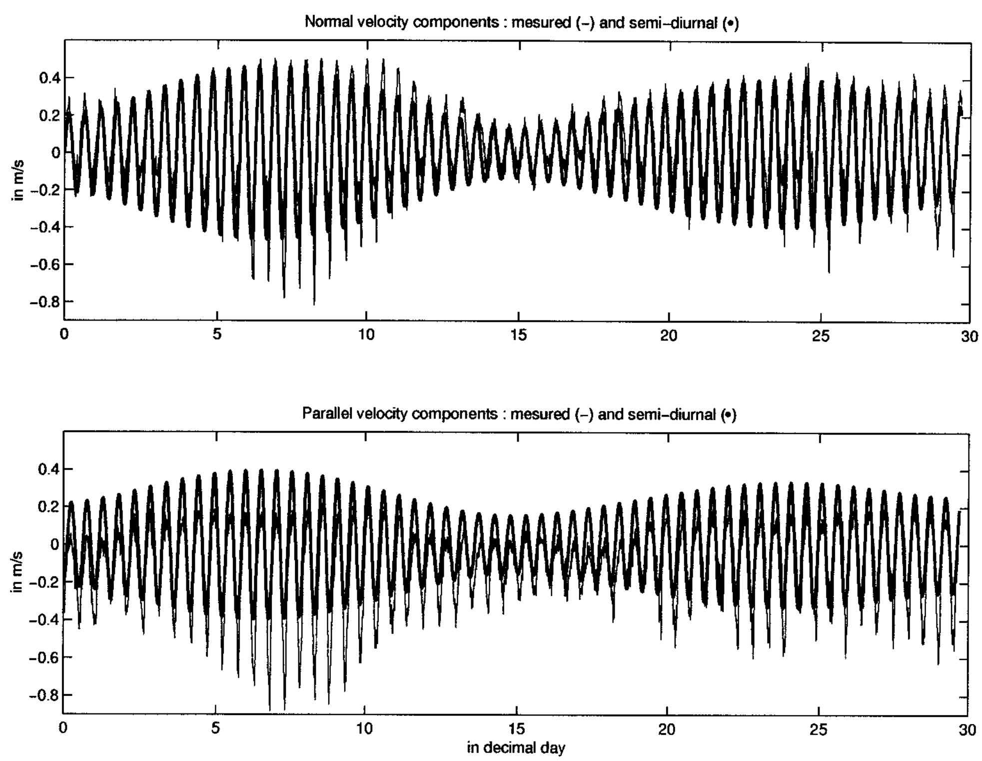

FIG. 4. As in Fig 3 but at depth h = 184 m.

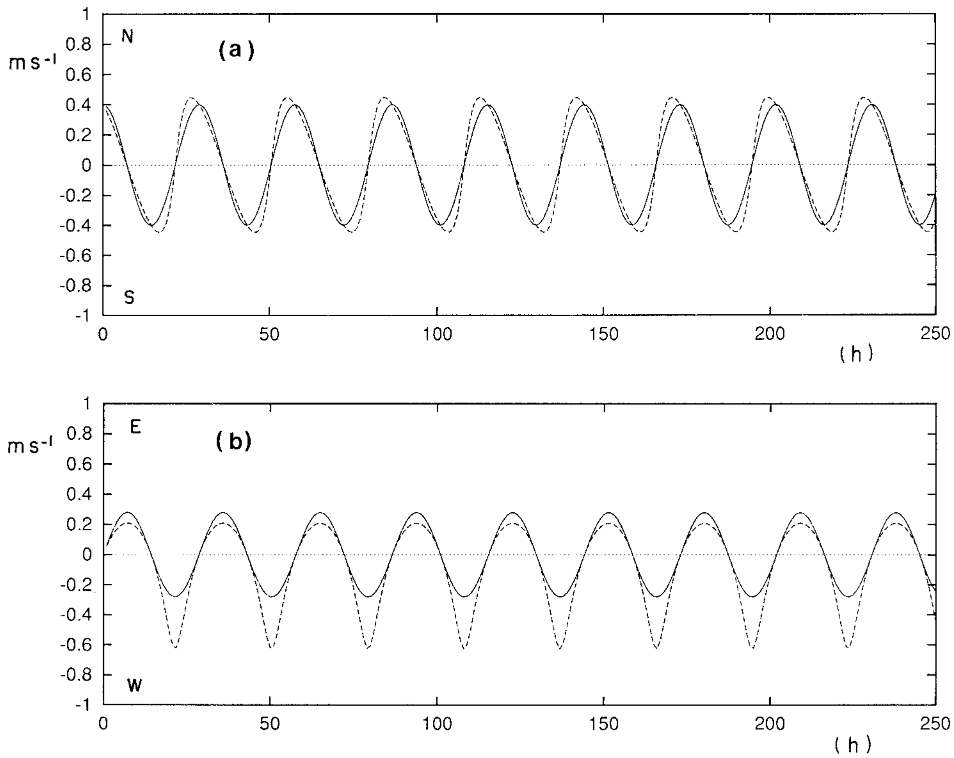

FIG. 5. Comparison between u_L and u^* and between v_L and v^* for a monochromatic current.

$\times 10^{-4}$ s^{-1}, $\Omega = 5 \times 10^{-6}$ s^{-1} ($T \simeq 15$ days), and $u_{11} = 0.35$ m s^{-1}, $u_{12} = 0.15$ m s^{-1}, $v_{11} = 0.3$ m s^{-1}, $v_{12} = 0.1$ m s^{-1}, $u_{21} = 0.3$ m s^{-1}, $u_{22} = 0.1$ m s^{-1}, $v_{21} = 0.24$ m s^{-1}, and $v_{22} = 0.08$ m s^{-1}. The topographic slope is $a = 2/3 \times 10^{-1}$.

2) The continental slope shape is linear (this is not the actual shape shown in Fig. 2).
3) The spatial evolution of the linear current is given by

$$u_l = \frac{D_x(t)}{H^*}\cos\omega t \quad \text{and} \quad v_l = \frac{D_y(t)}{H^*}\sin\omega t,$$

where

$$H^* = H\left(1 + \frac{2a}{H}\int_0^T D_x(t)\cos\omega t\, dt\right)^{1/2} \quad \text{if } H \geq 200 \text{ m}$$

and $H^* = 200$ m if the above calculated value is less than $= 200$ m. This last restriction is connected to the fact that H^* represents the actual water depth associated with a parcel moving from location x^*, where it is at time $t = t_1$ [$v_L(t_1) = 0$ so $t_1 = 0$], to location x, which is reached at time t with a velocity (u^*, v^*). Hence, H^* cannot be less than $H_1 = 200$ m if a continental shelf of constant depth is assumed beyond the 200-m isobath.

The current amplitude increases with depth because of the stratification. This feature implies baroclinic components that are not described in the expression of u^* and v^*. The ''apparent'' water displacement can then be theoretically calculated:

$$x - x_0 = \int_{t_0}^{t} u^*\, dt \quad \text{and} \quad y - y_0 = \int_{t_0}^{t} v^*\, dt.$$

To smooth this water parcel trajectory only one value for each tidal period is plotted. Hence, the M_2 tidal component is eliminated. The same procedure is adopted for the representation of in situ data.

The vertical structure of the ''measured'' trajectory is shown on Fig. 6. As with the semidiurnal current amplitude, the mean current increases with depth. This current practically flows in the alongslope direction toward the west. The ''apparent'' trajectories computed at 88 and 184 m (Fig. 7) are quite similar in order of magnitude. The assumptions restrict the generality of the above comparisons. However, it appears that the theoretical view of this problem allows specification of the effect of tidal rectification on the development of the alongslope current. Nevertheless, the spatial characteristics of the linear tide in the continental slope area must be exactly known. The use of a schematic tidal solution of the hydrodynamic equations allows numerical simulations of the ''mean'' current induced by this process (Mazé et al. 1998). Analysis of in situ data suggests that a ''linear tide'' very close to the actual

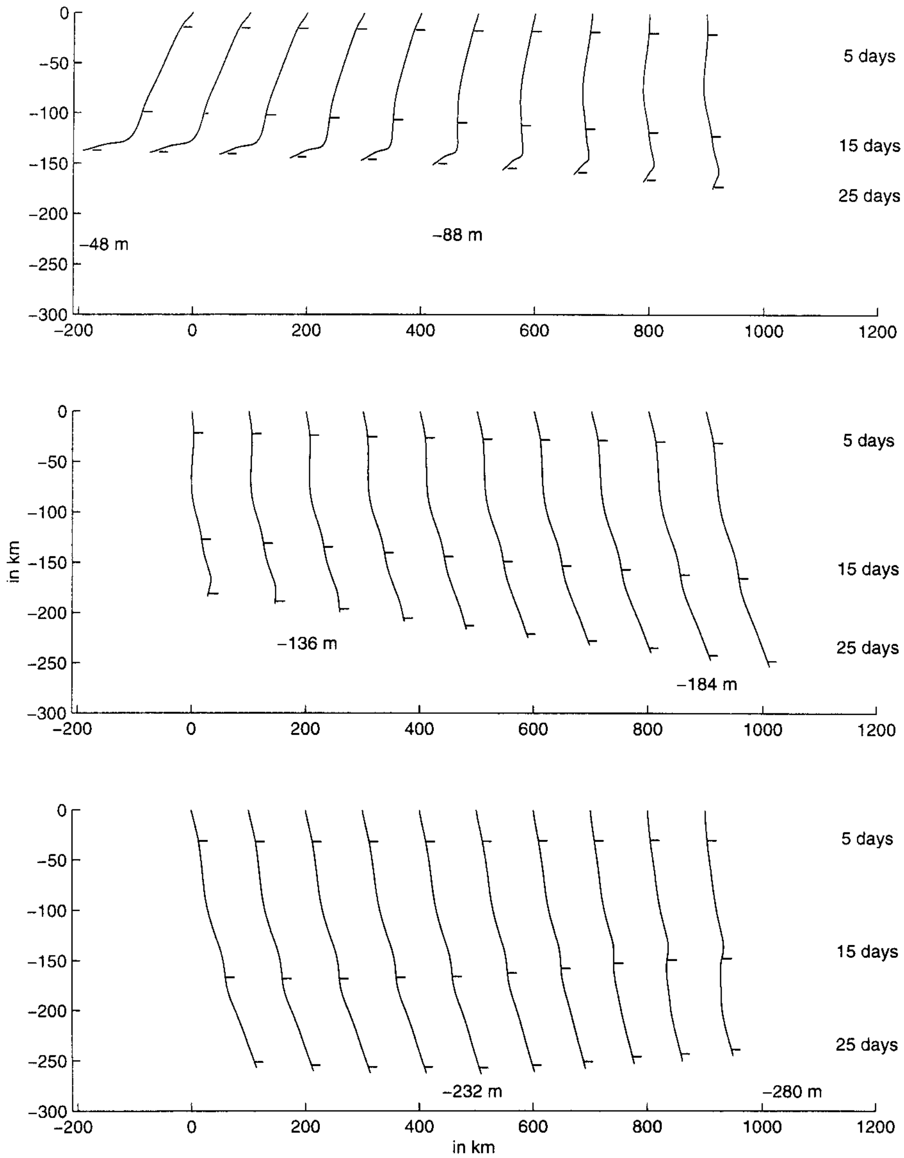

FIG. 6. Apparent trajectories of a water parcel obtained from data at different depths.

tide must be used; hence, this linear tide must be expressed from the tide-producing force associated with both the lunar and solar tidal potentials.

6. Conclusions

This study helps to clarify the problem of residual motion due to tidal rectification on a continental shelf break. It clearly leads to three conclusions:

1) The geostrophic degeneracy is only due to excessive approximations and can certainly not be considered as ''*a fundamental feature of the dynamics involved*,'' as stated in V94.
2) The friction strength ($-k\mathbf{u}$) used in the tidal stress theory is in reality an external force applied to an inviscid fluid. It is of no interest and can certainly not define the strengths balance of the residual motion in the Lagrangian reference frame under the form (V94, p. 2199):

$$\overline{k^\bullet(t)v^\bullet(t)} = \overline{k^\bullet(t)}\overline{v} + \overline{k^\bullet(t)v_T^\bullet(t)}. \qquad (87)$$

3) The development in series, followed by truncation of the series at first order, is a method that leads to unacceptable results if the residual component $\overline{u}$ perpendicular to the continental shelf break (i.e., as a matter of fact, the complete solution) is unknown.
4) The true reason of the indeterminacy of $\overline{\mathbf{u}}(\overline{u}, \overline{v})$ lies in the nonlinear character of the equations, which

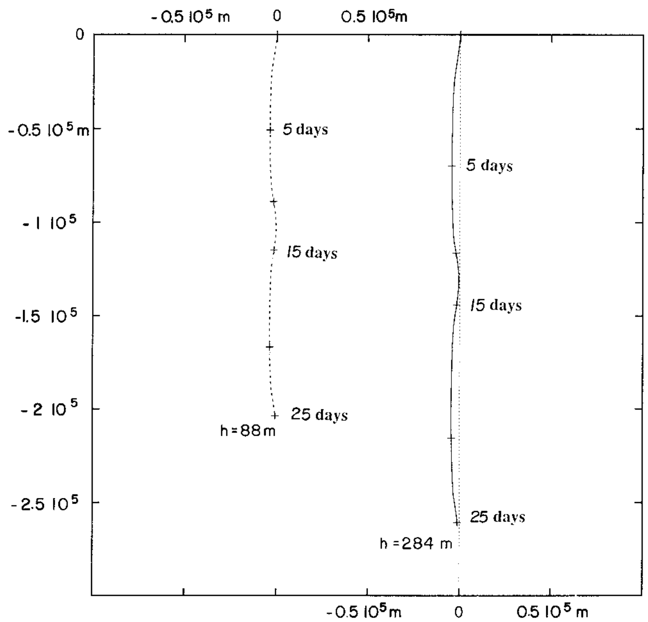

FIG. 7. Apparent trajectories resulting from time integration of u^* and v^* at depths $h = 88$ m and $h = 184$ m.

expresses the physics of this mechanism. It appears that the approximate residual current can be obtained by linearizing these equations, for example, with $u^*\partial/\partial x = u_0\partial/\partial x$. As the result obtained remains an approximation, it is necessary to state precisely its validity by using an accurate numerical solution of the nonlinear problem and with, of course, $k(x) = 0$.

5) The analysis of field data clearly explains the tidal rectification mechanism: when the alongslope component of the current has shallow water on the right (in the Northern Hemisphere), its intensity is highly amplified. In the opposite case, the current intensity is reduced. Thus, the forcing of a residual current results. This residual current presents the characteristic structure of the tidally rectified current with a periodic component associated with the fortnightly period due to the M_2–S_2 interaction. Qualitative and quantitative comparisons between theory and data seem very satisfactory. Nevertheless, this conclusion must be moderated because of uncertainties due to the various assumptions used in the expression of the spatial evolution of the tidal current above the continental slope.

REFERENCES

Garreau, P., and R. Mazé, 1992: Tidal rectification and mass transport over a shelf break: A barotropic frictionless model. *J. Phys. Oceanogr.,* **22,** 719–731.

Huthnance, J. M., 1973: Tidal current asymmetries over the Norfolk sandbanks. *Estuarine Coastal Mar. Sci.,* **1,** 89–90.

Loder, J. W., 1980: Topographic rectification of tidal currents on the sides of Georges Bank. *J. Phys. Oceanogr.,* **10,** 1399–1416.

Mazé, R., G. Langlois, and F. Grosjean, 1998: Tidal Eulerian residual currents over a slope: Analytical and numerical frictionless models. *J. Phys. Oceanogr.,* **28,** 1321–1332.

Outré, M., and A. Pichon, 1995: Données de courantométrie Döppler LADCP recueillies au cours de la campagne MINT-94. Rapport détude 39/EPSHOM, 40 pp. [Available from EPSHOM, B. P. 426, 29275 Brest, France.]

Pichon, A., 1997: Présentation et analyse des données d'hydrologie de la campagne MINT-94. Rapport détude EPSHOM/CMO, 212 pp. [Available from EPSHOM, B. P. 426, 29275 Brest, France.]

Pingree, R., and B. Le Cann, 1989: Celtic and Armorican slope and shelf residual currents. *Progress in Oceanography*, Vol. 23, Pergamon, 303–338.

Prandle, D., 1982: The vertical structure of tidal currents and other oscillatory flows. *Contin. Shelf Res.,* **1,** 191–207.

Visser, A. W., 1994: On tidal rectification, friction, and geostrophic degeneracy. *J. Phys. Oceanogr.,* **24,** 2196–2220.