On the Numerical Implementation of Advection Schemes for Use in Conjunction with Various Mixing Parameterizations in the GFDL Ocean Model

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ABSTRACT

The results from ocean model experiments conducted with isopycnal and isopycnal thickness diffusion parameterizations for subgrid-scale mixing associated with mesoscale eddies are examined from a numerical standpoint. It is shown that when the mixing tensor is rotated, so that mixing is primarily along isopycnals, numerical problems may occur and non-monotonic solutions, which violate the second law of thermodynamics, may arise when standard centered difference advection algorithms are used. These numerical problems can be reduced or eliminated if sufficient explicit (unphysical) background horizontal diffusion is added to the mixing scheme. A more appropriate solution is the use of more sophisticated numerical advection algorithms, such as the flux-corrected transport algorithm. This choice of advection scheme adds additional mixing only where it is needed to preserve monotonicty and so retains the physically desirable aspects of the isopycnal and isopycnal thickness diffusion parameterizations, while removing the undesirable numerical noise. The price for this improvement is a computational increase.

1. Introduction

It has long been thought that mixing of tracers by mesoscale eddies occurs primarily along isopycnal surfaces (Iselin 1939; Montgomery 1940; Gent and McWilliams 1990). Large-scale ocean models, however, have not yet been able to resolve the formation and effects of these eddies on long-climatic timescales (hundreds of years) over the global domain. While significant progress toward this direction has been made (e.g., Semtner and Chervin 1988, 1992), computational costs still preclude long-timescale equilibrium integrations of fully eddy-resolving ocean general circulation models (OGCMs). As such, it is necessary to develop and test parameterizations of subgrid-scale mixing processes in coarse-resolution ocean models.

Traditionally, subgrid-scale mixing in z-coordinate ocean models has been parameterized by either Laplacian or biharmonic horizontal/vertical diffusion. While this approach is very appealing from a numerical solution standpoint, it has a very weak physical basis and can lead to large, and spurious, diapycnal mixing in regions where the isopycnals have a large slope. Redi (1982) showed how the mixing tensor could be rotated for use in ocean models. This parameterization, while a step in the right direction, only partially accounted for the effects of mesoscale eddy-induced mixing of tracers. Unfortunately, the implementation of this parameterization into the GFDL OGCM (Cox 1987) still required a specified background horizontal mixing in order to maintain numerical stability. While this background horizontal diffusivity can be as weak as a factor of 10 smaller in models with sufficiently high horizontal resolution, it still can lead to unphysically large diapycnal fluxes in regions of steeply sloping isopycnals.

More recently Gent and McWilliams (1990) introduced a new parameterization of mesoscale eddy-induced mixing for use in coarse-resolution ocean models. The idea was to improve ocean models by adding isopycnal thickness diffusion to account for the removal of potential energy from the stratification due to baroclinic instability. While there is some question as to whether the implicit assumption of viscous dissipation of the released potential energy is appropriate (Tandon and Garrett 1996), the new parameterization has shown great success in improving the climatology of ocean models (Danabasoglu et al. 1994; Boning et al. 1995; Danabasoglu and McWilliams 1995; Robitaille and Weaver 1995; England 1995).

As shown by Gent et al. (1995), isopycnal thickness diffusion in the Gent and McWilliams (1990) parameterization is equivalent to the addition of eddy-induced ''bolus'' transport velocities

$$\mathbf{u}^* = \frac{\partial}{\partial z}\left(k_T\frac{\boldsymbol{\nabla}\rho}{\rho_z}\right), \qquad w^* = -\boldsymbol{\nabla}\left(k_T\frac{\boldsymbol{\nabla}\rho}{\rho_z}\right) \tag{1}$$

to the mean velocity in the tracer equations. Here k_T is

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the thickness diffusivity, ρ is the potential density, and $\mathbf{u}^* = (u^*, v^*)$ and w^* are the horizontal and vertical components of the bolus transport velocity, respectively. Danabasoglu et al. (1994) and Danabasoglu and McWilliams (1995) claim that one of the strengths of this parameterization is that the usual background horizontal diffusivity, needed to maintain computational stability in the Redi–Cox isopycnal mixing scheme, is no longer required in the GFDL z-coordinate OGCM. This leads to less diapycnal mixing and may thereby lead to more physically realistic simulations. Nevertheless, other numerical problems associated with implementation of this parameterization are alluded to in Danabasoglu and McWilliams (1995), where they state that fairly severe filtering and limiting of topography is needed to ''prevent numerical instabilities and inaccuracies associated with topographic roughness.''

From the definition of the bolus transport velocities [Eq. (1)] one might expect somewhat noisy fields in regions of steeply sloping isopycnals, as $\mathbf{u}^*$ and w^* depend on second derivatives of the potential density field. Since these transport velocities are added to the usual mean-flow advection velocities, the model's tracer advection scheme must be robust enough to be able to handle these potentially noisy fields. We shall show that the implementation of this parameterization using the traditional centered-difference advection scheme employed in the GFDL OGCM leads to non-monotonic tracer fields that violate the second law of thermodynamics, even in flat-bottom OGCM simulations. As we shall illustrate, artificial sources of unphysical water masses, which degrade the solution over large regions, may occur through numerical problems. The use of more sophisticated numerical advection algorithms designed to preserve monotonicity removes these problems as does the implementation of background horizontal diffusivity (see also Hirst and McDougall 1996). While others have noted the dispersion errors associated with standard centered finite difference advection schemes and have described new numerical methods in order to largely suppress them (e.g., Farrow and Stevens 1995; Hecht et al. 1995), there is still a need for further analysis in models that use isopycnal or Gent and McWilliams (1990) mixing schemes.

The purpose of this note is therefore to elucidate the cause of the numerical problems outlined above in a systematic manner and to provide practical solutions to overcome these deficiencies. In addition, we are able to explore the extent to which the benefits gained by the implementation of the Gent and McWilliams (1990) mixing scheme are linked to the use of isopycnal mixing (with no background diffusion) alone or to the addition of the bolus transport velocities. The outline of the rest of this note is as follows: In the next section we describe the model that is used and detail the set of experiments that we perform. In section 3 we discuss the results of the numerical experiments, illustrating how the numerical problems arise and demonstrating how they can be overcome. We summarize our results in section 4.

2. Description of the model

The model used in our experiments is Version 2 of the GFDL Modular Ocean Model (Pacanowski 1995). This version of the GFDL OGCM now includes options for a free surface formulation, complete convection, the Gent and McWilliams (1990) parameterization (discussed above), and the flux-corrected transport advection scheme of Gerdes et al. (1991). In our experiments we employ the rigid-lid approximation and the complete convection scheme of Rahmstorf (1993).

The model domain consists of a flat-bottomed box 4000 m deep extending from 1°S to 61°N, 1°W to 51°E. The horizontal resolution is 2° by 2° with 80 equally spaced (50 m) levels in the vertical. Uniform and relatively high resolution grid spacing was chosen everywhere to ensure that there was no numerical diffusion associated with the standard centered difference advection scheme (Yin and Fung 1991) and to reduce potential grid Peclet violations (Weaver and Sarachik 1990). In an attempt to minimize the complexity of the model we chose a linear equation of state depending only on temperature with no salinity dependence, as it was not carried as a prognostic variable (i.e., $\rho = 1 - 2 \times 10^{-4} T$). A Dirichlet (specified) surface boundary condition on temperature (T), ranging linearly from 25°C at the equator to 0°C at 60° N, was used. In addition, no wind stress forcing was applied, no-slip ($u=v=0$) boundary conditions were used at all lateral boundaries and the terms involving momentum advection removed. This latter approximation is valid since the Rossby number in coarse-resolution ocean models is $\ll 1$. Since there is no bottom friction, topography, wind forcing, nor momentum advection in our model, there is no way to excite a barotropic flow. Hence the barotropic mode is initialized to be zero everywhere and the code used to prognostically calculate it removed.

Split time steps were used (Bryan 1984), with the tracer and velocity time steps at all depths being 1 day and 1 hour, respectively. The integrations were carried out for 2000 years by which time near-equilibrium conditions were met.

Four primary experiments were carried out in this analysis. In experiment HOR we used the standard leapfrog horizontal advection scheme (centered differencing) together with a forward Euler treatment of horizontal/vertical diffusion. Experiment GM is the same as HOR except that we use the Redi–Cox isopycnal rotation of the tracer diffusion tensor and the isopycnal thickness diffusion parameterization to modify tracer advection (Gent and McWilliams 1990; Danabasoglu et al. 1994; Danabasoglu and McWilliams 1995). In experiment GMFCT we replace the leapfrog tracer advection scheme in GM by the flux-corrected transport advection scheme of Gerdes et al. (1991). Finally, ex-

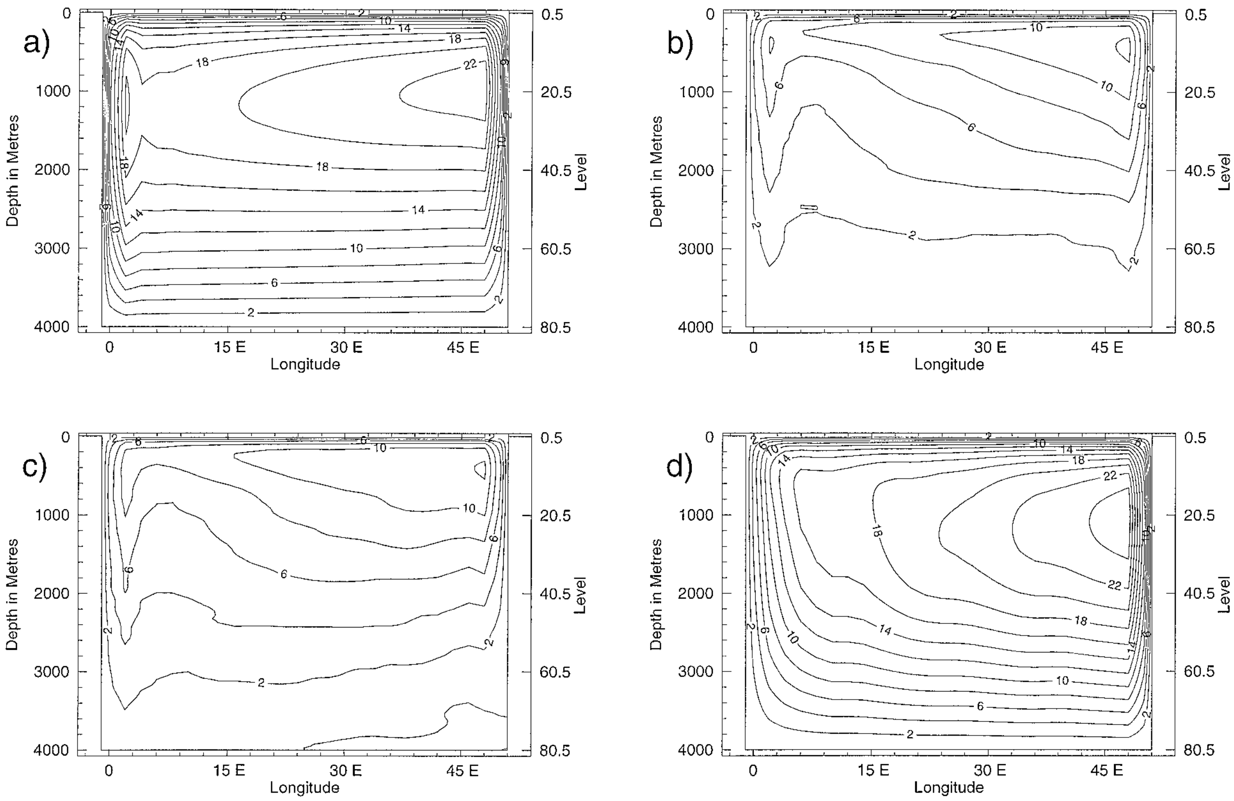


FIG. 1. Zonal overturning streamfunction from experiments (a) HOR, (b) GM, (c) GMFCT, and (d) ISOFCT. Positive values indicate clockwise flow. The contour interval is 2 Sv (1 Sv $\equiv 10^6$ m^3 s^{-1}).

periment ISOFCT is the same as GMFCT but without the addition of the bolus transport velocities. For all experiments the horizontal and vertical viscosity and the vertical diffusivity were set to 5×10^4 m^2 s^{-1}, 5×10^{-3} m^2 s^{-1}, and 1×10^{-4} m^2 s^{-1}, respectively. In HOR a horizontal diffusivity of 1×10^3 m^2 s^{-1} was used, while for all other experiments this horizontal diffusivity was zero and the isopycnal diffusivity was set to 1×10^3 m^2 s^{-1}. In the experiments involving the Gent and McWilliams (1990) parameterization, the isopycnal thickness diffusivity [k_T in Eq. (1)] was taken to be the same as the isopycnal diffusivity (1×10^3 m^2 s^{-1}).

3. Results

The horizontal mixing experiment (HOR) was carried out as the control run against which our other experiments are compared. The equilibrium zonal and meridional overturning streamfunctions are shown for HOR in Figs. 1a and 2a and for GM in Figs. 1b and 2b, respectively. Since our main interest is the equilibrium temperature fields from the different experiments, the streamfunction plots for GM and GMFCT (Figs. 1b,c and 2b,c) are calculated from the sum of the mean and bolus transport velocities as this is what gives rise to heat advection.

The GM experiment has a general reduction in the strength of both zonal and meridional overturning cells compared to HOR. The strong recirculation in the HOR case (Fig. 2a) is a consequence of unrealistically large diapycnal mixing, in the western boundary where isopycnals are steeply sloping, which requires intense western boundary current upwelling to remove this heat (Fig. 1a). This ''Veronis effect'' (Veronis 1975) no longer exists in the GM experiment (Figs. 1b, 2b) as already noted by Danabasoglu et al. (1994) and Danabasoglu and McWilliams (1995). Figures 3a and 3b show sections of temperature for GM at 3975 m and 26°E respectively, revealing large areas where the temperature is slightly below zero. These temperatures are unphysical and clearly violate the second law of thermodynamics since the lowest temperature in the initial condition and in the forcing is zero. In contrast, no negative temperatures are found in HOR. Thus, while the implementation of the Gent and McWilliams (1990) parameterization has the desirable feature of eliminating the unphysical Veronis effect as well as accounting for a mechanism by which available potential energy can be removed from the stratification through baroclinic instability, it has its own set of numerical problems, even in our idealized flat-bottomed simulations.

Figure 4 shows the horizontal bolus transport velocities (**u***) in the northern portion of the domain at 1975 m. Strong bolus transport velocities appear in the region

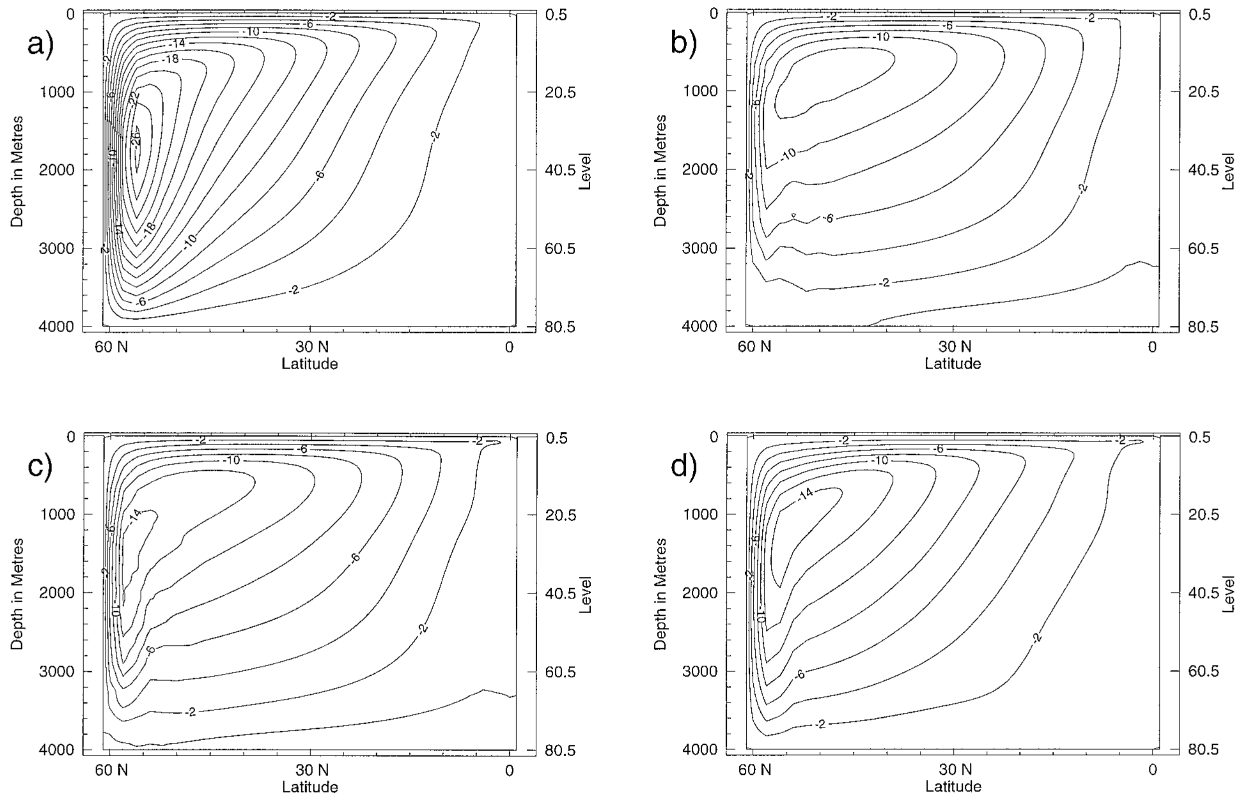
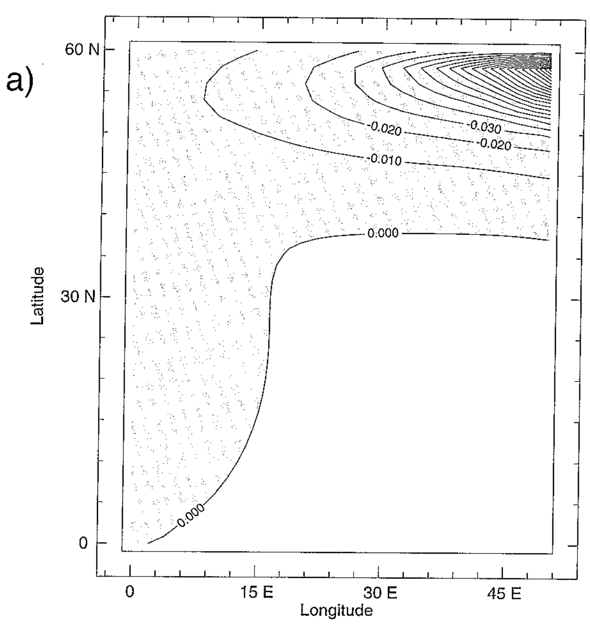


FIG. 2. Meridional overturning streamfunction from experiments (a) HOR, (b) GM, (c) GMFCT, and (d) ISOFCT. Shading indicates negative values and counterclockwise flow. The contour interval is 2 Sv.

where isopycnals are moving from being vertical to being more horizontal, as expected from Eq. (1). The existence of strong horizontal divergence and convergence within the field, implying large eddy-induced vertical velocities, is also apparent. These vertical velocities (w^*) are readily evident in Fig. 5, which shows the mean advective velocities (Fig. 5a), bolus transport velocities (Fig. 5b), and total tracer advective velocities (Fig. 5c)



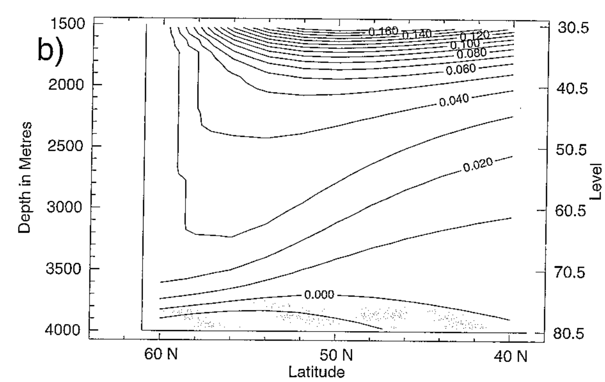


FIG. 3. Sections of temperature from experiment GM at (a) 3975 m and (b) at 26°E. The contour interval is 0.01°C.

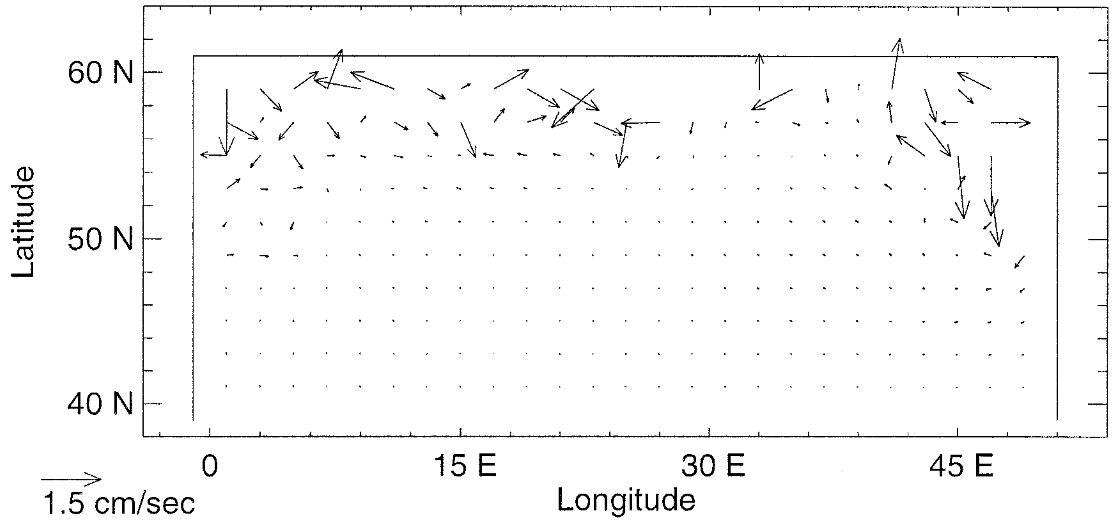


FIG. 4. Section of horizontal eddy-induced bolus transport velocities from experiment GM at 1975 m.

at 27°E. The noisy nature of those fields, which include **u*** and w*, would clearly be difficult for most advection schemes to handle accurately.

One way to remove the numerical problems associated with centered finite difference schemes is to change to a more robust advection scheme. The flux-corrected transport algorithm combines (in a nonlinear way) the more accurate (but not always monotonic) second-order centered difference scheme with the less accurate (but monotonic) first-order diffusive upstream scheme (Rood 1987). It uses as much of the centered difference solution as possible, while requiring that the advected field remain monotonic (Gerdes et al. 1991).

In experiment GMFCT, which is identical to GM except that we now use the flux-corrected transport scheme to handle advection, there are no values of temperature below zero. Figures 1c and 2c show the zonal and meridional overturning streamfunction for GMFCT, respectively. It is encouraging that the solution is not very different from the GM case. There is, of course, a cost to pay in using the flux-corrected transport advection scheme as the computation time for the entire model (not just the advection) increased by a factor of 2 to 3 over GM. Nevertheless, since we have successfully eliminated the artificial sources of spurious water masses, yet retained the positive aspects of the Gent and McWilliams (1990) parameterization mentioned earlier, we feel that this cost is well justified.

In order to find out where the flux-corrected transport algorithm is providing additional implicit mixing, we took the equilibrium of the GMFCT experiment and integrated it for one further time step (1 day), both with the flux-corrected transport and the regular centered difference advection scheme. The mean advection velocities in this flux-corrected transport experiment were similar to those shown in Fig. 5a for GM. Figure 6a shows the total vertical velocity field ($w + w$*) at the end of the time step for the flux-corrected transport experiment along a section at 26°E. Figure 6b gives the temperature difference (flux-corrected transport minus centered difference) at the end of this time step along this same section. The areas where the flux-corrected transport scheme is actively adding numerical mixing (i.e., resorting to more upstream differencing) corresponds well with the areas of large vertical velocity, as was to be expected.

At this stage it is still difficult to determine if it is the lack of background diffusivity, the transport velocity fields **u*** and w* themselves, or a combination of both that is causing the obviously unphysical temperatures to exist in the GM experiment. The area in which the flux-corrected transport algorithm is most active tends to correspond with the area of largest w (in the northeast corner of the model) where **u*** and w* are relatively weak. This also appears to be the largest source of negative temperature water (Fig. 3a). In addition, the flux-corrected transport algorithm tends to add numerical mixing in regions of strong w*. It is possible that we do not see violations of the second law of thermodynamics in HOR as the horizontal diffusion present (which would have a large diapycnal component in the northeast corner) may be enough to stabilize the advection. In order to further elucidate this issue, three short (10 year) integrations were continued from the equilibrium fields of GMFCT (which has no negative temperatures). These experiments were similar to GM, but in one case the background horizontal diffusivity and the isopycnal diffusivity were set to one-half the value of the original isopycnal diffusivity, and in another case the background horizontal diffusivity was set to be the same value as the isopycnal diffusivity. By 10 years the case with no background diffusion generated negative temperatures, while the cases with background diffusion did not.

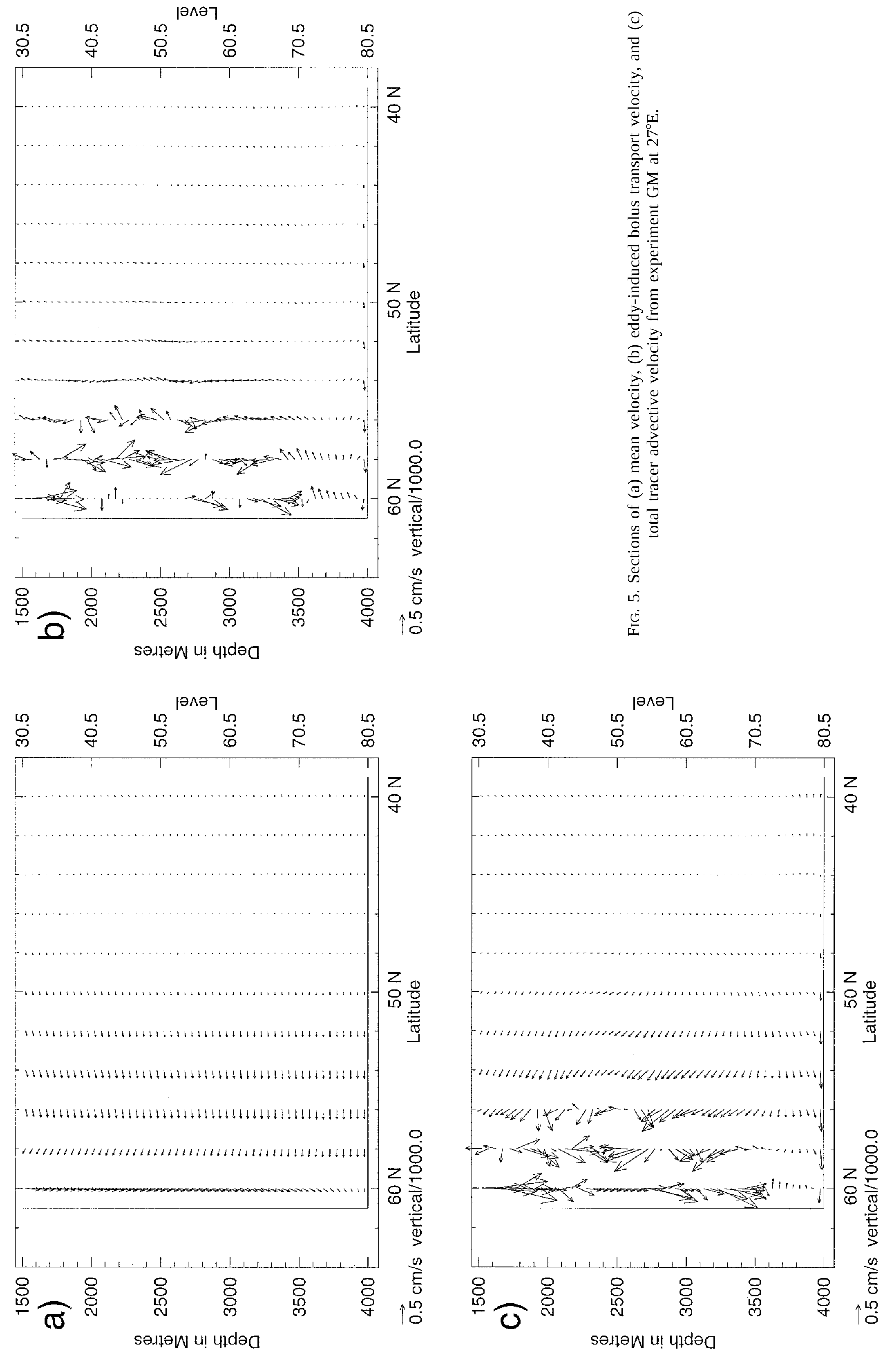


FIG. 5. Sections of (a) mean velocity, (b) eddy-induced bolus transport velocity, and (c) total tracer advective velocity from experiment GM at 27°E.

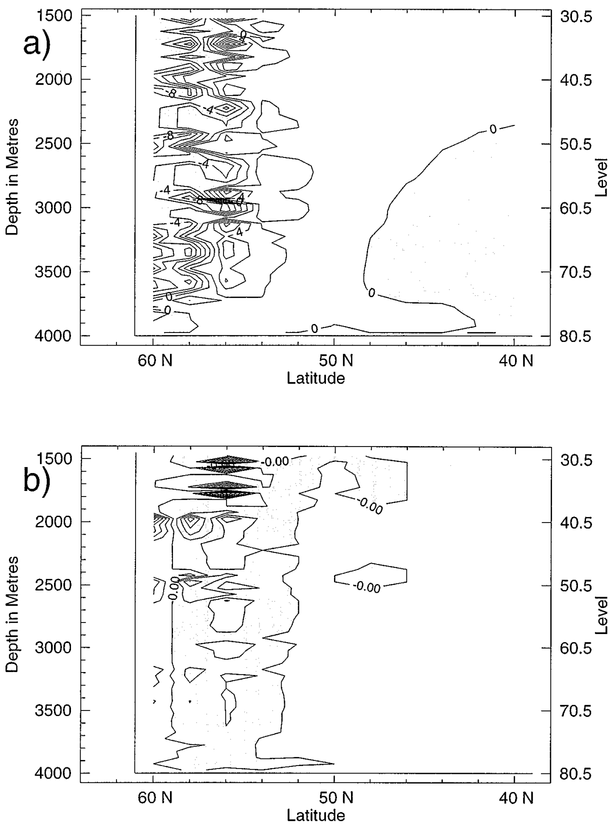


FIG. 6. Sections at 26°E of (a) total vertical tracer advective velocity and (b) difference in temperature when using flux-corrected transport instead of centered differences (flux-corrected transport–centered difference) for one time step, from GMFCT. Stippling indicates negative values. The contour intervals are 2×10^{-6} m s^{-1} in (a) and 1×10^{-4} °C in (b).

Figure 7 shows sections of temperature at 60°N for the three cases and the equilibrium state of GMFCT. The case with no background diffusion (Fig. 7a) clearly demonstrates that numerical problems are occurring at this latitude (either by the mean velocities **u*** and w* or a combination of both). The addition of background diffusion appears to be enough to remove the negative temperatures, with increasing background diffusion making the field smoother (Figs. 7b,c). The equilibrium state of GMFCT (Fig. 7d) looks somewhere between GM and those cases that specifically include background diffusion. Background horizontal diffusion smooths the field through two mechanisms: The first, and direct mechanism, is that increased diffusion tends to smooth out small perturbations in the temperature field. The second, and indirect mechanism, is that background diffusion can add a strong diapycnal flux of heat from low to high latitudes, thereby promoting convection from the top to the bottom of the ocean. This convection then mixes heat in the vertical, eliminating numerical perturbations. In the case with no background diffusion, small negative values can remain and grow, artificially stopping convection from continuing to the bottom. Thus, while the Gent and McWilliams (1990) scheme allows convection to occur in more isolated areas (Danabasoglu et al. 1994), this has the potential drawback of allowing numerical problems to develop, which may in fact be partially responsible for the existence of reduced regions of convection. HOR, on the other hand, has broad regions of convection that tend to occur in the same locations as strong downwelling, thus avoiding nonmonotonic solutions. As pointed out by Weaver and Sarachik (1990), it is the equatorial upwelling regions, where convection is not present, that tend to exhibit grid Peclet violations and subsequent numerical problems in HOR-type experiments.

In order to assess whether the bolus transport velocities alone may contribute to the generation of unphysical tracer values, we once more took the equilibrium of the GMFCT experiment and integrated it for one further time step (1 day), both with the flux-corrected transport and the regular centered difference advection scheme. Unlike in our earlier experiment where we used the total velocity field, we now used only the equilibrium values of **u*** and w* (from GMFCT) with the mean velocities set to zero. The results from these two experiments are very similar to those seen in Fig. 6. The largest single difference in temperature due to the use of the flux-corrected transport algorithm was 0.02°C. This large difference indicates that the flux-corrected transport algorithm had to add numerical mixing in order to preserve monotonicity that was otherwise not preserved by the centered difference advection scheme. It would therefore appear that the bolus transport velocities themselves can indeed contribute to unphysical advection when centered differencing is used.

Another possible cause of unphysical temperatures is negative diffusion associated with the rotation of the diffusion tensor [recall our regular grid does not allow numerical diffusion as discussed by Yin and Fung (1991)]. The numerical representation of isopycnal diffusion in the GFDL model does not guarantee that the effective diffusivity is always positive (R. Gerdes 1996, personal communication). As implemented, the rotation of the diffusion tensor implies that a tracer is first diffused vertically and then horizontally. Since the heat being diffused across a grid face does not depend solely on the temperatures of the grid boxes on either side of the face, there is no guarantee that the net diffusion is always positive. To see if negative diffusion is important in these integrations two additional short runs were carried out in which only isopycnal and vertical (with no background horizontal) diffusion acted on tracers (all advective and convective terms were set to zero). A 10-year run from the end state of GMFCT (which had no negative temperatures) showed that diffusion, by itself, did not generate negative values. A similar integration from the end state of GM (which had negative temperatures) indicated that diffusion was slowly reducing the maximum negative temperature. We thus conclude that it is indeed the numerical treatment of

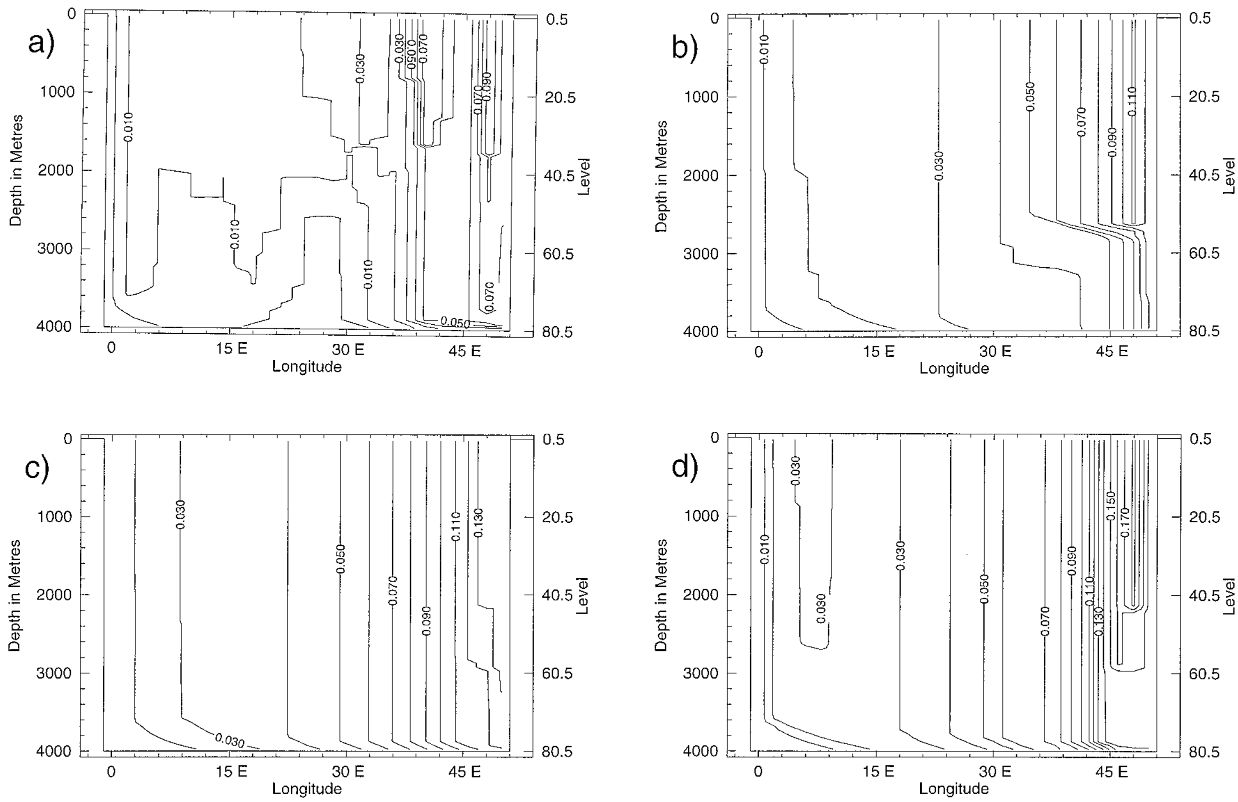


FIG. 7. Sections of temperature at 60°N for a case with (a) no background diffusion, (b) half background diffusion, (c) full background diffusion, and (d) for GMFCT. Stippling indicates negative values. The contour interval is 0.01°C.

advection that is responsible for the unphysical values of temperature in GM.

We are now in a position to ask the question as to what extent the differences between HOR and GM/GMFCT are due to the rotation of the diffusion tensor and to what extent they are due to the addition of the bolus transport velocities. Figures 1d and 2d show the zonal and meridional overturning streamfunction for ISOFCT, in which there is no background horizontal diffusion as it uses the flux-corrected transport algorithm to preserve monotonicity. The solution for the zonal streamfunction is much closer to HOR than GMFCT but the opposite is true for the meridional overturning. This suggests that the ''Veronis effect'' in the western boundary is partly eliminated by the rotation of the diffusion tensor (Fig. 2), while the reduction of the strong downwelling in the northeast corner is a consequence of the inclusion of isopycnal thickness diffusion (Fig. 1).

4. Conclusions

The rotation of the diffusion tensor and the implementation of Gent and McWilliams (1990) bolus transport velocities in ocean models that employ standard centered difference advection algorithms, while physically desirable, has been shown to have its own set of numerical problems. The generation of spurious water mass sources, which violate the second law of thermodynamics, can potentially degrade solutions over large areas. Two possible solutions to these problems are suggested: In the first, explicit background diffusion can be added to the model and, in the second, more sophisticated advection schemes can be employed. The idea of adding background diffusion is not very appealing since one has to know a priori how much diffusion will be required to eliminate the buildup of spurious water masses. In addition, a uniform background diffusion will also add diapycnal diffusion over large areas even when it is not needed, ironically creating other spurious water masses. The flux-corrected transport algorithm, on the other hand, only adds diffusion in those regions where it is required to preserve monotonicity of the tracer fields. This occurs predominantly along the northern boundary and specifically in the northeast corner of the model domain where vertical velocities are large. In our integrations, which include the flux-corrected transport algorithm, we found that we preserved the desirable features of the Gent and McWilliams (1990) parameterization yet managed to eliminate the spurious water mass sources. Of course, as with any more sophisticated numerical scheme, there is a price that must be paid. In our model and on our scalar IBM RS6000 computers, this price is a computational increase by a factor of 2–3 over the centered difference advection scheme. Combined with a computational increase of about a factor of 2 when the Gent and McWilliams (1990) scheme replaces the traditional horizontal mixing scheme, this leads to a total computational increase of about 4–6. We feel that such a price is justified by the substantial improvement in the numerical solution, especially since faster computers are becoming more readily available.

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