Int. Agrophysics, 2002, 16, 23-28

INTERNATIONAL
Agrophysics
www.ipan.lublin.pl/int-agrophysics

Pressure ratio of cereal grains determined in a uniaxial compression test**

J. Horabik and R. Rusinek*

Institute of Agrophysics, Polish Academy of Sciences, Doświadczalna 4, P.O. Box 201, 20-290 Lublin 27, Poland

Received November 16, 2001; accepted January 8, 2002

A b s t r a c t. The pressure ratio of cereal grains was determined in a uniaxial compression test. Experiments were performed according to Eurocode 1 recommendations. The tester was 210 mm in diameter and 100 mm high. The specimen was loaded to the reference vertical stress of 100 kPa using a universal loading frame at a constant displacement rate of 0.35 mm min^{-1}. Lateral to vertical pressure ratio was found dependent on procedure of the sample deposition. The pressure ratio of cereal grain generally decreased with an increase in moisture content. Experimental results were compared with theoretical consideration based on Janssen's method of pressure calculation in grain bins and with simplified approximation recommended by Eurocode 1. Significant differences between theoretical and experimental values were obtained.

K e y w o r d s: pressure ratio, cereal grain, storage structures

INTRODUCTION

Cereal grain and oilseed are a major food source. Storage, handling and processing of grain constitute a considerable part of operations in the food industry. A better understanding of the mechanical behaviour of granular materials is of fundamental importance in the design and operation of facilities for the storage and processing of granular materials. The lateral to vertical pressure ratio, the bulk density and the friction coefficient are the three most important mechanical parameters commonly used to calculate the loads exerted by grain on storage structures.

The recommended value of the lateral to vertical pressure ratio varies somewhat but the use of a value of approximately 0.4 is common. The pressure ratio depends on the type of grain, moisture content, bulk density and bedding structure of grain formed during the filling process. The angle of internal friction and the angle of friction on the wall material increase with the increase in the moisture content of the grain while the bulk density decreases or increases depending on the pressure level (£ukaszuk *et al*., 2001; Molenda and Horabik, 1994; Molenda *et al*., 1995; Thompson and Ross, 1983). The angle of friction at the interface of the grain and the wall material depends strongly on the roughness of the wall surface. All three properties of the grain's bulk influence the pressure ratio. Silo design standards recommend determining, experimentally, the parameters of stored materials for each specific silo design (Eurocode 1, 1996; Dyduch *et al*., 2000).

PRESSURE RATIO ACCORDING TO THE JANSSEN APPROACH

Almost all design codes use a Janssen-type pressure distribution to predict silo pressures (Wilms, 1991). The well known Janssen (1895) formula uses the equilibrium of a horizontal slice of the granular material to estimate pressures in deep silos. The fundamental assumption of Janssen's method involves a relationship between the average stresses acting on the finite dimension of a slice, and stresses that act on the walls of a silo. Janssen assumed that the ratio between the average vertical stress σ_z and the stress normal to the wall, σ_x is a constant for a given bulk material stored in a silo:

$$\frac{\sigma_x}{\sigma_z} = k = \text{const} \tag{1}$$

and k is to be determined from measurements. Other Janssen assumptions are: fully mobilized friction at the interface of the bulk material and the walls of the silo, and constant bulk density (Drescher, 1991).

Since the work of Janssen, several attempts aimed at an expression for k, based on postulating a mechanical model for bulk solids, have been proposed (Cowin, 1977;

*Corresponding author's e-mail: jhorabik@demeter.ipan.lublin.pl
**This work is supported by the State Committee for Scientific Research, Poland under the Grant No. 5 P06F 021 17.

Drescher, 1991; Moysey, 1979). The majority of estimations are based on the assumption that the bulk material stored in or discharged from a silo is at a limiting state of stress. Another important assumption concerning location of a region inside the slice of material where the yielding conditions occur involves relations between local stresses and the stresses averaged over the area or the perimeter of a slice (Drescher, 1991).

Two stress cases are commonly considered for a deep silo: active for filling and storage mode and passive for discharge mode. In the active case the vertical stress is higher than the lateral stress while in the passive case the lateral stress is higher than the vertical one.

Yielding at the silo centre

Considering yielding at the silo centre, the stress ratio k can be easily obtained from Mohr's circle construction (Kezdi, 1974; Moysey, 1979) for the active case (Fig. 1a):

$$k = \frac{1-\sin\varphi}{1+\sin\varphi}, \tag{2}$$

and for the passive case (Fig. 1b):

$$k = \frac{1+\sin\varphi}{1-\sin\varphi}, \tag{3}$$

where φ is the angle of internal friction.

A common assumption is that the lateral stress is constant along the slice resulting in the location of Mohr's circle for stresses at the wall outside the yield locus (Drescher, 1991). Probably this assumption does not adequately represent pressure distribution of the granular material in a silo.

Yielding at the silo wall

Assuming that yielding occurs at the silo wall, then stress ratio k can be determined from Mohr's circle construction as the function of the angle of internal friction and the angle of wall friction φ_w:

- for the active case (Fig. 1c):

$$k = \frac{1-\sin\varphi\cos\alpha}{1+\sin\varphi\cos\alpha}, \tag{4}$$

where:

$$\alpha = arc\sin\frac{\sin\varphi_w}{\sin\varphi} - \varphi_w, \tag{5}$$

- and for the passive case (Fig. 1d):

$$k = \frac{1+\sin\varphi\cos\alpha}{1-\sin\varphi\cos\alpha}, \tag{6}$$

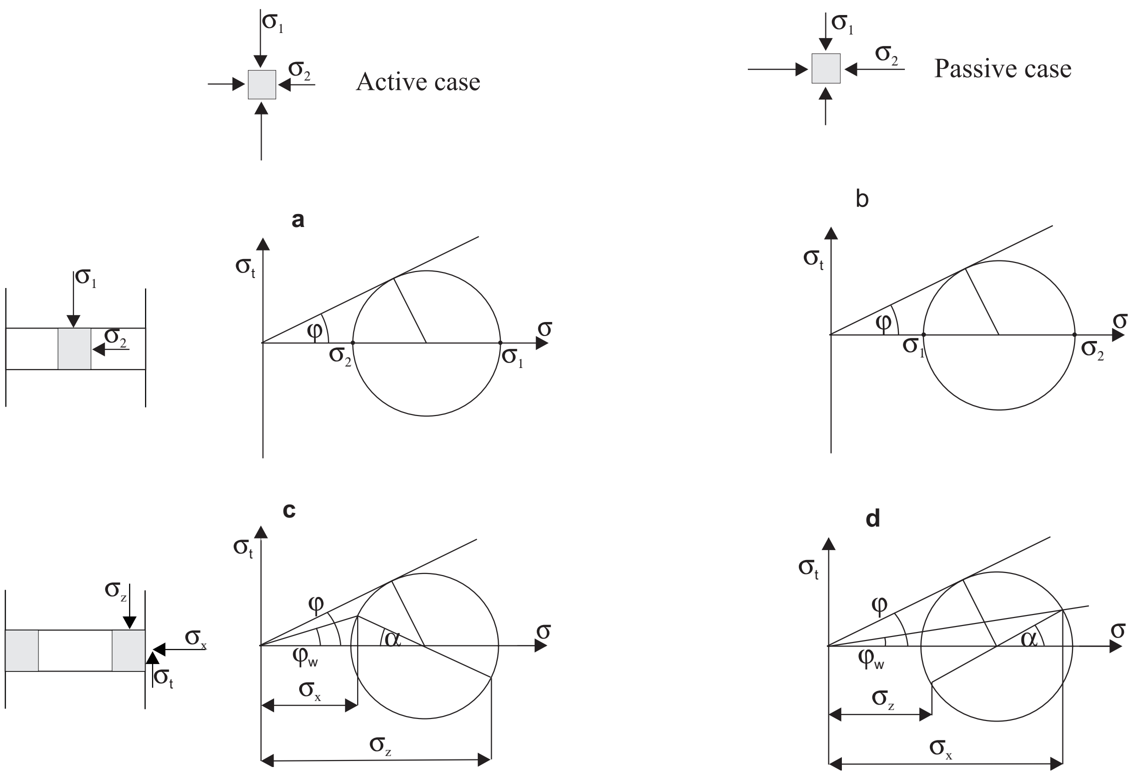

Fig. 1. Mohr's stress circles for: active case, yielding at the centre (a); passive case, yielding at the centre (b); active case, yielding at the wall (c); passive case, yielding at the wall (d).

where:

$$\alpha = arc\sin\frac{\sin\varphi_w}{\sin\varphi}+\varphi_w. \qquad (7)$$

A plot of the pressure ratio for yielding at the silo wall and the active and the passive stress cases for a typical range of values of the angle of internal friction and the angle of wall friction is shown in Fig. 2. The pressure ratio observed in practice varies in a considerably smaller range (Kwade *et al*., 1994b). A plot of the stress ratio calculated according to the following formula recommended by Eurocode 1 (1996):

$$k_\varphi = 1.1\left(1-\sin\varphi\right), \qquad (8)$$

is shown in Fig. 2. The angle of internal friction φ used in this formula should to be determined experimentally in the direct shear test or in the triaxial compression test. The plot of the pressure ratio obtained from Eq. (8) is located in the upper limit of theoretical values obtained for the active stress case and yielding at the wall (i.e., for the wall friction angle φ_w close to the internal friction angle φ).

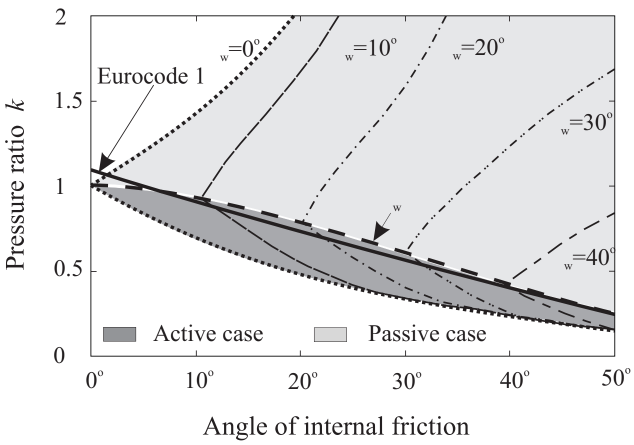

Fig. 2. Pressure ratio as the function of the angle of internal friction and the angle of wall friction for yielding at the wall (Horabik and Rusinek, 2000); — the pressure ratio according to Eurocode 1 (1996).

EQUIPMENT, PROCEDURE AND MATERIAL

The most popular method of experimental determination of the lateral to vertical pressure ratio is the uniaxial compression test (Kwade *et al*., 1994a; Molenda and Horabik, 1998). An experimental set for uniaxial compression (Fig. 3) was built according to the general guideline of Eurocode 1 (1996) standard. The wall of the chamber consisted of two semicircular halves cut along the axis. The two semicircular halves were connected with four load cells installed in pairs on the two connection lines, restoring the cylindrical shape of the chamber wall. One half of the wall was settled directly on the base and the other was allowed to move horizontally to follow the deflection of load cells. The

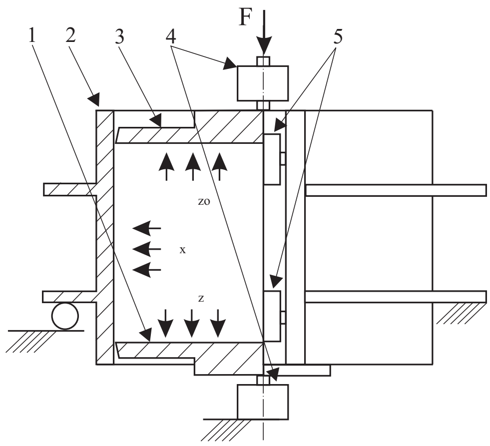

Fig. 3. Uniaxial compression tester: 1 – bottom plate, 2 – wall of the tester, 3 – top plate, 4 – load cells (vertical pressure), 5 – load cells (lateral pressure).

bottom and top plates of the chamber transmitted the vertical load through the load cells. The surface of the wall was smooth whereas the surfaces of the top and bottom plates were rough. The experimental set allowed the mean lateral pressure σ_x, the mean vertical pressure on the bottom σ_z, and the mean vertical pressure on the top plate σ_{zo}, to be determined (Horabik and Molenda, 2000).

The sample was poured into the test chamber through a centrally located spout, without vibration or other compacting actions. The specimen was loaded to the reference vertical stress of 100 kPa using a universal loading frame at a constant displacement rate of 0.35 mm min^{-1}. The top plate was rotated backwards and forwards three times through an angle of 10 degrees to consolidate the sample. Next the sample was reloaded to the reference vertical stress and slope of the lateral to the vertical pressure increase was determined. The pressure ratio k_s appropriate for filling and storing was determined as (Eurocode 1, 1996):

$$k_s=1.1\ k_{so}, \qquad (9)$$

where:

$$k_{so}=\Delta\sigma_x/\Delta\sigma_{zm} \qquad (10)$$

at the reference vertical stress σ_{zm} = 100 kPa, $\sigma_{zm} = (\sigma_z+\sigma_{zo})/2$ (Fig. 4).

Preliminary experiments were performed for samples of seeds of different size and shape and typical storage moisture content. An extended range of moisture content was applied for tests performed for cereal grain (10-20% (w.b.) for barley – of the variety Rudnik; corn – of the variety Mieszko; wheat – of the variety Banti and oats – of the variety Borowiecki) and 6-15% (w.b.) for rape seeds – of the

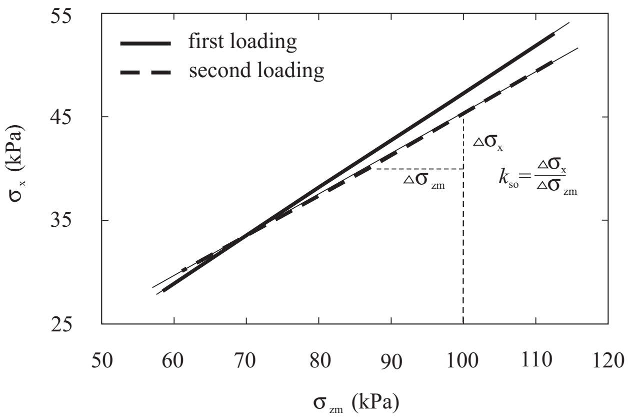

Fig. 4. Typical plot of the lateral pressure σ_x versus the mean vertical pressure σ_{zm} during first and second loading of the sample of seeds (rape seeds of moisture content of 6% w.b.).

variety Licosmos. Each variant of the experiment was performed in three replications. The angle of internal friction of the tested seeds was measured using a Jenike shear tester of 210 mm in diameter according to procedure recommended by Eurocode 1 (1996).

RESULTS

A typical plot of lateral pressure σ_x versus mean vertical pressure σ_{zm} during the first and second loading of the sample of seeds is shown in Fig. 4. A course of the relationship $\sigma_x(\sigma_{zm})$ during reloading of the sample was found to be linear for lower moisture contents and non-linear for higher moisture contents and the slope of the tangent to the $\sigma_x(\sigma_{zm})$ relationship was generally lower than during the first loading. The pressure ratio was found to be influenced by the procedure of the sample preparation. The most reproducible structure of grain deposition was obtained for the central filling and almost spherical grains. The shape, size and roughness of the seeds were found to influence the pressure ratio (Table 1). The lowest value of the pressure ratio was found for soybean (0.37) and the highest for lentil seeds (0.74). The reason is not only the difference in the angle of internal friction but also the different shape of seeds. The spherical shape of soybeans as compared to the lenticular shape of lentil seeds results in a different distribution of contact points in the settling of material. The almost vertical orientation of the shortest axis of lentil seeds obtained during filling the tester resulted in the easier transmission of the vertical load into the lateral direction than was the case for spherical (soybean) or corrugated seeds (pea).

With the moisture content increase, the friction force and the cohesion between grains increase. As a result, a smaller part of the vertical loading is transmitted into the lateral direction. Consequently, the lateral to vertical pressure ratio should decrease with an increase in moisture content. Tests performed for rape seed, corn, oat and wheat grains confirm this relationship (Table 2). An almost linear decrease in pressure ratio with an increase in moisture content was obtained for rape seed and corn. Another course of changes was obtained for barley: the pressure ratio was almost constant in the range of moisture content, up to 17.5% and then decreased. This indicates that other factors could probably also participate in the vertical to horizontal stress transmission.

The values of pressure ratio for tested seeds for all moisture content levels were found to be significantly lower than the range of 0.50-0.53 recommended by the Polish Standard (PN-89/B-03262, 1989) for filling and storing and lower than values obtained from Eq. (8) according to Eurocode 1. Experimental values of the pressure ratio are located near the lower limit of theoretical values obtained for yielding at the wall in the active stress case (i.e., for the wall friction angle φ_w approaching zero, Fig. 5). The lower limit of the theoretical values of the pressure ratio (dashed line in Fig. 5 indicated as $\varphi_w = 0^o$) comprise the case of yielding at the centre of a silo in the active case when directions of the principal stresses are vertical and horizontal. This confirms the opinion that during the uniaxial compression test, horizontal and vertical stresses are

Table 1. Measured (k_s) and calculated (k_φ) values of the pressure ratio of seeds and corresponding values the angle of internal friction φ (mean ± st. dev.)

Seeds	k_s	k_φ	φ (deg)
Amaranthus	0.62 ± 0.02	0.70 ± 0.02	21.3 ± 0.8
Pea	0.53 ± 0.01	0.59 ± 0.01	27.3 ± 0.6
White mustard	0.43 ± 0.01	0.64 ± 0.01	24.7 ± 0.4
Buckwheat	0.59 ± 0.02	0.68 ± 0.02	22.0 ± 0.8
Soybean	0.37 ± 0.02	0.55 ± 0.01	30.1 ± 0.9
Lentil	0.74 ± 0.01	0.80 ± 0.02	15.5 ± 0.6

T a b l e 2. Measured (k_s) and calculated (k_φ) values of the pressure ratio of seeds and corresponding values the angle of internal friction φ for different levels of the moisture content (mean ± st. dev.)

Grain	Moisture content (w.b.) (%)	k_s	k_φ	φ (deg)
Barley	10	0.45 ± 0.02	0.59 ± 0.01	27.8 ± 0.4
	12.5	0.47 ± 0.03	0.57 ± 0.01	28.5 ± 0.5
	15	0.43 ± 0.02	0.53 ± 0.01	31.2 ± 0.3
	17.5	0.45 ± 0.03	0.54 ± 0.02	30.6 ± 1.0
	20	0.39 ± 0.03	0.51 ± 0.01	33.2 ± 0.5
Corn	10	0.48 ± 0.04	0.60 ± 0.01	26.7 ± 0.6
	12.5	0.40 ± 0.03	0.52 ± 0.01	31.7 ± 0.5
	15	0.36 ± 0.05	0.51 ± 0.02	32.0 ± 1.4
	17.5	0.34 ± 0.03	0.50 ± 0.02	33.4 ± 0.8
	20	0.30 ± 0.05	0.49 ± 0.03	33.6 ± 1.5
Oat	10	0.49 ± 0.03	0.68 ± 0.02	22.1 ± 1.1
	12.5	0.44 ± 0.04	0.68 ± 0.02	22.4 ± 0.9
	15	0.45 ± 0.03	0.65 ± 0.01	24.0 ± 0.5
	17.5	0.40 ± 0.03	0.65 ± 0.02	23.9 ± 1.0
	20	0.41 ± 0.06	0.61 ± 0.03	26.4 ± 1.7
Wheat	10	0.44 ± 0.02	0.62 ± 0.01	25.7 ± 0.3
	12.5	0.38 ± 0.01	0.61 ± 0.01	26.2 ± 0.4
	15	0.34 ± 0.02	0.60 ± 0.01	27.0 ± 0.5
	17.5	0.31 ± 0.02	0.50 ± 0.02	33.0 ± 1.0
	20	0.35 ± 0.01	0.46 ± 0.01	35.5 ± 0.5
Rape seeds	6	0.46 ± 0.02	0.64 ± 0.02	24.7 ± 0.5
	9	0.28 ± 0.04	0.54 ± 0.01	30.6 ± 0.4
	12	0.27 ± 0.02	0.52 ± 0.01	31.7 ± 0.7
	15	0.24 ± 0.02	0.47 ± 0.01	34.8 ± 0.7

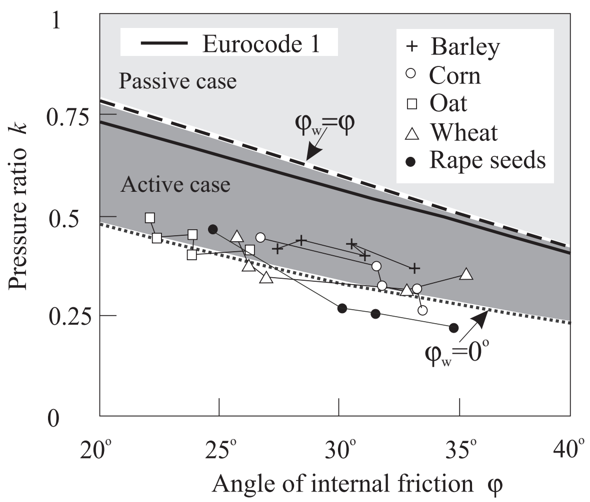

Fig. 5. Comparison of the experimental and theoretical values of the pressure ratio.

approximately principal stresses in the test sample whereas they may not be in the silo (Eurocode 1, 1996).

CONCLUSIONS

1. The lateral to vertical pressure ratio determined in the uniaxial compression test depends on the procedure of sample deposition, shape, size and the angle of internal friction of seeds. The most reproducible structure of seed deposition was obtained for the central filling. The lowest value of the pressure ratio was found for soybean (0.37) and the highest for lentil seeds (0.74). The reason is not only the difference in the angle of internal friction but also different shape of the seeds.

2. The pressure ratio was found to decrease with the increase in the moisture content of seeds. The strongest decrease was obtained for rape seeds.

3. Experimental values of the pressure ratio are similar to theoretical values obtained for yielding at the centre of a silo in the active case.

REFERENCES

Cowin S.C., 1977. The theory of static loads in bins. J. Applied Mechanics, 44(3), 409-412.

Drescher A., 1991. Analytical Methods in Bin-load Analysis. Elsevier, Amsterdam-Oxford-New York-Tokyo.

Dyduch M., Kamiński M., Łapko A., and Runkiewicz L., 2000. Amendment of Polish Standard PN-89/B-03262 'Concrete bins for storing granular materials and silags. Design rules' (in Polish). XI Conf. 'Reinforced and prestressed concrete containers for liquids and granular materials', 65-69, Świeradów Zdrój.

Eurocode 1, **1996.** Basis of design and actions on structures. Part 4. Actions in silos and tanks. DD ENV 1991-4.

Horabik J. and Molenda M., 2000. Device for determination of pressure ratio of granular materials (in Polish). Patent Application No. P-340014, Bulletin of Patent Office No. 22(700), A1(21) 340017.

Horabik J. and Rusinek R., 2000. Determination of the pressure ratio of plant granular solids. Part I. Theoretical considerations. Part II. Experimental investigations (in Polish). Acta Agrophysica, 37, 61-81.

Janssen H.A., 1895. Versuche über Getreidedruck in Silozellen. Verein Deutscher Ingenieure, Zetschrift (Düsseldorf), 39, 1045-1049.

Kezdi A., 1974. Handbook of soil mechanics. Akademia Kiado, Budapest.

Kwade A., Schulze D., and Schwedes J., 1994a. Determination of the stress ratio in uniaxial compression tests-Part 1. Powder Handling and Processing, 6(1), 61-65.

Kwade A., Schulze D., and Schwedes J., 1994b. Determination of the stress ratio in uniaxial compression tests-Part 2. Powder Handling and Processing, 6(2), 199-203.

Łukaszuk J., Stasiak M., Rusinek R., and Horabik J., 2001. Effect of moisture content on the angle of internal friction of cereal grain (in Polish). Acta Agrophysica, 46, 105-113.

Molenda M. and Horabik J., 1994. Normal and tangential stresses in the frictional contact area between wheat grain and flat surface. Int. Agrophysics, 8, 277-282.

Molenda M. and Horabik J., 1998. Lateral to vertical pressure ratio in a model grain silo. Proc. Seminar 'Operations on granular materials', Lublin, 31-39.

Molenda M., Horabik J., Grochowicz M., and Szot B., 1995. Friction of wheat grain (in Polish). Acta Agrophysica, 4.

Moysey E.B., 1979. Active and passive pressures in deep grain bin. Transactions of the ASAE, 22(6), 1409-1413.

PN-89/B-03262, **1989.** Polish Standard. Concrete bins for storing granular materials and silage. Design rules (in Polish).

Thompson S.A. and Ross I.J., 1983. Compressibility and frictional coefficient of wheat. Transactions of the ASAE, 26(4), 1171-1176.

Wilms H., 1991. Criteria for evaluation of silo design codes. Bulk Solids Handling, 11(1), 55-61.