Explosive Instability of Vorticity Waves

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ABSTRACT

The weakly nonlinear dynamics of ''vorticity waves'' (VW), specific wavelike motions occurring nearshore in the presence of an alongshore shear current is examined. By means of a standard asymptotic technique starting with the shallow-water equations, the authors derive the equations governing field evolution due to resonant interactions for the arbitrary current and bottom profiles and show that the VW interactions occur in the lowest order. Among them there are always explosive interactions; that is, the resonant triplets where all interacting waves growing synchronously tend to infinity in a finite time. The explosive instability is studied as a potential mechanism for VW generation, their main implications being the following: 1) The range of explosively excited scales appears to be much wider than the domain of linear instability, with the low-frequency cutoff absent; 2) the explosive instability occurs even when all linear perturbations are damped due to bottom friction, provided the initial amplitudes of disturbances exceed a certain threshold; and 3) the weakly nonlinear evolution most likely results in the emergence of strongly nonlinear motions. The dependence of the explosive processes on the background parameters is analyzed for the simplest model of alongshore current and topography.

1. Introduction

Vorticity or shear waves (VW) are specific wavelike motions with frequencies in the range of 10^{-3} – 10^{-2} Hz, that is, well below the traditional low-frequency limit of the infragravity band,[1] and wavelengths of the order 10^2 m occurring nearshore in the presence of a strong alongshore current. These motions, manifesting themselves as intense variations of the longshore component of the current velocity, were first observed by Oltman-Shay et al. (1989) in the 1986 SUPERDUCK field study. Their frequency dependence on the wavenumber was found to be almost linear, their celerity closely related to the velocity of the mean current.

The milestone work in the theoretical study of these motions was done by Bowen and Holman (1989), who identified the principal mechanism of VW to be potential vorticity conservation, with the background vorticity field supplied both by the shear structure of the longshore current and the bottom slope. They studied a simplified model with piecewise constant background potential vorticity field and assumed VW to be unstable modes of the Rayleigh-type boundary value problem governing the dynamics of linear mean current perturbations. Their analysis was extended by Dodd and Thornton (1990) who explored a somewhat more realistic model of the mean current and the bottom cross-shore structure. The numerical analysis performed by Dodd et al. (1992) by using mean flow and bathimetry data from the two real beaches at Duck, North Carolina (SUPERDUCK study) and Santa Barbara, California [Nearshore Sediment Transport Studies (NSTS)] stressed the importance of taking into account the influence of bottom friction on VW dynamics, which results in a decrease in the span of unstable wavenumbers and of growth rates. The comparison of numerical simulation results and field data exhibits good quantitative agreement at least for SUPERDUCK. The numerical study was further continued by Putrevu and Svendsen (1992) and by Falques and Iranzo (1994) who ran a similar model without the ''rigid-lid approximation'' and took into account eddy viscosity. The effect of varying bottom shear stress on the stability of wave-driven shear current was investigated by Dodd (1994) who found that the current does not become unstable unless its offshore shear exceeds some critical value. Thus, all these theoretical studies were aimed at establishing more accurately the dependence of linear instability characteristics on various parameters specifying the alongshore current and topography of the coastal zone.

The experimental data, however, provide a strong motivation for developing a nonlinear theory as the observed perturbations of horizontal velocity were quite

[1] That is why these motions are often called far infragravity waves (Mei and Liu 1993).

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large (up to 30 cm s^{-1}) and often comparable with the magnitude of the mean current, thus ensuring the nonlinear character of VW dynamics during most of the field studies. The only attempt to investigate the nonlinear stage of VW evolution was made by Dodd and Thornton (1992), who considered self-interaction of the fastest growing mode in the so-called near-critical condition (when the current shear just slightly exceeds the threshold due to the bottom friction and thus allows the unstable modes to develop). Their study showed that the alongshore current is supercritical; therefore nonlinear saturation of shear instability is expected and disturbances are likely to evolve to a finite, steady amplitude. The resulting amplitude was found to be comparatively small (at least ten times smaller than the mean current). The principal limitation of such an approach is that it allows one to address only narrowband wave trains centered near the fastest growing mode, which might be justified for the comparatively narrow class of marginally stable situations. Meanwhile, a robust feature of the motions under consideration, which has not received an explanation yet, even a qualitative one, is that the field data indicate no low-frequency cutoff in VW spectra, while the linear instability analysis (see Dodd 1994) predicts global low-frequency cutoff below which the instability does not develop. The aim of the present work is to develop a weakly nonlinear theory of VW interactions with the emphasis on the explosive interactions, which might contribute essentially to VW generation and dynamics.

The linear instability, considered up to now to be the only mechanism of VW generation, can be most naturally interpreted in terms of the linear interaction of two eigenmodes of different energy. The concept of *negative energy waves*, first developed in plasma physics in the context of electronic beam instabilities in the early 1950s and later introduced into hydrodynamics by Landahl (1962) and Benjamin (1963) in their studies of shear flows over flexible boundaries, plays an essential role in our work. A formal mathematical definition of the sign of energy for the VW will be given below. Basically, a wave has negative energy if exciting it lowers the total energy of the system: the amplitude of a negative energy wave grows when it loses energy in contrast to the usual positive energy wave (see, e.g., Cairns 1979; Craik 1985; Ostrovsky et al. 1986). The essential point we would like to stress is that *the sign of the wave energy is specified entirely by the linearized problem*. If we consider from this viewpoint any of the known dispersion relations of VW, we may easily distinguish the domains of negative and positive energy, as well as those of unspecified energy sign. A sketch of a typical VW dispersion curve with marked domains of different energy sign is depicted in Fig. 1a. [For the detailed derivation, for a particular example, see section 5 where the simplest model of Bowen and Holman (1989) is analyzed.] Within the linear theory in the absence of dissipation, there is no difference in behavior of waves of positive and negative energy. However, this difference becomes principal when one considers nonlinear interactions among these waves. It is easy to show that the VW dispersion relation always permits nonlinear resonant interactions in the lowest order, that is, three-wave resonant interactions among the wave packets of frequencies and wavenumbers ω_i, k_i satisfying the well-known ''resonance conditions'' (e.g., see Craik 1985)

$$k_n = k_l + k_j$$
$$\omega_n = \omega_l + \omega_j. \tag{1}$$

We can see that among the permitted triads are those comprising waves of different signs of energy (Fig. 1b) and those involving waves of the unspecified energy sign (Fig. 1c).[2] It is well known that triad interaction between waves of different energy sign may lead to the specific nonlinear instability of VW, namely, the so-called *explosive* instability (Craik 1985). Some general properties of explosive instabilities are well known (e.g., Weiland and Wilhelmsson 1977; Craik 1985). Such nonlinear instability is, in a sense, a more powerful mechanism for VW generation than the linear one, as the interacting wave amplitudes become infinite in finite time rather than growing exponentially as linearly unstable modes do. Moreover, even linearly damped waves can grow explosively provided their initial amplitudes exceed a certain threshold.

The main goal of our work is to specify the conditions when the various types of explosive processes can occur and analyze their main consequences for VW dynamics.

In section 2, we discuss the problem statement starting with the classical shallow-water equations modified by taking into account bottom friction. Description of the general weakly nonlinear three-wave resonant interactions of VW is the subject of section 3. The equations derived obviously have the universal form. The novelty is accumulated in the specific expressions for the interaction coefficients, which are valid for arbitrary mean current and cross-shore depth dependence and might be used in a number of various problems. In section 4, we confine ourselves to the consideration of isolated triads. In section 5, we focus our attention on the explosive interactions taking as the illustrative example the simplest model of the physical background, that of Bowen and Holman (1989), modified by taking into account weak bottom friction. We show explicitly the existence of explosive triplets of VW and investigate the influence of dissipation on their dynamics. In particular, we estimate the amplitude thresholds necessary for the explosive instability to commence. The implications of the results obtained for VW dynamics in gen-

[2] In principle, triads including waves of the same sign of energy might be possible as well; however, we have not found such interactions in the particular models we analyzed.

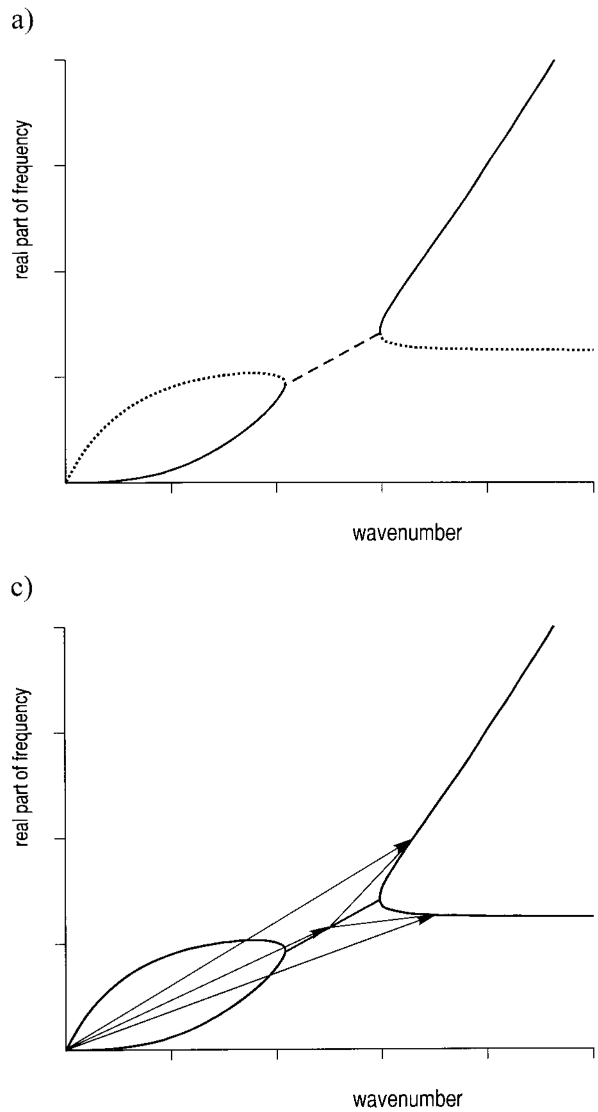

FIG. 1. A sketch of typical VW dispersion curves and the resonant triads. (a) Marked are the regions of waves with positive (solid line), negative (dotted line), and unspecified (dashed line) energy; (b) resonant triad for *S* processes; (c) triads corresponding to *U* processes.

eral, and for the interpretation of SUPERDUCK and NSTS 1983 field studies, are discussed in section 6.

2. Basic equations

We shall consider finite-amplitude VW dynamics in the nearshore in the coordinate frame with the x and y axes directed offshore and alongshore respectively and the shoreline situated at $x = 0$. As the typical timescale of VW observed in the field experiments was found to be much smaller than the Coriolis timescale, the influence of the earth's rotation on the VW dynamics is neglected. The total velocity field is assumed to consist of the mean longshore current and perturbations

$$\mathbf{u}_* = \{u(x, y, t), V(x) + v(x, y, t)\}, \qquad (2)$$

where $V(x)$ represents the mean steady current and $\mathbf{u} = \{u, v\}$ the perturbed velocity field. The basic equations governing VW dynamics in the nearshore are then the standard shallow-water equations

$$u_t + Vu_y = -g\zeta_x - \frac{\mu}{h}u - (uu_x + vu_y) \qquad (3a)$$

$$v_t + Vu_y + uV_x = -g\zeta_y - \frac{\mu}{h}v - (uv_x + vv_y) \qquad (3b)$$

$$\zeta_t + V\zeta_y + [(\zeta + h)\, u]_x + [(\zeta + h)v]_y = 0. \qquad (3c)$$

Here ζ is the free surface elevation, g is gravity acceleration, $h = h(x)$ is the depth presumed to be uniform alongshore, while the parameter μ represents dissipation. The dissipative terms can be derived from first principles by considering the full nearshore momentum balance. Their particular form used in (3) corresponds to bottom friction rather than to eddy viscosity and is based on the standard set of assumptions (e.g., see Dodd 1994). It is known that the main VW features are weakly sensitive to the type of dissipation term used, the phenomenological coefficients being the source of the largest uncertainty (Dodd 1994; Thornton and Guza 1986). So we accept the set (3) as the basic one, supposing in the further analysis that the dissipation is relatively mild and, thus, modifies only slightly the nondissipative dynamics of linear VW. This assumption proves to be quite realistic for both SUPERDUCK and NSTS conditions.

The scalings

$$\{u, v\} = V_0\{u', v'\}, \qquad V = V_0V'$$
$$\zeta = \zeta_0\zeta', \qquad h = h_0h'$$
$$(x, y) = x_0(x', y'), \qquad t = \frac{x_0}{V_0}t' \tag{4}$$

are now introduced, where V_0 is the typical magnitude of the mean current velocity, say its maximum value in the nearshore, x_0 is the typical mean current cross-shore width, h_0 is the typical depth within the current domain, and where primed quantities are nondimensional. With scaling (4) the nondimensional equations of motion take the form

$$u_t + Vu_y = -\frac{g\zeta_0}{V_0^2}\zeta_x - \frac{\mu x_0}{V_0h_0}\frac{u}{h} - (uu_x + vu_y) \tag{5a}$$

$$v_t + Vu_y + uV_x = -\frac{g\zeta_0}{V_0^2}\zeta_y - \frac{\mu x_0}{V_0h_0}\frac{v}{h} - (uv_x + vv_y), \tag{5b}$$

where primes have been dropped for convenience. The Eqs. (5a–c) indicate that the proper scale of the free surface elevation ζ is

$$\zeta_0 = \frac{V_0^2}{g}. \tag{6}$$

The nondimensional continuity equation with (6) taken into account is

$$(hu)_x + (hv)_y = - F[\zeta_t + V\zeta_y + (\zeta u)_x + (\zeta v_{y)}] \tag{5c}$$

where the parameter

$$F = \frac{V_0^2}{gh_0} \tag{7}$$

is the Froude number, typically much smaller than unity for real field conditions.

Thus, the problem, in principle, contains three nondimensional parameters, namely, the Froude number F, the ratio of the advective timescale x_0/V_0 to the frictional time scale h_0/μ

$$\epsilon = \frac{\mu x_0}{h_0V_0}, \tag{8}$$

and the amplitude parameter ϵ_n, which represents the ratio of the typical velocity perturbation magnitude to the typical magnitude of the mean current velocity. The dynamics of the system depends, of course, on their comparative values. Here we shall study probably the most interesting case when the effects of nonlinearity, friction, and divergence are all of the same order of smallness, that is,

$$\epsilon_n = F = \epsilon \ll 1, \tag{9}$$

and shall look for the solutions to the Eqs. (5a–c) in the form of power series in ϵ

$$u = \sum_{n=1}^{\infty} \epsilon^n u_n$$
$$v = \sum_{n=1}^{\infty} \epsilon^n v_n$$
$$\zeta = \sum_{n=1}^{\infty} \epsilon^n \zeta_n. \tag{10}$$

For future use we also define the material derivative operator D_t,

$$D_t \equiv \partial_t + V(x)\partial_y \tag{11}$$

and its formal derivative with respect to the cross-shore variable

$$\partial_x^p D_t \equiv \partial_x^p V \cdot \partial_y. \tag{12}$$

In the first order in ϵ the motion is nondivergent; therefore, it is convenient to describe the motion via the streamfunction, introduced by setting

$$hu_1 = -(D_t\Phi)_y$$
$$hv_1 = (D_t\Phi)_x. \tag{13}$$

Cross differentiating (3a′), (3b′) to exclude surface elevation ζ, we obtain a linear self-conjugate equation for the function Φ

$$\left(\frac{D_t^2\Phi'}{h}\right)' + D_t^2\frac{\Phi_{yy}}{h} = 0. \tag{14}$$

Hereinafter, prime denotes the derivative with respect to cross-shore variable x. The surface elevation turns out to be unambiguously related to Φ through (3b′)

$$\frac{D_t^2\Phi'}{h} = -\zeta_y. \tag{15}$$

The effects of the divergence contribution in (3′) are second order in ϵ. To describe them, we introduce two auxiliary functions $\Psi(x,y,t)$ and $S(x,y,t)$, the velocity perturbation components are expressed in their terms as

$$hu_2 = -(D_t\Psi)_y$$
$$hv_2 = (D_t\Psi)' + S. \tag{16}$$

The second-order terms of the continuity equation (5c) lead to the relation between the first-order free-surface elevation ζ_1 and the new function S:

$$S_y + D_t\zeta_1 = 0. \qquad (17)$$

Thus, S is also unambiguously expressed in terms of the first-order streamfunction [see (15)].

3. Nonlinear resonant interactions of VW: General consideration

From the general theory viewpoint, all the diverse problems for the dynamics of weakly nonlinear waves in any medium can be reduced to a rather limited number of universal equations with more or less known properties. To start the study of weakly nonlinear dynamics for a particular type of motions, one should first specify the types of resonances permitted. The specifics of a particular medium lies just in the dispersion relations and in the particular expressions for the interaction coefficients. Examining the linear dispersion relations known from the aforementioned studies, it is easy to see that three-wave resonant interactions of VW are permitted to occur and therefore are expected to dominate the weakly nonlinear dynamics. The aim of this section is to derive simplified equations describing the dynamics. The particular choice of equations to be derived is dictated by the physical problem of interest and will be discussed below. First, we derive the most general equations.

The standard way to obtain the governing equations is to represent the perturbation field, as a superposition, to first order, of free quasi-harmonic waves

$$\Phi(x, y, t) = \sum_{k,n} A_k^n\phi_k^n(x)\exp\{iky - i\omega_k^n t\} + \text{c.c.}$$

$$\zeta_1 = \sum_{k,n} \hat{\zeta}_k^n\exp\{iky - i\omega_k^n t\} + \text{c.c.}$$

$$S = \sum_{k,n} \hat{f}_k^n\exp\{iky - i\omega_k^n t\} + \text{c.c.}, \qquad (18)$$

permitting the amplitudes A_k^n of harmonics to vary slowly with time and space. According to (14), the mode function $\phi_k^n(x)$ is then the solution of the linear boundary value problem

$$\left(\frac{\sigma^2\phi'}{h}\right)' - \sigma^2k^2\frac{\phi}{h} = 0 \qquad (19a)$$

$$\phi = 0, \quad x = 0 \qquad (19b)$$

$$\phi \to 0, \quad x \to \infty, \qquad (19c)$$

where $\sigma(x) = \omega - kV(x)$ is the local Doppler frequency of the wave. The boundary condition $\phi(0) = 0$ implies that there is no flux normal to the beach, while $\phi(\infty) = 0$ is the condition for a wave to be trapped nearshore.

From (19) the orthogonality relation for eigenfunctions $\phi_k^n(x)$ is easily found to be

$$\int_0^\infty \frac{\sigma_n + \sigma_m}{h}(\phi_n'\phi_m' + k^2\phi_n\phi_m)\,dx = J_n\delta_{nm}, \qquad (20)$$

where δ_{nm} is the Kronecker delta. Besides, hereinafter we use a joint mode index $n = (n, k_n)$. Both the first-order free-surface elevation ζ_1 and S function are linearly related to the first-order streamfunction, and their Fourier components $\hat{\zeta}$ and $\hat{f}$ are found to be given by the expressions

$$\hat{\zeta}_n = -\frac{i}{k}\frac{\sigma^2}{h}\,\phi_n'A_n$$

$$\hat{f}_n = -\frac{i}{k^2}\frac{\sigma^3}{h}\,\phi_n'A_n. \qquad (21)$$

The slow temporal evolution of a harmonic amplitude A_n occurs owing to nonlinear resonant interaction with other harmonics, linear dissipation, and divergence effects. For simplicity we confine ourselves to the study of spatially uniform wave trains; that is, we allow the amplitudes to depend on "slow" time $\tau = \epsilon t$ only. To obtain the governing equations, we represent the second-order streamfunction Ψ as superposition of linear harmonics with space–time-dependent amplitudes $\psi_n(\tau, x)$ and by substituting (18) and (21) into (5a–c) obtain, to the second order in ϵ, the inhomogeneous boundary problem for ψ_n:

$$\left(\frac{\sigma^2\psi_n'}{h}\right)' - k^2\,\frac{\sigma^2\psi_n}{h} = -\frac{1}{k^2}\left(\frac{\sigma^4\phi_n'}{h^2}\right)'A_n - 2i\left[\left(\frac{\sigma\phi_n'}{h}\right)' - k^2\,\frac{\sigma\phi_n}{h}\right]\partial_\tau A_n - i\left[\left(\frac{\sigma\phi_n'}{h^2}\right)' - k^2\,\frac{\sigma\phi_n}{h^2}\right]A_n - (\hat{N}[\Phi])_k \qquad (22)$$

$$\phi = 0, \quad x = 0$$

$$\phi \to 0, \quad x \to \infty,$$

where $(N[\Phi])_k$ is the corresponding Fourier component of the nonlinear operator, which is specified below. Multiplying (22) by ϕ_n and integrating over space domain $0 < x < \infty$, we obtain the solvability condition

$$(J_n\partial_t + I_n - i\mathcal{L}_n)A_n = -i\int_0^\infty \phi_n(\hat{N}[\Phi])_k\,dx, \quad (23)$$

with the coefficient J_n determined through (20) and I_n and L_n having the form

$$I_n = \int_0^\infty \frac{\sigma_n}{h^2}[(\phi_n')^2 + k^2\phi_n^2]\,dx - \frac{1}{2}\int_0^\infty \left(\frac{\sigma'}{h^2}\right)'\phi_n^2\,dx \quad (24)$$

$$\mathcal{L}_n = \frac{1}{k^2}\int_0^\infty \left(\frac{\sigma_n^2\phi_n'}{h}\right)^2 dx. \quad (25)$$

Since the nonlinearity is quadratic, only those harmonics that satisfy the well-known 'resonance conditions' (e.g., see Craik 1985)

$$k_n = k_l + k_j$$
$$\omega_n = \omega_l + \omega_j \quad (26)$$

contribute to the kth Fourier component of the nonlinear operator $(\hat{N}[\Phi])_k$

$$(\hat{N}[\Phi])_{kn} = i\sum_{k_l,k_j}\left\{\left[-\left(\frac{(\sigma_j\phi_j)'}{h}\right)'k_l\frac{\sigma_l\phi_l}{h} + \frac{(\sigma_j\phi_j)'}{h}k_l\frac{(\sigma_l\phi_l)'}{h}\right]' + k_n\left[\frac{\sigma_j\phi_j}{h}\left(\frac{\sigma_l\phi_l}{h}\right)'k_lk_j - \frac{\sigma_l\phi_l}{h}k_l^2\frac{(\sigma_j\phi_j)'}{h}\right]\right\}A_jA_l. \quad (27)$$

The equations are quite general. Any initial distribution of the wave field can be decomposed into the set of harmonics (18) with the desired accuracy, and the field dynamics at timescales ϵ^{-1} will be adequately described. It is easy and worthwhile to trace the contributions to the field dynamics due to different factors. For example, the effects of friction are accumulated in the I_n terms. It is obvious that adopting a different model for dissipation will result only in changing the particular values of the coefficients I_n. If we put viscosity to zero, the system becomes Hamiltonian. The influence of nondivergence is accumulated in L_n and results in an $O(\epsilon)$ correction to the dispersion curve. The nontrivial dynamics is caused by nonlinear terms due to the resonant interactions grouped on the rhs. Although it is difficult to infer any direct conjectures from these equations in their general form, they are nonetheless of interest. Their first advantage compared to the shallow-water equations is rather obvious: They are much simpler and more convenient for numerical simulation. Second, in situations where the initial field distribution could be well approximated by a few narrowbanded packets, the system could be drastically simplified in a straightforward manner. An analysis of the most simple system of this kind will be the subject of the next section.

4. Explosive processes

The key concept about explosive processes is that of wave energy. We define energy E due to a harmonic wave perturbation of amplitude A as $E = \mathrm{sgn}\,J\,|A|^2$, the expression for J given by (20). The interaction among the waves with the same sign of energy results in spreading of wave energy among different wave components. In the absence of dissipation, neither the energy of the field composed of an arbitrary number of harmonics nor a particular wave amplitude can grow infinitely with time. On the contrary, in the process of resonant interaction among waves of different energy signs, the conservation of the wave field energy does not necessarily imply a limitation on the growth of wave components. Growth of waves of different energy can occur without violating the conservation of energy. Provided some special conditions specified below are fulfilled, all waves can grow synchronously, resulting in the so-called explosive interaction, which changes the picture of the field evolution drastically.

Two qualitatively different types of explosive processes are possible: those involving only waves linearly stable in the absence of dissipation and those embracing two stable (in the same sense) waves and one unstable wave. We shall refer to them as ''S'' and ''U processes'' for brevity (see Figs. 1b and 1c, respectively).

a. *Dynamics of an isolated triad: Explosive S processes*

To elucidate the new mechanism of instability, we start by considering the elementary processes involving only *linearly stable* waves, that is, S processes. For simplicity, we study a single resonant triad satisfying (26). Straightforward calculations yield the classical equations of three-wave resonant interaction

$$(\partial_\tau + \upsilon_j)a_j = s_ja_na_l\cos\Theta$$
$$(\partial_\tau + \upsilon_l)a_l = s_la_na_j\cos\Theta$$
$$(\partial_\tau + \upsilon_n)a_n = -s_na_ja_l\cos\Theta$$
$$\partial_\tau\Theta = \Delta + \left(s_n\frac{a_ja_l}{a_n} - s_j\frac{a_na_l}{a_j} - s_l\frac{a_ja_n}{a_l}\right)\sin\Theta, \quad (28)$$

where Θ and Δ are the phase difference and the phase mismatch; correspondingly

$$\Theta = \theta_n - \theta_j - \theta_l \qquad \Delta = \delta_n - \delta_j - \delta_l,$$

where a_i, θ_i are the normalized real amplitude and phase of the complex amplitudes A_n

$$A_n = \frac{1}{P}\sqrt{|J_j||J_l|}\, a_n \exp\{i\theta_n\}, \qquad (n, j, l = 1, 2, 3). \tag{29}$$

The dissipation coefficients and the mismatches are given by the expressions

$$\upsilon_i = \frac{I_i}{J_i}, \quad \text{and} \quad \delta_i = \frac{\mathcal{L}_i}{J_i}, \tag{30}$$

and s_i stands for the sign of the normalizing coefficient J_i.

Thus, in general, the dissipation due to the effect of bottom friction on the VW dynamics and mismatch caused by the surface motion varies for different space–time harmonics because of the bottom cross-shore variability. This greatly hampers the investigation of the triad (28) dynamics as no analytical solution is available in this case. The coupling coefficient P is common for all members of the resonant triad in accordance with the general theory (e.g., Weiland and Wilhelmsson 1977; Craik 1985) regardless of the particular type of bottom profile. The explicit expression of the coefficient is found to be

$$\begin{aligned} P = \frac{1}{2}\int_0^\infty dx \\ &\times \left\{ -k_j k_l k_n \phi_j \phi_l \phi_n \left[\left(\frac{V'}{h}\right)^2\right]' \right. \\ &+ \frac{1}{h}\left(\frac{V'}{h}\right)' (k_j k_l \phi_j \phi_l \sigma_n \phi_n' \\ &+ k_j k_n \phi_j \phi_n \sigma_l \phi_l' \\ &\left. + k_l k_n \phi_l \phi_n \sigma_j \phi_j') \right\}. \end{aligned} \tag{31}$$

The set (28) was intensively studied, the equations becoming exactly solvable in the absence of dissipative terms [both the partial differential equations (PDE) governing spatially localized wave packets (Zakharov and Manakov 1975; Kaup et al. 1979) and ordinary ones (ODE) governing spatially uniform wave trains]. This solvability is not the case for triplets with dissipation unless all dissipative terms are equal. Still the general features of their dynamics were studied in a number of numerical experiments (e.g., see Weiland and Wilhelmsson 1977). Here we only sketch the main results for the simplest, that is, spatially uniform, case.

If dissipation is absent (all $\nu_i \equiv 0$), the system (28) is conservative and subject to various conservation laws including the so-called Manley–Rowe relations

$$\begin{aligned} \partial_\tau (s_j a_j^2 - s_l a_l^2) &= 0 \\ \partial_\tau (s_n a_n^2 + s_{j,l} a_{j,l}^2) &= 0. \end{aligned} \tag{32}$$

If the coefficients J_i for all triad members have the same sign, the second relation (32) imposes a restriction on the amplitude growth. The energy exchange among different harmonics results in the growth of a_j, a_l and simultaneous decrease of a_n or vice versa. This type of interaction is called ''*decay*,'' and its output is slow periodical changes of wave amplitudes. On the contrary, when the signs of the coefficients J_j, J_l are the same and opposite to that of J_n, all three wave amplitudes can grow simultaneously as it follows from (32). This means that, only if the wave with maximum eigenfrequency has energy of sign different from that of two other triplet members, the negative energy wave loses energy and grows in amplitude, while positive energy waves acquire energy and also grow. All amplitudes tend to become infinite in finite time, that is, they grow faster than exponentially. This type of nonlinear resonant interaction is called ''explosive.'' Another conservation law following from (28) and governing the phase difference Θ dynamics in the process of interaction is

$$a_j a_l a_n \sin\Theta + s_j \frac{\Delta}{2} a_j^2 = \Gamma_j, \tag{33}$$

where Γ_j is a constant. Hence, while all wave amplitudes a_i grow to infinity in the process of explosive interaction, the phase difference Θ diminishes to zero. This effect is called ''phase locking.''

Though Manley–Rowe relations do not hold in a nonconservative medium, the principal result remains intact: The explosive instability in the system (28) always occurs when the initial harmonic amplitudes exceed some threshold value depending on the dissipation strength and the value of phase mismatch Δ. Obviously this singular growth is limited by the physical factors lying beyond the framework of weakly nonlinear models: either by the next order nonlinear terms corresponding to four-wave interaction or by nonlinear dissipation. The saturation does not occur within the weakly nonlinear stage of instability unless the three-wave interaction coefficient contains some external small parameter, while the four-wave one does not (see Romanova and Shrira 1988). If this is not the case, the final state has interacting amplitudes of the order of unity, which means wave breaking or intense vortex formation are the most likely outcome of the explosive instability process.

Here we restrict our consideration to the initial stage of instability when weakly nonlinear theory does hold. Within its framework one could hope to find out how the explosion time, amplitude threshold, and the instability domain in wavenumber space depend on the set of ''external'' parameters. The answers can be derived for any specific model of the nearshore zone. A particular example will be given below in section 5.

b. Nonlinear dynamics of linearly unstable waves (U-processes)

Linearly stable space–time harmonics (18) act as elementary excitations and their response to external forcing is prescribed by their energy sign, which in this case is quite definite. If one considers the dynamics of linearly unstable modes, the elementary excitation consists of a pair of modes with the same real part of the frequency but with imaginary parts of opposite sign. These unstable waves do not possess a definite energy sign (Ignatov 1989). Nonlinear triad interactions involving such pairs, with resonant conditions satisfied only for real parts of ω

$$k_n = k_l + k_j$$
$$\mathrm{Re}\omega_n = \mathrm{Re}\omega_l + \mathrm{Re}\omega_j \qquad (34)$$

are governed by more complex equations than (28), these being nonintegrable in principle though they have some conservation laws. We do not present these equations, as they are fairly difficult to treat. The resonant interactions of this type have been studied in plasma physics; however, not too much can be stated in quantitative terms. The main qualitative conclusion relevant to our context is that explosive processes *are always plausible* for the triad (34) though it is no longer possible to obtain any explicit estimations of the explosion time and the amplitude threshold (Ignatov 1984). The condition for three-wave resonance for U processes (34) involve only the real parts of eigenfrequencies and are not exact in a sense. However, they can still be considered as such provided the initial amplitudes of interacting waves are sufficiently large, while linear growth rates are sufficiently small. From the viewpoint of the general theory of nonlinear resonant interactions, the resonance should be exact within the accuracy of the small amplitude parameter O(ϵ). If the ratio of linear growth rate to eigenfrequency is of the same order, the resonance (34) is as accurate as (26). Therefore, U processes occur only for the waves with amplitudes of the order of Imω/Reω; that is, they come into play only *after* a period of initial perturbation growth due to linear instability.

5. An illustrative example

To exemplify the general theory discussed above we choose the simplest model of the physical background, that of Bowen and Holman (1989). Their model neglects free-surface movements. The latter assumption is valid when the Froude number is sufficiently small ($F \ll \epsilon$), which is possible in real conditions. As we have shown in section 3, taking into account the free-surface effects results in a slight correction to the dispersion relation and does not affect the nonlinear terms; therefore it is not essential for the estimates that we intended to derive in this section. Here we show that explosive triplets do exist within a realistic range of model parameters and outline the span of unstable wavenumbers, and their dependence on the basic current geometry.

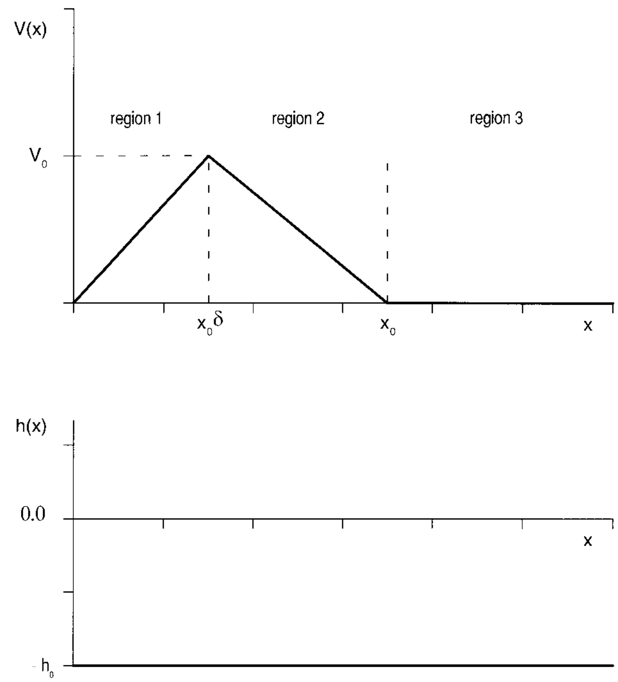

FIG. 2. Depth–longshore current profiles for the illustrative example.

The mean current velocity cross-shore dependence is taken to be of the form (see Fig. 2)

$$V(x) = \begin{cases} \delta^{-1}x, \\ \quad 0 \le x \le \delta \quad \text{(region 1)} \\ (1-\delta)^{-1}(1-x), \\ \quad \delta \le x \le 1 \quad \text{(region 2)} \\ 0, \\ \quad 1 \le x < \infty \quad \text{(region 3)}, \end{cases} \qquad (35)$$

while the bottom is presumed to be flat: $h(x) = 1$ in nondimensional units.

The linear problem (19) is easily solved and its solution is

$$\phi = \begin{cases} A\dfrac{\sinh(kx-\omega\delta)}{kx-\omega\delta} + B\dfrac{\cosh(kx-\omega\delta)}{kx-\omega\delta} & \text{(region 1)} \\ D\dfrac{\sinh[k(x-1)+\omega(1-\delta)]}{k(x-1)+\omega(1-\delta)} & \\ \quad + E\dfrac{\cosh[k(x-1)+\omega(1-\delta)]}{k(x-1)+\omega(1-\delta)} & \text{(region 2)} \\ F\exp[k(1-x)]. & \text{(region 3)} \end{cases} \qquad (36)$$

The solutions (36) obtained should be matched at the boundaries of the regions 1–3 (at $x = \delta$ and $x = 1$), where the jumps of background vorticity are present, subject to matching conditions, these being the continuity of the eigenfunction and its first derivative

$$[\phi] = 0,\ [\phi'] = 0 \text{ at } x = \delta,\ x = 1. \tag{37}$$

Here square brackets are used to designate the jump of the value inside at the given point. Four matching conditions (37) plus the boundary condition at the coast (19b) constitute the set of five equations specifying four amplitude constants and eigenfrequency ω for any wavenumber value k, the fifth amplitude, say A, being free in the linear problem. Performing the necessary calculations, we obtain the linear dispersion relation for VW:

$$\omega\left(\cosh(k) + \frac{1}{\omega - k}\frac{1}{\delta(1-\delta)}\sinh(k\delta)\cosh[k(1-\delta)]\right) = \left(\frac{1}{1-\delta} - \omega\right)\left(\sinh(k) + \frac{1}{\omega - k}\frac{1}{\delta(1-\delta)}\sinh(k\delta)\sinh[k(1-\delta)]\right). \tag{38}$$

Obviously, (38) represents a quadratic equation with respect to ω and coincides with that obtained by Bowen and Holman (1989). The corresponding dependence of Reω on wavenumber k for the mean current backshear parameter $\delta = 0.3$ is shown in Fig. 3, with the region of negative energy waves (those with $J < 0$) marked. Also shown are the explosive triads of the S type representing *two different families*. Fixing the arbitrary Xmultiplicative constant A,

$$A = \frac{\delta}{\delta - 1}\cosh(\omega\delta), \tag{39}$$

we obtain the explicit expressions for the other amplitude coefficients depending on wavenumber k, wave frequency ω, and backshear parameter δ:

$$B = \frac{\delta}{\delta - 1}\sinh(\omega\delta)$$

$$D = \cosh[(1-\delta)\omega - k] + \frac{1}{\omega - k}\frac{1}{\delta(1-\delta)}\sinh(k\delta)\cosh[(1-\delta)(\omega - k)]$$

$$E = \sinh[k - (1-\delta)\omega] - \frac{1}{\omega - k}\frac{1}{\delta(1-\delta)}\sinh(k\delta)\sinh[(1-\delta)(\omega - k)]$$

$$F = \frac{1}{\omega(1-\delta)}\left\{\sinh(k) + \frac{1}{\omega - k}\frac{1}{\delta(1-\delta)}\sinh(k\delta)\sinh[k(1-\delta)]\right\}. \tag{40}$$

With the dispersion equation solved and the eigenfunctions of the triad members found, the normalizing coefficients J, I and the coupling coefficient P could be easily calculated for a chosen resonant triad. A particular advantage of the model is that for an arbitrary mean current profile accounting for the dissipation is extremely simple, namely, the dissipation equally affects all space–time harmonics of VW due to the absence of cross-shore depth variability. This simplification is closely related to the particular form of bottom friction (1) and is not likely to hold for a different type of dissipation. Thus, all normalized friction coefficients ν_i in (30) are equal to each other and, in our scaling, to unity. The set of equations (28) governing the evolution of a triad composed of linearly stable waves then reduces by means of the nonlinear ansatz

$$U_i(T) = a_i \exp\{\tau\}$$
$$T = 1 - \exp\{-\tau\} \tag{41}$$

to an equivalent set of equations without dissipation

$$\partial_T U_j = s_j U_n U_l \cos\Theta$$
$$\partial_T U_l = s_l U_n U_j \cos\Theta$$
$$\partial_T U_n = -s_n U_j U_l \cos\Theta$$
$$\partial_T \Theta = \left(s_n\frac{U_j U_l}{U_n} - s_j\frac{U_n U_l}{U_j} - s_l\frac{U_j U_n}{U_l}\right)\sin\Theta. \tag{42}$$

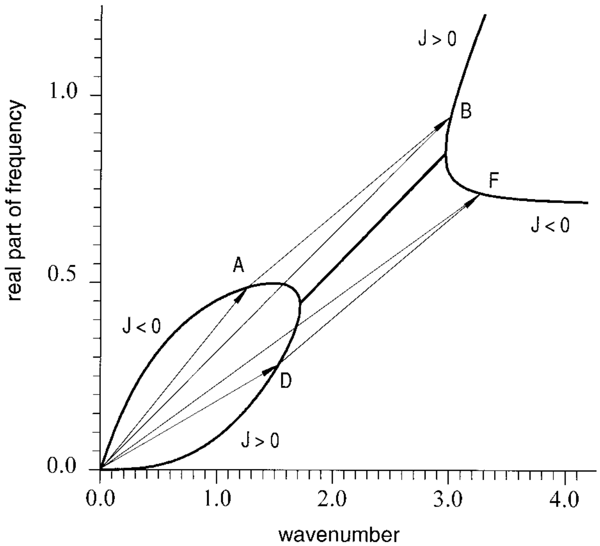

FIG. 3. Real part of wave frequency vs wavenumber for $\delta = 0.3$. Regions of positive and negative energy waves (those with $J > 0$ or < 0) are marked. Explosive resonant triplets $\boldsymbol{OA} + \boldsymbol{AB} = \boldsymbol{OB}$ and $\boldsymbol{OD} + \boldsymbol{DF} = \boldsymbol{OF}$ are representatives of two different families.

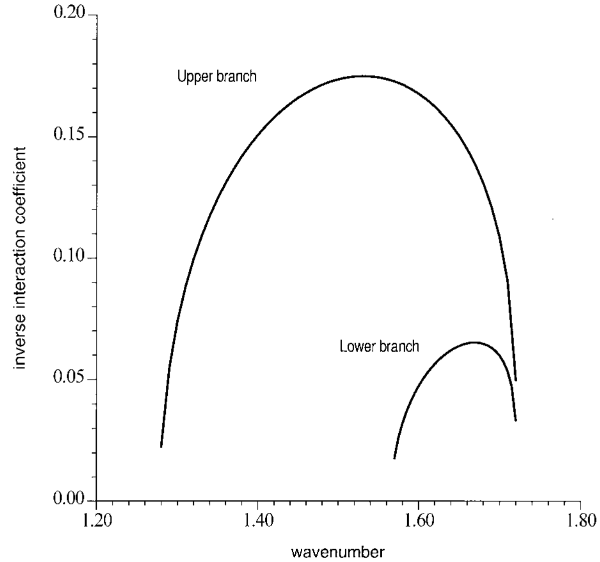

FIG. 4. Inverse interaction coefficient $\sqrt{|J_nJ_jJ_l|}P^{-1}$ versus wavenumber of resonant harmonic.

Given the initial harmonics amplitudes U_i (0), (42) can be solved analytically and the amplitude slow time dependence can be found explicitly in terms of elliptic functions (Weiland and Wilhelmsson 1977; Craik 1985). The exact solution could also be presented for the spatially localized wave trains governed by the PDE analogue of (42). Here we mention that, in the generic case, the amplitudes grow faster than exponentially and develop a singularity in finite time T_∞ depending on initial amplitudes and phases. In the simplest case, that of equal initial amplitudes and zero initial phase difference, this relationship has the form

$$T_\infty = \frac{1}{U_i(0)}. \tag{43}$$

For the ''explosion time'' in the system (28) we, therefore, obtain the expression

$$\tau_\infty = -\ln(1 - T_\infty). \tag{44}$$

As the logarithmic argument must be positive, we conclude that the explosive instability in the system with dissipation develops only if the initial amplitudes are larger than the threshold value, which in our scaling is unity. Alongshore and cross-shore velocity perturbations corresponding to nth harmonic in dimensional units are found to be

$$u_n = -\mu\frac{x_0}{h_0}k_n\frac{\sigma_n\phi_n}{\sqrt{|J_n|}}\frac{\sqrt{|J_nJ_jJ_l|}}{P}a_n\exp\{i(k_ny - \omega_nt)\} + \text{c.c.}$$

$$v_n = -i\mu\frac{x_0}{h_0}\frac{(\sigma_n\phi_n)'}{\sqrt{|J_n|}}\frac{\sqrt{|J_nJ_jJ_l|}}{P}a_n\exp\{i(k_ny - \omega_nt)\} + \text{c.c.} \tag{45}$$

Equations (45) indicate that the velocity perturbation amplitude depends both on the nearshore environment features, such as dissipation strength, depth, or mean current cross-shore width, and on the spectral characteristics of the corresponding harmonic. Most of the latter dependence is accumulated in the normalized interaction coefficient $P/\sqrt{|J_nJ_jJ_l|}$, which is a single-parameter function and varies significantly through the span of wavenumbers of nonlinearly unstable VW. The dependence of the inverse coefficient ($\sqrt{|J_nJ_jJ_l|}P^{-1}$) on the smallest triplet wavenumber k_j is plotted in Fig. 4 for two families of explosive triads of VW. The backshear parameter δ was taken to be equal to 0.3, which is close to the experimentally observed values. Substantial variability of the coefficient leads to a broad range of amplitude thresholds for explosive instability, and therefore the real wave field is most likely to exhibit that part of the spectrum with smaller threshold values. The main properties of the explosive instability are closely, though implicitly, related to the mean current backshear. The most sensitive is the span of nonlinear instability in wavenumber space. The boundaries of the regions of linear and nonlinear instability in k–δ space are depicted in Fig. 5. Evidently, the influence of an increasing current backshear on the span of wavenumbers of unstable VW is opposite for linear and nonlinear instability: the former extends with the growth of δ, while the latter constricts. The conclusions thus made concern the S processes only, which alone are significant at the initial stage of perturbation growth. Thus, one might expect the linear instability to prevail for currents with strong backshear in the initial period.

Obviously, for the later stage nonexact resonance involving linearly unstable waves (34) should be taken

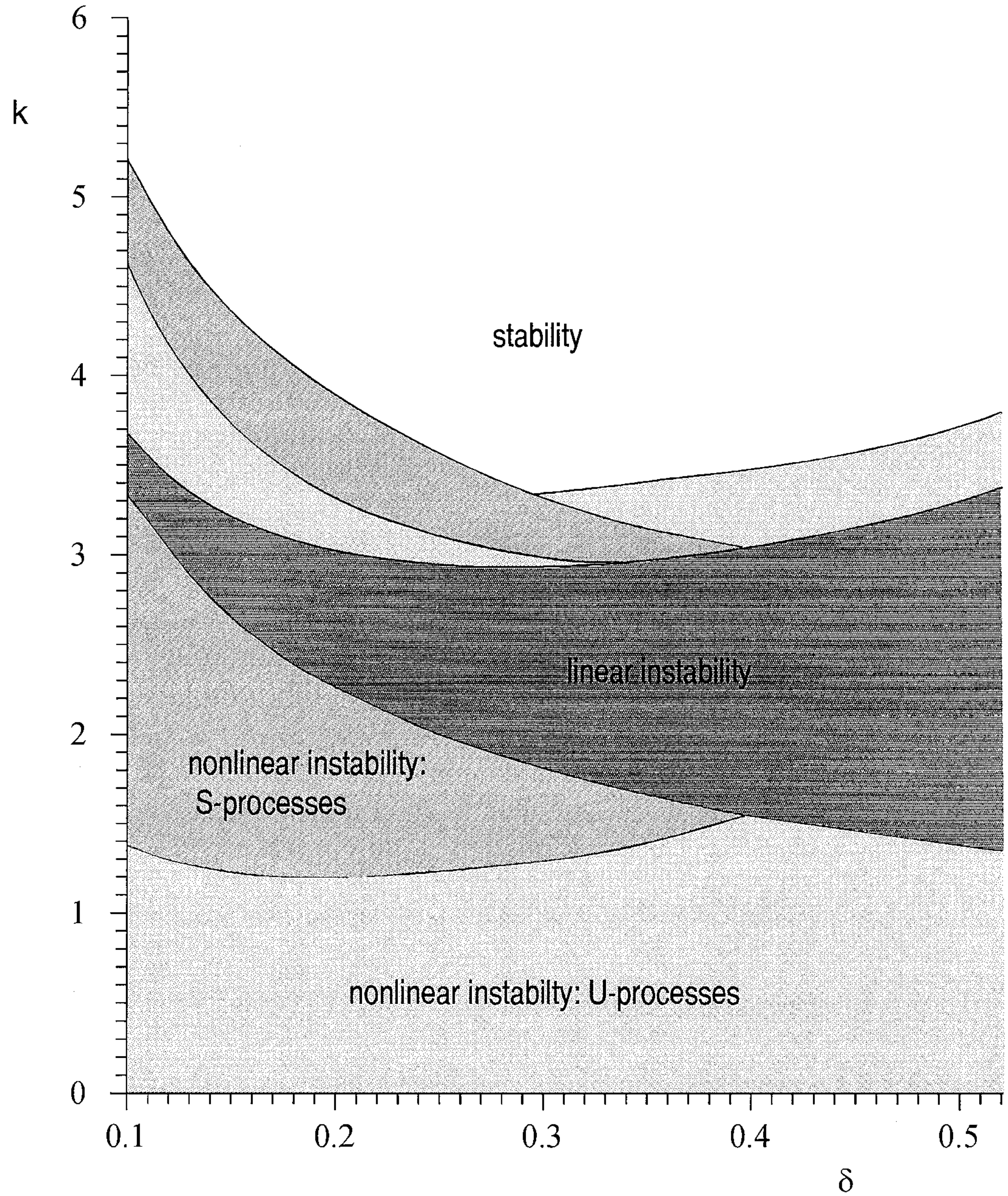

FIG. 5. Stability diagram in $k-\delta$ space. Darkest shading represents region of linear instability.

into account. Their main effect is that the span of instability expands in small wavenumber region down to zero as the dispersion curves indicate and, we stress this point, they are possible at any value of the mean current backshear. The absence of the *S* processes at high values of backshear does not mean that the current becomes more stable. The total domain of instability (linear plus nonlinear) expands with increase of backshear.

As for other environmental parameters, their influence is quite evident from (45): smaller dissipation and current width and larger depth moderate the amplitude threshold and so make explosive processes more plausible. The direct calculations of the amplitude threshold for velocity perturbations via (45) based upon the measured data for depth, mean current width and backshear, and friction coefficient yield values in the range 7–15 cm s^{-1} for SUPERDUCK and 4–11 cm s^{-1} for NSTS. Though large, these values seem to be realistic and the initial velocity perturbations of such magnitude could be spontaneously generated by some inhomogeneity in the flow to trigger explosive *S*-type instability even in the absence of linear instability.

6. Discussion

The above consideration within the framework of weakly nonlinear theory certainly shows that explosive processes can occur. Although the role of explosive interactions in the dynamics of vorticity waves is difficult to quantify at the present time, it is already possible to formulate some robust qualitative conclusions on the role of explosive processes in VW dynamics, which might be verified experimentally.

The most robust and easily detectable consequence of the explosive instability is the expansion of the instability range in k space: *there is no low-frequency cutoff* while the high-frequency bound is also noticeably shifted upward compared to the linear limit. This conclusion seems to resolve the difficulty of the linear theory formulated by Dodd (1994) who found theoretically on the basis of linear instability analysis that a global frequency cutoff exists below which instability does not develop, in contrast with the results of measurements where no cutoff was detected.

The difference between exponential and explosive growth seems to be difficult to detect not only in field data but in laboratory experiments as well. The problem is that, if in a laboratory experiment the initial amplitudes just slightly exceed the threshold, then the explosion time will be logarithmically large according to (43) and no peculiar features of explosive process could be registered during the relatively small observation time typical for the laboratory experiments (e.g., see Reniers et al. 1994). To make the decisive conclusion, one should investigate situations where all linear modes are damped by introducing calibrated incoming perturbations as is common in experiments on boundary layer instability. The growth is expected to start upon exceeding a certain threshold. The distinction of ''growth versus damping'' is obviously much easier to detect.

From the point of view of field observations, the distinction between the two types of explosive processes discussed here looks rather subtle, but still detectable. The S processes are expected to be more important for smaller backshear. The data on the mean current cross-shore profiles obtained during both SUPERDUCK and NSTS studies ensure that, at least in some runs, the backshear was sufficiently small, which is crucial for the existence of these explosive triads. The parameter δ was in the range 0.25–0.4 sometimes being as small as 0.15 (see Thornton and Guza 1986; Oltman-Shay et al. 1989). In these conditions, the S processes allow, in principle, the instability to develop starting from the primordial noise in the spectral range different from that of linear instability. A particular feature of this extension of the instability domains is the shift of the *high-frequency cutoff.* For the flows with the strong backshear, with δ exceeding the critical value 0.39, these processes disappear. The absence of S-processes at higher values of backshear does not mean that the current becomes more stable, it means just the disappearance of a particular type of explosive instability and the prevalence of linear instability. The U processes involving linearly unstable waves are always present although being confined to a smaller domain in k space. They amplify the growth of already growing perturbations. Their main contribution is extension of the instability domain down to zero frequency. We note that despite the disappearance of explosive instability at large values of backshear, its increase leads to the enhancement of instability strength and expansion of its total (both linear and nonlinear) domain in Fourier space.

The classical theory of nonlinear resonant interactions, including the explosive ones, is a *weakly nonlinear* theory and is valid only if the medium under consideration is *weakly* unstable. To be precise, the growth rates of interacting waves due to linear instability should be small in comparison with their eigenfrequencies. Yet the results of both Bowen and Holman (1989) and Dodd and Thornton (1990) indicate that already at $\delta \approx 0.5$ the imaginary and real parts of linear VW eigenfrequencies become comparable in magnitude. Under such conditions there is no sense in studying weakly nonlinear wave dynamics as the motions become strongly nonlinear within a few wave periods unless there is very strong friction. In situations characterized by strong backshear and strong, $O(1)$, dissipation there is a room for weakly nonlinear regimes, but of quite different types without explosive processes playing a noticeable role. In such regimes one may expect formation of a narrowband wave field subjected to cubic self-interaction of the Ginzburg–Landau type. It seems that such regimes, along with some strongly nonlinear ones, were reproduced in the quite recent extensive direct numerical study by Allen et al. (1996).[3]

One of the main qualitative conclusions of the present study is that the explosive processes of both types strongly amplify either primordial background noise or linearly growing perturbations and, once activated, lead inevitably to strongly nonlinear motions. These developed motions are most likely to be more adequately described in terms of *vortices* rather than waves. This point of view is indirectly supported by the quite recent results of direct numerical simulation by Deigaard et al. (1994), where formation of vortices was found to be one of the typical outcomes of the shear instability.

It should be mentioned that the theory we propose does not exclude the existence of other nonlinear mechanisms of VW generation; for example, the processes described by Shemer et al. (1991) may contribute by creating initial perturbations. The absence of a low-fre-

[3] The authors are grateful to J. Allen for the possibility to read the manuscript before its publication. We became aware of his study upon accomplishing and submitting the present work. This, along with the fact that the models of nearshore zones used by Allen et al. (1996) and us are different, makes any immediate intercomparison of the results rather difficult.

quency cutoff might be alternatively interpreted as the manifestation of the inverse cascade of 2D turbulence, although the relatively regular character of the motion at first sight contradicts this viewpoint. Still, summarizing, we consider the explosive interactions to be an important *intermediate* process in the evolution of nearshore current instability. To quantify their importance, much yet remains to be done. The theoretical models should first take into account realistic mean current profiles and the cross-shore depth variability — the easiest part of the problem. We already can say, for instance, that barred beaches have wider domains of explosive instabilities. Second, one should handle realistic noisy initial distributions, which may be far from being a set of narrowband wavetrains; this situation requires the investigation of the interactions among multiple resonant triads simultaneously, both explosive and nonexplosive ones. The latter problem is quite general and complicated and has not been solved yet. All this inevitably implies a heavy emphasis on numerical simulation in further studies of the VW explosive processes. We also hope that progress with laboratory experiments will supply the data allowing decisive judgements in the near future.

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