On Janssen's Model for Surface Wave Generation by a Gusty Wind

JOHN MILES

Institute of Geophysics and Planetary Physics, University of California, San Diego, La Jolla, California

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ABSTRACT

Janssen's model for the effect of gustiness on the mean energy transfer from a turbulent wind to a surface wave is simplified through a Gauss–Hermite approximation to his probability integral. Two examples are considered.

1. Janssen's model

Janssen (Komen et al. 1994) and Nikolayeva and Tsimring (1986) have shown that gustiness may increase the energy transfer from a turbulent wind to surface waves. Following Janssen, we assume that this gustiness can be described by a slowly varying (compared with the surface wave) friction velocity $u_*(t)$ with the Gaussian distribution

$$P(u_*;\,\overline{u}_*,\sigma) = (2\pi)^{-1/2}\sigma^{-1}\exp\left[-\frac{1}{2}\left(\frac{u_* - \overline{u}_*}{\sigma}\right)^2\right], \quad (1)$$

where $\overline{u}_*$ and σ are the mean value and standard deviation of u_*. We seek the mean, dimensionless growth rate ($\sigma = \sigma_u$ and $\zeta = \gamma/\omega$ in Janssen's notation)

$$\overline{\zeta}(\overline{u}_*) = \int_{-\infty}^{\infty} P(u_*;\,\overline{u}_*,\sigma)\zeta(u_*)\,du_* \quad (2)$$

on the assumption that the dimensionless growth rate for $\sigma = 0$,

$$\zeta(u_*) \equiv (\omega E)^{-1}(\partial E/\partial t), \quad (3)$$

is known for a monochromatic wave of energy E and angular frequency ω.

Substituting (1) into (2) and introducing $x = (u_* - \overline{u}_*)/\sqrt{2}\sigma$, we obtain

$$\overline{\zeta}(\overline{u}_*) = \pi^{-1/2}\int_{-\infty}^{\infty} e^{-x^2}\zeta(\overline{u}_* + \sqrt{2}\sigma x)\,dx. \quad (4)$$

If, as for most theoretical models, $\zeta(u_*) = 0$ for $u_* < 0$, we may replace the lower limit in (4) by $-\overline{u}_*/\sqrt{2}\sigma$. It follows, as remarked by Janssen, that gustiness may imply a positive energy transfer to a wave moving against the mean wind if $-\overline{u}_* = O(\sigma)$.

Corresponding author address: John W. Miles, Institute of Geophysics and Planetary Physics, Scripps Institution of Oceanography, University of California, San Diego, La Jolla, CA 92093-0225.

2. Gauss–Hermite approximation

The integral in (4) may be expressed in terms of the error function and its derivatives through the expansion of $\zeta(\overline{u}_* + \sqrt{2}\sigma x)$ in powers of x. However, this requires differentiation of $\zeta(u_*)$, which may be avoided through Gauss–Hermite quadrature to obtain ($n = 3$ in Abramowitz and Stegun 1964, § 25.4.46)

$$\overline{\zeta}(\overline{u}_*) = \frac{1}{6}\zeta(\overline{u}_* + \sqrt{3}\sigma) + \frac{2}{3}\zeta(\overline{u}_*) + \frac{1}{6}\zeta(\overline{u}_* - \sqrt{3}\sigma). \quad (5)$$

This approximation implies a positive energy transfer for $\overline{u}_* > -\sqrt{3}\sigma$; higher-order members of the Gauss–Hermite sequence would extend the tail of $\overline{\zeta}$ in $\overline{u}_* < 0$, but this tail is unlikely to be significant for $\overline{u}_* < -\sqrt{3}\sigma$.

3. Quadratic fit

The quasi-laminar (Miles 1957) and related models yield ζ in the form

$$\zeta(u_*) = \begin{cases}\beta(u_*/c)^2 \\ 0\end{cases} \quad (u_* \gtrless 0), \quad (6)$$

where c is the wave speed and β (rescaled from Miles 1957) is a slowly varying function of c/u_*. Substituting (6) into (4) and neglecting the variation of β, we obtain

$$\overline{\zeta} = \zeta[(\overline{u_*^2})^{1/2}]Z(\upsilon),\quad \overline{u_*^2} = \overline{u}_*^2 + \sigma^2,\quad \upsilon \equiv \overline{u}_*/\sqrt{2}\sigma, \quad (7a\text{–}c)$$

where $\overline{u_*^2}$ is the mean square of u_*,

$$Z(\upsilon) = \frac{1}{2}[1 + \mathrm{erf}(\upsilon)] + \pi^{-1/2}\left(\frac{\upsilon}{1 + 2\upsilon^2}\right)e^{-\upsilon^2}, \quad (8)$$

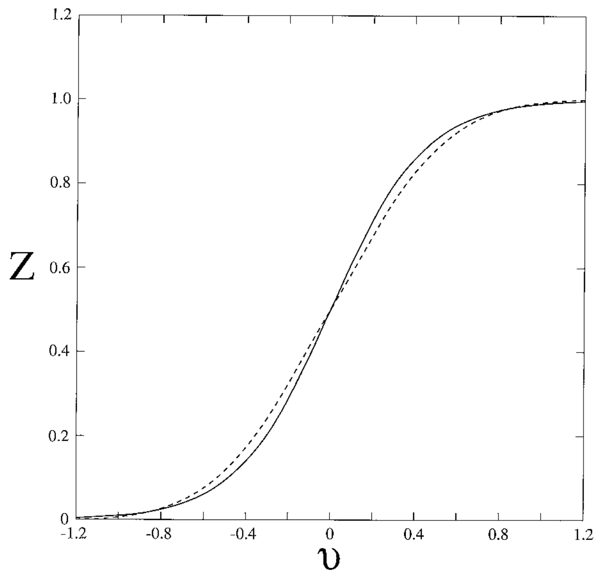

FIG. 1. The function $Z(\upsilon)$, as calculated from (8) (——) and (9) (– – –).

and erf is the error function. Alternatively, the approximation (5) yields (7) with

$$Z(\upsilon) = \begin{cases} 1, \; [\upsilon \geq \upsilon_0 \equiv (3/2)^{1/2}] \\ (1 + 2\upsilon^2)^{-1}[2/3\upsilon^2(1 + \mathrm{sgn}\,\upsilon) \\ \quad + 1/3\,(\upsilon + \upsilon_0)^2], \; (|\upsilon| \leq \upsilon_0), \\ 0, \; (\upsilon \leq -\upsilon_0), \end{cases} \qquad (9a\text{–}c)$$

which coincides with (8) at $Z(0) = ½$ and is almost indistinguishable therefrom in the plot of Fig. 1. A correspondingly close approximation to (4) by (5) may be expected for any smoothly varying $\zeta(u_*)$.

4. Empirical fit to ζ

Janssen adopts the empirical fit (Snyder et al. 1981)

$$\zeta(u_*) = \begin{cases} A[C(u_*/c) - 1] \\ 0 \end{cases} \left(Cu_*/c \begin{matrix} > 1 \\ < 1 \end{matrix} \right), \qquad (10a,b)$$

where $A = 0.2$ and $C = 28$. Since (10) appears to have been derived from observations of ζ and u_* it presumably is a fit to $\overline{\zeta}(\overline{u}_*)$, rather than $\zeta(u_*)$; however, it provides an example of the importance of the smoothness of $\zeta(u_*)$ for the approximation (5).

The substitution of (10) into (4) yields Fig. 2.6 in Komen et al. (1994), in which the sharp corner of $\zeta(u_*)$ at $Cu_*/c = 1$ is replaced by a smooth curve between the upper and lower asymptotes $\overline{\zeta} \sim \zeta(\overline{u}_*)$ derived from (10a,b). The Gauss–Hermite approximation (5) replaces this curve by the segments of two straight lines and is less satisfactory than in the example of section 3, presumably in consequence of the discontinuity in slope of (10) at $\overline{u}_*/c = 1/C = 0.036$, where (10), (4), and (5) yield $\overline{\zeta} = 0$, $0.080\overline{u}_*/\sigma$ and $0.058\overline{u}_*/\sigma$, respectively.

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REFERENCES

Abramowitz, M., and I. A. Stegun, 1964: *Handbook of Mathematical Functions.* U.S. Natl. Bur. of Stand., 1046 pp.

Komen, G. J., and Coauthors, 1994: *Dynamics and Modelling of Ocean Waves.* Cambridge University Press.

Miles, J. W., 1957: On the generation of surface waves by shear flows. *J. Fluid Mech.,* **3,** 185–204.

Nikolayeva, Y. I., and L. S. Tsimring, 1986: Kinetic model of the wind generation of waves by a turbulent wind. *Izv. Atmos. Oceanic Phys.,* **22,** 102–107.

Snyder, R. L., F. W. Dobson, J. A. Elliott, and R. B. Long, 1981: Array measurements of atmospheric pressure fluctuations above surface gravity waves. *J. Fluid Mech.,* **102,** 1–59.