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Determination of the elastic constants of cereal grains in a uniaxial compression test

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Abstract. Stress-strain relationships of bulk of seeds were examined under conditions of uniaxial compression with additional measurement of lateral stress. Tests were performed on wheat, rye, barley, oats and rapeseed for five levels of moisture content in a range from 6 to 20%. A constitutive stress-strain relationship formulated for sand was found to describe adequately the 'en masse' mechanical behavior of grain. The elastic constants of the material (Young's modulus E and Poisson's ratio ν) were determined using the linear phase of the unloading of the sample. Values of the modulus of elasticity E of cereal grains were found in a range from 10.4 to 23.6 MPa and decreasing with an increase in moisture content. Values of E for rapeseed ranged from 9 to 6.6 MPa for moisture contents of 6 and 15%, respectively. Values of Poisson's ratio ν of cereal grains were found in a range from 0.22 (wheat - 10.8% of m.c.) to 0.15 (barley – 15.3% of m.c.), while in the case of rapeseed, values of ν ranged from 0.24 for 6% to 0.10 for 15% of m.c.

Keywords: grain, elastic parameters, uniaxial compression

INTRODUCTION

Cereal grain is a granular material and as such exhibits mechanical behavior different to liquids and solids. The mechanical properties of grain settling depend on the properties of the single grain, friction between particles, inter-particle contact geometry and load history. Grain as a material of biological origin reveals the strong dependence of its stress-strain behavior on the moisture content affecting the properties of the seed coat as well as of the endosperm of single grain. Properties of the material serve as parameters for engineers designing storage systems or processing plants.

Recent demands in the industry resulted in the revision of several silo design codes, which included standardization of methods for determining the mechanical properties of granular materials. European Standard Eurocode 1 (1996) and Polish Standard (Dyduch *et al*., 2000) recommends the determination of properties under load conditions similar to operating loads. Some properties are to be determined using the shear test (Jenike tester or triaxial compression test) others, using the uniaxial compression test. Cereal and rapeseed are also important raw materials for the food industry. Knowledge of their mechanical properties will improve the processes involving these materials within the food industry. Today, as in the past, adaptation of testing methods from soil mechanics to describe the mechanical behavior of other granular materials remains a valuable option.

In this paper, the stress-strain relationship proposed for sand by Sawicki (1994) was adopted to model the stress-strain relationships of cereal grain and rapeseed. A model curve was fitted to the experimental data from the uniaxial compression test during loading and unloading performed on material with several moisture content levels. Values of the most important mechanical constants characterizing the elastic deformation of grain settling under load (modulus of elasticity E and Poisson's ratio ν) were determined.

EQUIPMENT AND PROCEDURE

An experimental set for the uniaxial compression (or oedometric) test was constructed (Horabik and Molenda, 2000) (Fig. 1). The wall of the oedometer consisted of two semicircular halves cut along the axis. The two semicircular halves were connected with four load cells installed in pairs on the two connection lines, restoring the cylindrical shape of the wall. One half of the wall rested directly upon the base. The bottom and top plates of the chamber transmitted the

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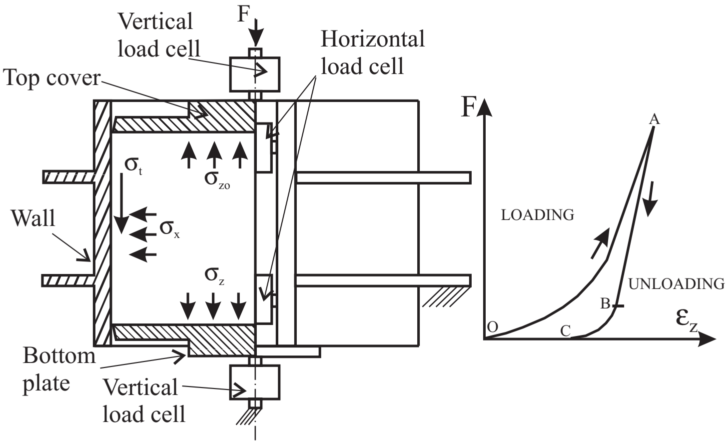

Fig. 1. Uniaxial compression test.

vertical load through the load cells. The experimental set allowed the mean lateral pressure σ_x, mean vertical pressure on the bottom σ_z, and the mean vertical pressure on the top plate σ_{z0} to be determined. The surface of the wall was smooth while the surfaces of the top and bottom plates were rough.

The granular material was poured into the test chamber, without vibration or other compacting action. The sample was 80 mm high and 21 cm in diameter. The settling was loaded to the reference vertical stress σ_{z0} of 100 kPa using a universal testing machine. The top cover of the apparatus descended with a constant speed of 0.35 mm min^{-1}, while the displacement was measured with an inductive sensor with an accuracy of 0.01 mm. Next, unloading took place with the same speed of deformation until to the 0 kPa of the stress level was reached. Tests were conducted on grains of wheat, barley, oats and rye for five levels of moisture content in a range from 10 to 20%, and on rapeseed having a moisture content ranging from 6 to 15%. Experiments were performed in three replications.

The model equation adopted after Sawicki (1994) is based on the elasto-plastic approach (see Fig. 1). During loading both reversible (elastic) and irreversible (plastic) strains develop in the sample. Plastic ε_z^p and elastic ε_z^e strains develop in the material in the phase of loading:

$$\varepsilon_z = \varepsilon_z^e + \varepsilon_z^p, \tag{1}$$

$$\varepsilon_z = D_1 \ln\left(1 + D_2 \sigma_{z0}^{\alpha}\right) + \frac{\sigma_{z0}}{E}\left(1 - \frac{2\nu^{*2}}{1-\nu^*}\right), \tag{2}$$

where: ε_z - total vertical strain; ε_z^p- plastic vertical strain; ε_z^e- elastic vertical strain; σ_{z0} - mean vertical pressure on the top cover; E - modulus of elasticity; ν^*- equivalent of Poisson's ratio for loading $\nu^* = K_0/(1+K_0)$; K_0 - slope of straight line $\sigma_x = K_0\sigma_z$; D_1, D_2, α - model parameters.

Constant K_0 upon which the equivalent of Poisson's ratio ν^* was estimated, is a ratio of horizontal stress σ_x and vertical stress σ_{z0} during consolidation of the sample. During this compression phase the horizontal deformation which is the sum of plastic and elastic horizontal strains is zero ($\varepsilon_x = \varepsilon_x^e + \varepsilon_x^p = 0$). D_1 and D_2 are compaction coefficients. Originally Sawicki (1994) applied the value of exponent α equal to 3/2 but in our examination, the value of α was treated as a variable parameter to obtain a better fit of experimental results to the model curve.

Two phases of the unloading can be observed (Fig. 1). The first phase is characterized by a purely elastic deformation and was used for determination of elastic constants, the modulus of elasticity E and Poisson's ratio ν. The second stage of unloading is characterized by both elastic and plastic deformations. It is assumed that the material reversible response is governed by Hook's law:

$$\varepsilon_x^e = \frac{1}{E}\left[(1-\nu)\,\sigma_x - \nu\,\sigma_{z0}\right], \tag{3}$$

$$\varepsilon_z^e = \frac{1}{E}\left[\,\sigma_{z0} - 2\nu\,\sigma_x\right]. \tag{4}$$

During the first phase of unloading (path AB) sample shows linear reaction which is characteristic for elastic deformation. Assuming that $\varepsilon_x = \varepsilon_x^e = \varepsilon_x^p = 0$ from Eq. (3) $\nu \dfrac{\sigma_x}{\sigma_{z0}} = \dfrac{\nu}{1-\nu}$ is obtained and applying assumption that $\varepsilon_z = \varepsilon_z^e$ to Eq. (4) ε_z may be expressed as below:

$$\varepsilon_z = \frac{\sigma_{z0}}{E}\left(1-\frac{2\nu^2}{1-\nu}\right). \tag{5}$$

Elastic constants were determined using experimental results from the linear phase of unloading. The ratio of horizontal stress σ_x to vertical stress σ_{z0} was assumed as constant (elastic state of stress) and the slope A of a straight line, $A = \sigma_x/\sigma_{z0} = \nu/(1-\nu)$, was estimated using the linear regression procedure applied to experimental values of stresses (see Fig. 3). Then, with A estimated, values of Poisson's ratio ν were calculated as:

$$\nu = \frac{A}{1+A}. \tag{6}$$

Values of modulus of elasticity E were estimated using relationship $\varepsilon_z(\sigma_{z0})$ (Eq. (5)) with experimental values of ε_z and σ_{z0}, and ν determined as described above.

Secondly – the nonlinear part of unloading starts when the vertical stress drops below the value of the horizontal stress. Elastic and plastic strains take place in the sample simultaneously. Sawicki (1994) assumed that in that phase of the unloading of the sample, the stress increments were equal, i.e., $d\sigma_x = d\sigma_z$. That means that the stress deviator remains constant. Vertical deformation is given as:

$$\varepsilon_z = D_3 \ln\left[1+D_4\left(\sigma_{z0}\right)^{\beta}\right]+D_5\sigma_{z0} \tag{7}$$

where: D_3, D_4, β - plastic parameters, D_5 - elastic parameter.

RESULTS

Figure 2 shows relationships of vertical stress σ_{z0} and total vertical strain ε_z for loading - unloading cycles for rapeseed with four levels of moisture content. The first part of the loading curve reflects consolidation of the sample with translation and rotation movements of particles, but without their deformations. Second, the steeper part of the curve shows an increase in the plastic and elastic stresses in the sample associated with deformation of particles. During this phase of loading, deformation takes place mainly in contact areas between grains. Variation of moisture content resulted in large differences between experimental stress – strain relationships as shown in Fig. 2, which were reflected in values of model parameters. Results of the estimation of elastic constants are presented in Table 1. Values of material constants were found dependent on species and on moisture contents of the material. For all samples tested modulus of elasticity E decreased with an increase in the content of the moisture in the seeds. The highest values of modulus of elasticity E, in a range from 15.1 to 23.6 MPa, were obtained in the case of rye while the lowest were values obtained for oats and barley. Modulus of the elasticity of oats ranged from 10.4 to 17.8 MPa and that of barley ranged from 10.4 to 14.2 MPa. The lowest of all were values of E obtained for rapeseed found in a range from 6.6 MPa for 15% moisture content to 9 MPa for 6% moisture content. Values for wheat found in a range from 11.1 to 22.4 MPa were lower than those of rye. Maximum normal pressure applied in reported tests (of 100 kPa) corresponds to pressure produced by a column of grain approximately 14 m high. For such a height of settling, Polish Standard - PN-89/B-03262 (1989) and the proposition of its amendments (Dyduch *et al.*, 2000) recommends applying modulus of elasticity for grain of 20 MPa, thus in good agreement with our results. Sawicki and Œwidziñski (1998) obtained E of wheat of 96 MPa. The reason for this difference is probably the distinctly higher maximum pressure of 800 kPa applied in the tests of these authors. The Sawicki and Œwidziñski sample (1998) was 40.5 mm high and 75.5 mm in diameter, thus distinctly smaller than ours, which may also contribute to the discrepancy observed in the test results. Values of modulus of elasicity for wheat reported by various authors ranged widely - from 0.7 MPa to 70 MPa (Thompson *et al.,* 1984; Zhang *et al.,* 1988). Such a large range in variation is probably a result of the differentiation in experimental methods, levels of pressures and the experimental material itself.

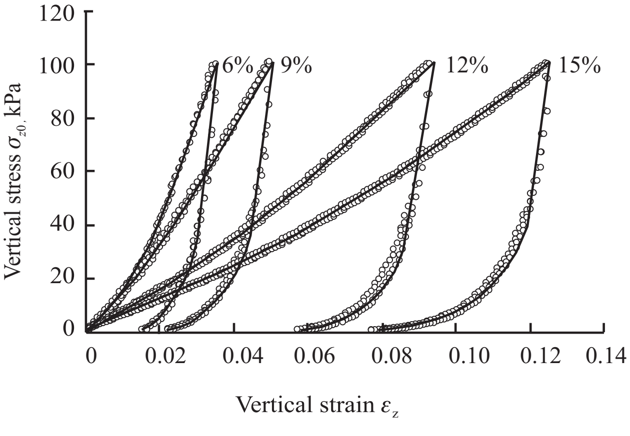

Fig. 2. Experimental data and fitted $\sigma_{z0}(\varepsilon_z)$ relationships for rapeseed of four levels of moisture content.

Maximum value of Poisson's ratio ν of 0.22 was obtained in the case of wheat of 10% of moisture content while a minimum of 0.15 was found for barley of 15% and for oats of 20%. Values of ν in the case of rapeseed were found in a range from 0.10 to 0.24 increasing with a decrease in moisture content from 15 to 6%. Sawicki and Œwidziñski (1998) found ν of 0.27 in the case of dry wheat. Zhang *et al.* (1988) reported Poisson's ratio of 0.29 for dry wheat. Polish Standard - PN-89/B-03262 (1989) recommends that value of Poisson's ratio for grains of 0.4 should be used for the

Table 1. Coefficients of determination and elastic parameters for cereal grains and rape seeds for first (linear) sector of unloading

Grain	Moisture content (%)	R^2	E (MPa)	ν
Barley	10	0.916	14.2	0.19
	12.5	0.854	14.0	0.16
	15	0.872	13.8	0.15
	17.5	0.866	12.3	0.17
	20	0.888	10.4	0.19
Oats	10	0.892	17.8	0.18
	12.5	0.891	16.0	0.20
	15	0.926	13.2	0.17
	17.5	0.855	10.7	0.17
	20	0.879	10.4	0.15
Wheat	10	0.818	22.4	0.22
	12.5	0.838	22.2	0.18
	15	0.824	19.3	0.20
	17.5	0.869	17.2	0.20
	20	0.918	11.1	0.19
Rye	10	0.887	23.6	0.19
	12.5	0.846	20.9	0.20
	15	0.875	20.2	0.21
	17.5	0.865	20.0	0.21
	20	0.886	15.1	0.21
Rapeseed	6	0.896	9.0	0.24
	9	0.865	8.7	0.17
	12	0.863	7.1	0.16
	15	0.868	6.6	0.10

design. The standard does not contain any information about the influence of moisture content on the two elastic constants of the material.

CONCLUSIONS

Uniaxial compression test was found useful for determination of elastic constants – modulus of elasticity E and Poisson's ratio ν of seeds of various moisture contents. Tests were performed with a maximum pressure of 100 kPa which is typical for many practical situations. Values of E were found in a range from 6.6 kPa to 23.6 kPa and increasing with an increase in the moisture content of seeds. Values of E reported here are in good agreement with those recommended by the Polish Standard as well as with those reported by various authors.

Values of elastic constants were strongly influenced by the method of determination, maximum pressure applied and the mode of consolidation of the sample.

Questions of determination of elastic constants of granular materials call for further investigation to find a standard procedure that would allow comparable results in various laboratories to be obtained.

REFERENCES

Dyduch M., Kamiński M., Łapko A., and Runkiewicz L., 2000. Amendment of Polish Standard PN-89/B-03262 'Concrete bins for storing granular materials and silags. Design rules' (in Polish). XI Conference 'Reinforced and prestressed concrete containers for liquids and granular materials', Świeradów Zdrój, 65-69.

Eurocode 1, **1996.** Basis of design and actions on structures - Part 4: Actions in silos and tanks. European Committee for Standarization. Central Secretariat: rue de Stassart 36, B-1050 Brussels.

Horabik J. and Molenda M., 2000. Device for determination of pressure ratio of granular materials (in Polish). Patent Application No. P-340014, Bulletin of Patent Office No. 22(700), A1(21) 340017.

PN-89/B-03262, **1989.** Polish Standard. Concrete bins for storing granular materials and silage. Design rules (in Polish).

Sawicki A., 1994. Elasto-plastic interpretation of oedometric test. Archives of Hydro-Engineering and Environmental Mechanics, 41 (1-2), 111-131.

Sawicki A. and Świdziński W., 1998. Elastic moduli of particulate materials. Powder Technology, 96, 24-32.

Thompson S.A. and Ross I.J., 1984. Thermal stresses in steel grain bins using the tangent modulus of grain. Transactions of the ASAE, (16)3, 165-168.

Zhang Q., Puri V.M., Manbeck H.B., and Wang M.C., 1988. Elastoplastic constitutive parameters of en masse wheat for a cyclic load model. Transactions of the ASAE, (31)3, 910-916.