Energy Diagnostics in a 1½-Layer, Nonisopycnic Model

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30 July 1996 and 9 December 1996

ABSTRACT

The development of a pointwise (in the horizontal) energy diagnostic scheme applicable to a 1½-layer, nonisopycnic, primitive equation model is presented. The scheme utilizes the concept of available gravitational energy to replace the conventional potential energy. This gives a total energy (kinetic plus potential) that is zero and a minimum with respect to a given reference state (a positive definite quantity) locally. Mean and eddy components of the kinetic and available gravitational energy forms are defined by introducing a thickness-weighted mean for velocity and density. Finally, mathematical formulations for the conversion terms, that is, those terms responsible for a reversible exchange of energy between the four energy compartments, are derived.

1. Introduction

We explore below a scheme whereby the traditional energy diagnosis (e.g., Holland and Lin 1975; Holland 1978; Bleck 1985; Pinardi and Robinson 1986) of ocean models can be applied to 1½-layer, nonisopycnic, primitive equation models (e.g., McCreary et al. 1989; Røed 1996). As such, the model is a simplified version of multilayer, nonisopycnic, models featuring variable layer densities (e.g., McCreary and Kundu 1988; Røed 1995). For the purpose of this note, we will consider the motion to be frictionless. Thus, the considered flow is, in essence, governed by the frictionless nonlinear shallow-water equations. This allows us to study the nature of the conservative conversion terms without having to consider conversion due to the nonconservative terms, that is, due to dissipation, diffusion, diapycnal mixing, and frictional drag.

As is common, the total energy is split into two parts to separate the energy contained in the motion, that is, kinetic energy, from the energy that can be converted into kinetic energy, that is, potential energy. We derive a potential energy that is zero and minimum with respect to a certain specified reference state such that it is positive everywhere for any deviation away from that state, that is, a locally positive definite quantity. That such a reference state exists is obvious since any fluid under gravity, and in the absence of any external forces and friction, strives toward an equilibrium state in which its center of gravity is at its minimum position, that is, a state in which there is no energy left for conversion into kinetic energy. The potential energy we derive, called the available gravitational energy [AGE: a name adopted from Pinardi and Robinson (1986) who used a similar potential energy in their study of energetics for open ocean quasigeostrophic models] bears a strong resemblance to the concept of available potential energy first introduced by Margules (see Gill 1982, p. 219) and is derived following the ideas of Holliday and McIntyre (1981) and Andrews (1981). Unlike the available potential energy, the AGE does not require that the fluid is contained within a closed system. Since AGE for a multilayer model is locally a positive definite quantity, it relaxes the containment requirement of the available potential energy concept and, hence, allows us to diagnose the energetics pointwise in the horizontal.

As has become common, we also derive the energetics associated with mean and eddy components of the two energy forms, where the mean kinetic and available gravitational energy is associated with the mean flow and the eddy kinetic and available gravitational energy is associated with the eddy motion. In this we have applied the traditional assumption that there is no exchange of mass between the mean and the eddy motion. This requires the introduction of two different averaging operators, as defined below.

We start by giving the equations of motion both for the total flow and the mean flow. Then we discuss the energy partition and the definition of the available gravitational energy, where the case of adjustment under gravity is used as an illustration. Finally, the development of mean and eddy energy components and their time rate of change are derived to reveal the various energy conversion terms.

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2. Equations of motions

The governing equations for the 1½-layer model neglecting diapycnal mixing, frictional, and driving forces are (e.g., McCreary et al. 1989; Røed 1996)

$$(h\mathbf{u})_t + \nabla\cdot\left(\frac{\mathsf{R}}{\rho_0}\right) + f\mathbf{k}\times h\mathbf{u} + \nabla\left(\frac{\varphi}{\rho_0}\right) = 0, \quad (1)$$

$$h_t + \nabla\cdot h\mathbf{u} = 0, \quad (2)$$

$$(hq)_t + \nabla\cdot\mathbf{P} = 0, \quad (3)$$

where

$$\mathsf{R} = \rho_0 h\mathbf{u}\mathbf{u}, \qquad \varphi = hp, \qquad \mathbf{P} = hq\mathbf{u},$$

$$q^2 = \frac{\rho_0 - \rho}{\rho_0}, \qquad p = \frac{1}{2}g\rho_0 q^2 h. \quad (4)$$

Here, h is the layer thickness, $\mathbf{u}$ the horizontal velocity, φ a measure of the pressure p, q a measure of the density contrast, ρ the density, and ρ_0 a reference density. Note that R is a tensor. In the remainder of this paper we will refer to q as "density" although it is actually a measure of the relative density contrast; that is, it is a dimensionless quantity.

To pave the way for a decomposition of the energy into mean and eddy components we now follow Bleck (1985) and define two averaging operators, one denoted by an overbar ($\overline{\psi}$) and a second denoted by a circumflex ($\hat{\psi}$). The first is an ordinary spatial or temporal averaging operator, while the second is a thickness weighted averaging operator; that is,

$$\hat{\psi} = \frac{\overline{h\psi}}{\overline{h}}. \quad (5)$$

We also assume that the averaging operators are interchangeable; that is,

$$\overline{A\hat{\psi}} = \overline{A}\hat{\psi}. \quad (6)$$

Then defining

$$h = \overline{h} + h'', \qquad \mathbf{u} = \hat{\mathbf{u}} + \mathbf{u}', \qquad q = \hat{q} + q', \quad (7)$$

it follows that

$$\overline{\varphi} = \varphi_M + \varphi_E, \qquad \varphi_M = \frac{1}{2}g\rho_0\hat{q}^2\overline{h}^2,$$

$$\varphi_E = \frac{1}{2}g\rho_0(\overline{hqh''}\hat{q} + \overline{hqhq'}),$$

$$\overline{\mathsf{R}} = \rho_0\overline{h\mathbf{u}\mathbf{u}} = \mathsf{R}_M + \mathsf{R}_E, \qquad \mathsf{R}_M = \rho_0\overline{h}\hat{\mathbf{u}}\hat{\mathbf{u}},$$

$$\mathsf{R}_E = \rho_0\overline{h\mathbf{u}'\mathbf{u}'}, \qquad \overline{\mathbf{P}} = \overline{hq\mathbf{u}} = \mathbf{P}_M + \mathbf{P}_E,$$

$$\mathbf{P}_M = \overline{h}\hat{q}\hat{\mathbf{u}}, \qquad \mathbf{P}_E = \overline{hq'\mathbf{u}'}. \quad (8)$$

Applying the ordinary averaging operator to (1)–(3) and making use of (5), (6), and (8) gives equations of motion governing the mean variables:

$$(\overline{h}\hat{\mathbf{u}})_t + \nabla\cdot\left(\frac{\mathsf{R}_M}{\rho_0}\right) + f\mathbf{k}\times\overline{h}\hat{\mathbf{u}} + \nabla\left(\frac{\varphi_M}{\rho_0}\right)$$

$$= -\nabla\left(\frac{\varphi_E}{\rho_0}\right) - \nabla\cdot\left(\frac{\mathsf{R}_E}{\rho_0}\right), \quad (9)$$

$$\overline{h}_t + \nabla\cdot\overline{h}\hat{\mathbf{u}} = 0, \quad (10)$$

$$(\overline{h}\hat{q})_t + \nabla\cdot\mathbf{P}_M = -\nabla\cdot\mathbf{P}_E. \quad (11)$$

As noted in the introductory section, there is no interaction term on the right-hand side of (10). This is a direct implication of the use of the thickness weighted averaging operator (5) to define the mean current $\hat{\mathbf{u}}$. The eddy motion does, however, interact with the mean motion through the terms on the right-hand sides of the remaining equations. As revealed by (9)–(11), the equations of motion for the mean flow have the appearance of being the equations of motion for a hypothetical fluid of thickness $\overline{h}$, velocity $\hat{\mathbf{u}}$, and density $\hat{q}$, with source terms determined by the eddy flow.

3. Energy partition

We start by specifying a reference state as being one at rest with constant layer thickness and layer density; that is,

$$h = h_r, \quad q = q_r. \quad (12)$$

The kinetic energy of the upper layer follows by integrating the kinetic energy vertically over the upper layer assuming that the velocity is depth independent; that is,

$$K = \frac{1}{2}\rho_0 h\mathbf{u}^2. \quad (13)$$

Note that the kinetic energy is a positive definite quantity; that is, it is always positive and attains its minimum (zero) value in the reference state. A development of the conventional potential energy, here denoted by Π, by a similar vertical integration gives (e.g., O'Brien 1967; McCreary and Yu 1992)

$$\prod = \frac{1}{2}g\rho_0 q^2 h^2, \quad (14)$$

a quantity that is indeed positive but is not a positive definite quantity in the above sense. To achieve the latter we need to derive an expression that is different, yet similar.

We propose to express the potential energy in the form

$$\Phi = \frac{1}{2}g\rho_0(qh - q_r h_r)^2 \quad (15)$$

and will refer to it as being the available gravitational energy. Note that AGE is strictly nonnegative, is zero (minimum) in the reference state, and has the dimension of energy. Moreover, with AGE defined as potential energy, the energy equation, as shown by (26) below, takes on a conservative form consistent with conventional theory. Note that (15) includes the possibility of a variable upper-layer density and thus energy can be stored in the form of lateral density deviations as well as upper-layer thickness deviations.

For illustration purposes let us consider the problem of adjustment under gravity of a rotational fluid (Rossby 1937, 1938; Gill 1976, 1982). We first assume that the density is constant (i.e., $q = q_r$) and that the initial state is at rest with an unbalanced (dynamically) upper-layer thickness that has a positive deviation Δh away from the equilibrium depth h_r for $x < 0$ and an equal but negative deviation for $x \geq 0$ (e.g., Gill 1976). The initial AGE is then equal to the initial energy and is everywhere (that is, for all x) given by the positive value

$$E = \frac{1}{2} g\rho_0 q_r^2 \Delta h^2. \qquad (16)$$

We may alternatively assume that the upper-layer thickness is constant and equal to the thickness of the reference state and let the initial density be unbalanced; that is, let the density deviate positively away from the reference density for $x < 0$ and negatively for $x \geq 0$. Let Δq denote the density deviation. Then the initial AGE is again positive and is everywhere given by the expression

$$E = \frac{1}{2} g\rho_0 h_r^2 \Delta q^2. \qquad (17)$$

It is also possible to let both the density and the thickness deviate initially, as for instance described above. In that case, the initial energy becomes

$$E = \frac{1}{2} g\rho_0 \begin{cases} (h_r\Delta q + q_r\Delta h + \Delta q\Delta h)^2, & x < 0 \\ (h_r\Delta q + q_r\Delta h - \Delta q\Delta h)^2, & x \geq 0, \end{cases} \qquad (18)$$

which, although different for $x < 0$ and $x \geq 0$, is still positive everywhere.

Under any of these circumstances a motion will ensue for $t > 0$ in which this initial AGE is converted into kinetic energy. Due to the positive definite character of the AGE the energy (in an integrated sense) remains constant for the ensuing frictionless flow and, hence, there is a maximum amount of kinetic energy that can be obtained by conversion of AGE. In turn, this imparts an upper limit on the possible attainable currents. We will return to this point once we have derived the energy conservation equation, that is, at the end of section 4.

4. Energy equations

The time rate of change of the two energy forms may now be developed based on the equations of motion. Taking the time derivative of K and Φ, and substituting the various terms from the equations of motion (1)–(3), gives

$$K_t + \nabla \cdot K\mathbf{u} = C, \qquad (19)$$

$$\Phi_t + \nabla \cdot \Phi\mathbf{u} + \nabla \cdot (\varphi - \varphi_r)\mathbf{u} = -C, \qquad (20)$$

where

$$C = -\mathbf{u} \cdot \nabla\varphi \qquad (21)$$

is a conversion term converting energy between the two energy forms and

$$\varphi_r = \frac{1}{2} g\rho_0 q_r^2 h_r^2 \qquad (22)$$

is a measure of the pressure in the reference state. Note that the third term on the left-hand side of (20) is the work done by the pressure excess, rather than the work done by the pressure as in conventional derivations of this equation. The energy equation now follows by adding (19) and (20); that is,

$$E_t + \nabla \cdot (E + \varphi - \varphi_r)\mathbf{u} = 0, \qquad (23)$$

where $E = K + \Phi$ is the energy.

At this stage it is important to note that there is an inherent ambiguity in the choice of the conversion term. This stems from the fact that the energy equation (23) does not provide us with any insight into how to split up the pressure excess work between the two energy parts. The choice underlying (19) and (20) gives a straightforward physical interpretation of the conversion term as being the advective pressure work. The other obvious choice, a choice commonly found in textbooks, would be to associate the pressure excess work with (19) in which case the conversion term becomes

$$C^* = (\varphi - \varphi_r)\nabla \cdot \mathbf{u}. \qquad (24)$$

With this, a conversion between potential and kinetic energy is measured by the divergence of the flow field (or the compressibility in a compressible fluid).

We now introduce the energy density e as the sum of the kinetic energy density e_k and the available gravitational energy density e_p, defined by

$$e_k = \frac{1}{2}\mathbf{u}^2, \qquad e_p = \frac{1}{2} g \frac{(qh - q_r h_r)^2}{h}. \qquad (25)$$

This allows us to rewrite the energy equation as

$$(he)_t + \nabla \cdot hB\mathbf{u} = 0, \qquad (26)$$

where

$$B = e + \frac{\varphi - \varphi_r}{\rho_0 h} \qquad (27)$$

is the Bernoulli function. As is well known, there is a strong resemblance between the motion of a compressible gas and a fluid motion governed by the shallow-water equations in that the thickness assumes the role of density. Indeed, in comparison with compressible gas theory, in which the Bernoulli function is defined as $e_k + e_p + (\pi/\mu)$ where π is pressure and μ is density, we may conclude that in a 1½-layer, nonisopycnic, model $\rho_0 h$ assumes the role of the density, while $\varphi - \varphi_r$ assumes the role of pressure. It is interesting to note that the latter bears a strong resemblance to the pressure excess in Holliday and McIntyre (1981) and Andrews (1981). Thus, the introduction of AGE as potential energy leads to a form of the energy equation for the 1½-layer nonisopycnic model consistent with conventional theory and in which the particular form of the Bernoulli function in (27) is a direct implication of the introduction of the AGE to replace the conventional expression for the potential energy.

We may now return to the Rossby adjustment problem. As is well known, the final adjusted state is stationary and consists of a geostrophic jet confined to the region near the initial thickness and/or density jump. Hence, the energy flux $\nabla \cdot hB\mathbf{u}$ is zero everywhere in the final state. During the adjustment phase the energy flux is, however, nonzero and is responsible for the radiation of energy away from the initial density and/or upper-layer thickness jump. Thus, at a particular location in space the sum of the kinetic energy and the AGE does not remain constant, but changes in response to the energy flux. This explains why not all of the initial AGE is converted into kinetic energy locally. If the fluid is assumed to be contained within solid walls, an integration of the energy equation reveals that the integrated sum of the kinetic and the available gravitational energy remains constant since then the integrated flux term vanishes. In this case the energy radiated by the waves cannot escape and all of the initial AGE remains in the closed system as available potential energy. This shows that the AGE is strongly related to the available potential energy concept.

5. Energy decomposition

Applying the conventional averaging operator to the kinetic energy (13), making use of (6) and (7), it follows that

$$\overline{K} = \frac{1}{2}\rho_0\overline{h\mathbf{u}^2} = \frac{1}{2}\rho_0\overline{h}\hat{\mathbf{u}}^2 + \frac{1}{2}\rho_0\overline{h\mathbf{u}'^2}, \qquad (28)$$

which shows that the average kinetic energy naturally decomposes into a component

$$K_M = \frac{1}{2}\rho_0\overline{h}\hat{\mathbf{u}}^2 \qquad (29)$$

associated with the kinetic energy of a hypothetical fluid of layer thickness $\overline{h}$, velocity $\hat{\mathbf{u}}$, and density $\hat{q}$ (henceforth *mean kinetic energy*), and a component

$$K_E = \frac{1}{2}\rho_0\overline{h\mathbf{u}'^2} \qquad (30)$$

associated with the eddy motion (henceforth *eddy kinetic energy*). The same procedure may be followed for the potential energy to give

$$\overline{\Phi} = \Phi_M + \Phi_E, \qquad \Phi_M = \frac{1}{2}g\rho_0(\hat{q}\overline{h} - q_r h_r)^2,$$

$$\Phi_E = \frac{1}{2}g\rho_0(\overline{hqh''}\hat{q} + \overline{hqhq'}), \qquad (31)$$

where Φ_M will be denoted the *mean AGE* and Φ_E the *eddy AGE*. Due to the variable density the eddy AGE has two parts, one associated with the deviations in the upper-layer thickness variations and a second associated with deviations in the upper-layer density.

Following the procedure used to derive (19) and (20), the time rate of change of the mean kinetic energy and mean AGE may be developed based on the equations of motion for the mean variables, that is, (9)–(11). Thus,

$$K_{M_t} + \nabla \cdot K_M\hat{\mathbf{u}} = C_M^M + C_M^E + C_{ME}, \qquad (32)$$

$$\Phi_{M_t} + \nabla \cdot \Phi_M\hat{\mathbf{u}} + \nabla(\varphi_M - \varphi_r)\hat{\mathbf{u}} = -C_M^M + C^{ME}, \qquad (33)$$

where

$$C_M^M = -\hat{\mathbf{u}} \cdot \nabla\varphi_M, \qquad C_M^E = -\hat{\mathbf{u}} \cdot \nabla\varphi_E,$$

$$C_{ME} = -\hat{\mathbf{u}} \cdot \nabla \cdot \mathbf{R}_E,$$

$$C^{ME} = -g\rho_0(\hat{q}\overline{h} - q_r h_r)\nabla \cdot \mathbf{P}_E. \qquad (34)$$

Adding (32) and (33) gives

$$E_{M_t} + \nabla \cdot (E_M + \varphi_M - \varphi_r)\hat{\mathbf{u}} = C_M^E + C_{ME} + C^{ME}, \qquad (35)$$

which then is the energy equation governing the mean motion, that is, the energy equation governing the motion of the hypothetical fluid. The three terms on the right-hand side of (35) represent source terms that are responsible for reversible (or conservative) conversions between the mean and the eddy energy parts. The first term is associated with conversion between mean kinetic energy and eddy available gravitational energy. The second is associated with conversion between the mean and eddy kinetic energy, whereas the last is associated with conversion between the mean and eddy available gravitational energy. Here C_M^M, which appears in both (32) and (33) with opposite sign, is the mean counterpart to C in (21). As discussed in the previous section, the choice of these terms is ambiguous. Thus, care must be exercised when interpreting these terms in isolation. Adding and subtracting an arbitrary term on the right-hand side of (35) may alter the meaning of the individual terms, but not their collective meaning. It should be noted that this result is independent of the expression chosen to represent potential energy. For instance, Plumb (1983) made a similar remark in his criticism of applying energy diagnostics to atmospheric models where a conventional expression for the potential energy was used.

Applying the conventional averaging operator to the energy equations (19) and (20), and subtracting, respectively, (32) and (33), gives equations for the time rate of change of the eddy energy parts; that is,

$$K_{E_t} + \nabla \cdot \mathbf{F}_{KE} = C_E^E - C_{ME}, \qquad (36)$$

$$\varphi_{E_t} + \nabla\cdot\left(\mathbf{F}_{\Phi E} + \frac{1}{2}g\rho_0\overline{(hqh''\hat{q} + hqhq')\mathbf{u}'}\right) = -C^E_E - C^E_M - C^{ME}, \quad (37)$$

where

$$C^E_E = -\overline{\mathbf{u}'\cdot\nabla\varphi}, \qquad \mathbf{F}_{KE} = \frac{1}{2}\overline{h\mathbf{u}'^2\mathbf{u}} + \hat{\mathbf{u}}\cdot\mathbf{R}_E,$$

$$\mathbf{F}_{\Phi E} = \frac{1}{2}g\rho_0\overline{(hqh''\hat{q} + hqhq')\mathbf{u}} + g\rho_0(\hat{q}\overline{h} - q_r h_r)\mathbf{P}_E. \quad (38)$$

The C^E_E term represents a conversion between the eddy kinetic and available gravitational energy and is the eddy counterpart of C^M_M. Note that $\mathbf{F}_{\Phi E}$ and $\mathbf{F}_{KE}$ are constructed in exactly the same manner, that is, as the average of those terms defining the eddy kinetic and available gravitational energy components, respectively, times the velocity. Thus, by adding the two eddy components we obtain

$$E_{E_t} + \nabla\cdot\left(\mathbf{F}_E + \frac{1}{2}g\rho_0\overline{(hqh''\hat{q} + hqhq')\mathbf{u}'}\right) = -C^E_M - C_{ME} - C^{ME}, \quad (39)$$

where $\mathbf{F}_E = \mathbf{F}_{KE} + \mathbf{F}_{\Phi E}$. Note that the source terms on the right-hand side of (39) are exactly those that appear as source terms in the equation governing the mean energy (35), but with opposite signs. As is obvious, the second gradient vector term on the left-hand side of (39) represents the eddy counterpart to the pressure excess work, that is, $\nabla\cdot(\varphi - \varphi_r)\mathbf{u}$, and hence may be denoted the eddy pressure excess work. Due to the inclusion of a possible lateral density variability also the eddy pressure excess has two terms associated with the density and thickness variability, respectively.

6. Concluding remarks

The energy diagnosis above has been developed to a stage where it is useful for application to a 1½-layer model. In particular, the introduction of the available gravitational energy concept, inspired by the works of Holliday and McIntyre (1981) and Andrews (1981), is shown to be useful. It allows us to define the energy in terms of a locally quadratic invariant and positive definite quantity, which bears a strong resemblance to the available potential energy concept (see Gill 1982, 219). However, in contrast to the available potential energy, it allows us to undertake a pointwise (in the horizontal) energy diagnosis, that is, relaxes the confinement requirement associated with the available potential energy concept. Thus, if the motion is frictionless, the time rate of change of the sum of the kinetic and available gravitational energy is balanced locally by a well-defined energy flux. The kinetic energy that can then be obtained by conversion of AGE at any location is equal to the initial available gravitational energy at that location plus the flux of energy toward that location.

The obvious next step is to extend the ideas above to yield useful mathematical expressions for multilayer models, both reduced gravity and finite depth, featuring a laterally variable density in all layers, and to employ the method to cases involving realistic mesoscale features as, for instance, those of McCreary and Yu (1992) and Røed (1996). Such work is underway and will be reported elsewhere.

Acknowledgments. I thank Jim O'Brien for his hospitality during my visit to the Center for Ocean–Atmosphere Prediction Studies, The Florida State University, at which time this note was begun. I also thank Bruce Hackett for painstakingly going through an earlier version of the manuscript and two anonymous reviewers for constructive criticisms. This research was supported by the Research Council of Norway through Contracts 101528/720, 109227/110, and 107613/410, the Nordic Council of Ministers through Contract 1.7.07, and the Norwegian Pollution Control Authority through Contract 95332.

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