INTERACTION OF INDIVIDUAL SURFACE MICRORIDGES IN THE COURSE OF FRICTION BETWEEN CONSTRUCTION WALLS AND PLANT MATERIAL*

J. Frączek, J. Kaczorowski, Z. Ślipek

Faculty of Engineering and Energetics in Agriculture, University of Agriculture
Balicka 104, 30-149 Cracow, Poland

A b s t r a c t. The paper presents the analysis of a typical example of plastic micro-contact in the course of mutual slide of processed vegetative material and construction surface. One of the theories of border load capacity was applied; i.e., the method of integration along-side the feature curves. In effect, the border load capacity of single surface microridges was defined, in the course of mutual friction between the construction and processed vegetative material.

K e y w o r d s: friction, single surface microridge

LIST OF SYMBOLS

v	– shift velocity,
v_x	– horizontal component of shift velocity,
P_x, P_y	– components of the force causing the wedge shift,
α, β, λ	– angles in Prandtl fan,
$\sigma < \theta < \lambda$	– slope of the active surface of the wedge in relation to the plastic medium,
$\tau = m\,k$	– the tangential strain at the contact surface,
$m \in [0, 1]$	– roughness coefficient,
k	– acceptable strain at simple cutting,
h	– plasticity depth of plant material,
N, T	– normal force and friction force,
μ	– friction coefficient,
$\xi = P_x / P_y$	– minimised coefficient.

FORMULATION OF RESEARCH PROBLEM

Current theories of friction, adhesion and molecular-mechanic mechanism were developed almost fifty years ago. Both fundamental theories in tribology have evolved substantially since. However, the amendments did not concern the physical interaction models of bodies during friction.

The physical model of kinetic friction based on the mechanical-molecular friction theory [5], consists in the mechanism by which the upper section of microridges in the harder substance (nonplastic one) penetrates the smooth surface of the softer body; the movement resistance results from the resisting force of elastic or plastic deformations and the molecular displacement resistance. During the course of kinetic friction between solid bodies the surface interface occurs mainly between the top sections of surface microridges in these bodies, while the movement resistance results from the simultaneous resisting force of elastic and plastic deformations. The closer analysis of the phenomenon of plant material shift along the construction material directly related to external friction provided the way to optimise technological processes applied in agriculture.

When interaction of individual surface microridges during friction between construction walls and plant material (the shifting of plant material over the construction walls) is treated as conforming to the scheme shown in Fig. 1 with certain simplifying assumptions, it is possible to consider the problem of shifting the rigid spike (the top section of microridge) loaded with vertical force P_y in relation to the plastic half-surface with a given velocity with a horizontal component v_x. It is assumed that the force P_y has a predefined constant value while the wedge horizontal shift is enforced kinematically. The stationary process is analysed assuming that vertical components of wedge velocity $v_y = 0$. The initial phase of undetermined floating is neglected in our study. It is assumed that the angle θ - (slope of the active surface of the wedge in relation to the plastic medium) is smaller than $\Pi/2$.

Such a formulation of the problem is useful because when the surface of the harder body in friction interface (construction walls) is shown as a series of wedge-shaped microridges (Fig. 2), it is possible to describe and interpret the phenomenon of friction with plant substances (ductile in character). For example, when the theoretical solution is known for one wedge -shaped section, it is possible to define, theoretically, the relation of friction coefficient μ that depends on the shape of microridge and adhesion forces at the interface.

In the present study, it is assumed that the substance of construction wall is isotropic, homogenous, non-compressible, rigid-plastic,
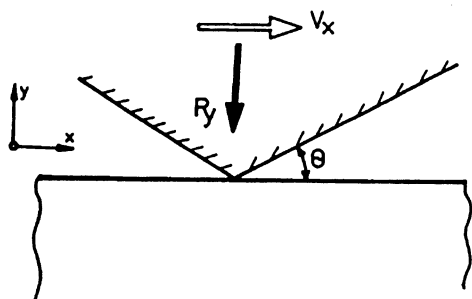
with no reinforcement. It is also assumed that the length of spike l measured perpendicularly to the surface (x,y) is sufficiently large to provoke the level deformation condition. The introduction of such assumptions allows to use the slip-line analysis [3,6]. Of course, it is necessary to accomodate all other assumptions recognised in the classic theory of plastic flow. Since the assumed model of the substance does not depend on the adopted time-scale, the solution is independent from the value of v_x velocity. The force value P_y has no impact upon the solution form; any dimension of linear solution remains in direct proportion to the value of P_y. Thus the border conditions of the studied problem are defined by the specification of two parameters: the angle θ of the inclination of impact section of the spike in relation to the medium surface and the roughness coefficient at the spike surface. For the sake of simplification, it is assumed here that the friction term at the spike surface is $|\tau| \leq mk$ where τ means the static tension affecting this surface, and $m \in [0,1]$ is the roughness coefficient. It is assumed that the slide along the spike surface is impossible, when $|\tau| = mk$, or when the normal stress upon the spike surface is null.

The characteristic feature of the studied problem that significantly prevents any straightforward analysis, is the fact that it is impossible to establish the position of the free surface at the stage of predefined flow beforehand. This category of problems may not have only one solution. Definite values of θ, m parameters,
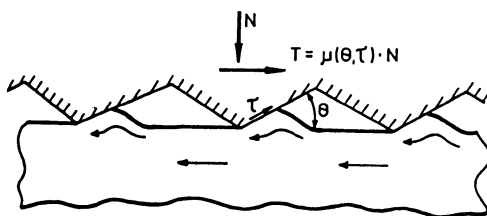

Fig. 1. Diagram of interaction between individual surface microridges during the friction of construction walls and plant material.

Fig. 2. Diagram of plastic micro-contact occurring in mutual sliding of plant material and construction walls.

i.e., definite border conditions, are usually related to the infinite class of various solutions that meet all the conditions for strain and deformation speed. The present study provides a solution scheme which is correct only for $\theta <$ 1/2 arc cos m, and describes the process related to plasticization of the outer layer of vegetative material to the depth of $h > 0$.

SOLUTION DIAGRAM

The suggested solution is complete when it meets all the conditions for strain and deformation speeds. The positive condition for dissipated power can be tested on the basis of Green's method [2] or Ford's method [1]. The diagram of slip-lines network is shown in Fig. 3. The elements of the free surface before and after deformation are at the same level. The adoption of this pattern of slip-lines network illustrates the shift (friction) process of vegetative substance and construction walls best. The adoption of other patterns (network diagrams) would reflect the processes of burnishing, cutting or shearing the outer layer of vegatative substance. According to the adopted condition of friction, the static strains τ upon the interface surface are:

$$\tau = k \cos 2\lambda = m\,k \qquad (1)$$

and thus:

$$\lambda = 1/2 \text{ arc cos } m. \qquad (2)$$

Construction that solves this problem is possible for $0 \le \theta < \lambda$. For any adopted value of α angle, the value of β angle must be selected in the following way: at any given value of θ angle, the points B and I must lie at the same level. It can be seen that for the same border conditions defined by m and θ, an infinite class of solutions is obtained; each of them contains different values of α and β angles. The question arises which of these solutions has physical meaning. This problem has yet not been solved. The „minimum force hypothesis" could be adopted at this point. Such a procedure follows engineering intuition but it is not sufficiently proved scientifically. According to this hypothesis, the correct solution is the one related to the minimum force necessary to implement the kinematically enforced process. In the phenomenon studied here, minimalization criterion should be applied to the value of horizontal force P_x necessary to shift the spike (wedge) loaded with vertical force P_y of definite value. In other words, it is necessary to minimalize the coefficient $\zeta = P_x / P_y$. However, the solution chosen in this way must meet the condition of not overloading the corners of the rigid areas. Hill's criteria [4], that takes into consideration also Hencky's rule [4,6] lead to the following inequalities limiting the value of β angle:

$$\beta \le \frac{1}{2}\delta = \frac{1}{2}\left(\frac{1}{4}\Pi + \lambda - \theta - \alpha \right)$$

for the B corner, $\qquad (3)$

$$\beta \le \frac{1}{2}\left(\frac{3}{4}\Pi - \lambda + \theta - \alpha \right)$$

for the I corner. $\qquad (4)$

Fig. 3. Diagram of slip-line network and speed hodograph.

Figure 3 shows diagramatically distribution of strains at the surface of the wedge (spike).

On the basis of the relations proposed by Hencky it is possible to find the value of strain p in any point of the slip-lines network. Thus :

$$p_A = p_E = k (1 + 2\alpha)$$
$$p_I = k (1 + 2\alpha + 4\beta). \qquad (5)$$

Therefore the normal strain σ_n along AE amounts to:

$$\sigma_{n\,AE} = k (1 + 2\alpha + \sin 2\lambda) \qquad (6)$$

and the maximum one equals :

$$\sigma_{n\,max} = k (1 + 2\alpha + 4\beta + \sin 2\lambda) \qquad (7)$$

obtained at the point I. In this way, the border load capacity was defined for the interaction of individual microridges on the surface during friction.

The BFGHI line (Fig. 3) is the discontinuity line of speed; in the *abc* and *eda* areas, the deformation does not occur. Since B and I points
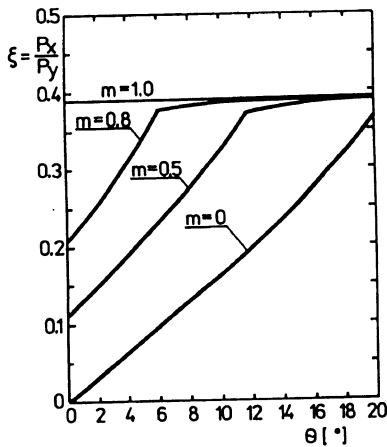
Fig. 4. Relation $\xi = P_x/P_y$ as a function of θ for various values of m.
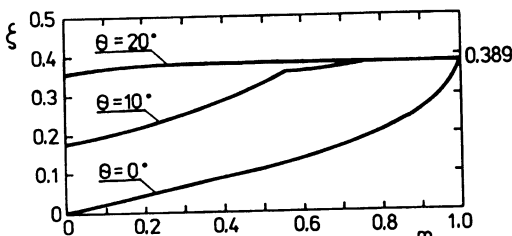

Fig. 5. Relation $\xi = P_x/P_y$ as a function of m for various values of θ.

lie at the same level the condition of non-compression results in the situation when the normal component of speed vector is zero along AB. Therefore, in fact the equation describes only the stationary condition. Only the h deep layer of material takes part in the deformation. The remaing part of the medium is rigid.

CONCLUSION

As a result of the unevenness of the meeting surfaces of plant material and construction walls, the actual contact consists of a large number of micro-contacts, the joint surface of which is but a small fraction of the nominal interface area.

The studied problem may be treated as one of the typical cases of plastic micro-contact occuring in mutual sliding of vegatative material and construction walls. Since one of these substances is significantly harder than the other, and its ridges may be considered as nondeforming. The presented solution allows for the theoretical determination of relation between friction coefficient and the angle θ of the ridge surface inclination and the roughness coefficient m. Figures 4 and 5 illustrate this relation in the interval $0 \le \theta \le 20°$. For $\theta > 20°$, other deformation mechanisms occur. The given numerical values must be treated as tentative, probably due to strong influence of simplifying assumptions. It should be remembered that the presented solution does not reflect the real properties of materials, and in particular, neglects the properties reinforcement and elasticity.

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