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# OSCILLATION CRITERIA FOR SYSTEMS OF NEUTRAL HYPERBOLIC DIFFERENTIAL EQUATIONS WITH CONTINUOUS DISTRIBUTED DEVIATING ARGUMENTS

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**Abstract**

Sufficient conditions are established for oscillation of systems of neutral hyperbolic differential equations with continuous distributed deviating arguments. The approach used is to reduce the multi-dimensional oscillation problems to one-dimensional problems for functional differential inequalities. The main results are illustrated by two examples.

**AMS Subject Classification:** 35B05; 35R10.

**Keywords**: Oscillation, system, hyperbolic differential equation, continuous distributed deviating argument, neutral type.

## 1 Introduction

In 2000, Li and Cui[1] studied the oscillation of the following systems of neutral hyperbolic differential equations

$$\frac{\partial^2}{\partial t^2}[u_i(x,t)+\mu(t)u_i(x,t-\rho)] = a_i(t)\Delta u_i(x,t) + \sum_{k=1}^{m}\sum_{j=1}^{d} a_{ikj}(t)\Delta u_k(x,t-\tau_j)$$

$$-p_i(x,t)u_i(x,t) - \sum_{k=1}^{m}\sum_{h=1}^{l}\int_a^b q_{ikh}(x,t,\xi)u_k(x,g_h(t,\xi))d\sigma(\xi), \qquad (E)$$

$$(x,t)\in\Omega\times[0,\infty)\equiv G, i=1,2,\ldots,m.$$

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In this paper, we study the oscillation of systems of neutral hyperbolic differential equations with continuous distributed deviating arguments of the form

$$\frac{\partial^2}{\partial t^2}\left(u_i(x,t)+\sum_{r=1}^{s}\mu_r(t)u_i(x,\rho_r(t))\right)=a_i(t)\Delta u_i(x,t)+\sum_{k=1}^{m}\sum_{j=1}^{d}a_{ikj}(t)\Delta u_k(x,\tau_j(t))$$

$$-p_i(x,t)u_i(x,t)-\sum_{k=1}^{m}\sum_{h=1}^{l}\int_a^b q_{ikh}(x,t,\xi)u_k(x,g_h(t,\xi))d\sigma(\xi), \qquad (1.1)$$

$$(x,t)\in\Omega\times[0,\infty)\equiv G, i=1,2,\ldots,m.$$

In (1.1) and $(E)$, $\Omega$ is a bounded domain in $R^n$ with a piecewise smooth boundary $\partial\Omega$, $\Delta u_i(x,t)=\sum_{r=1}^{n}\dfrac{\partial^2 u_i(x,t)}{\partial x_r^2}, i=1,2,\ldots,m$, and the integral is Stieltjes integral.

It is obvious that system $(E)$ is a particular case of system (1.1). Therefore, our work extends the results of [1].

Throughout this paper, suppose that the following conditions hold:

**(A1)** $\mu_r\in C^2([0,\infty);[0,\infty))$, $a_i\in C([0,\infty);[0,\infty)), a_{ikj}\in C([0,\infty);R), a_{iij}(t)>0$, and

$$A_j(t)=\min_{1\le i\le m}\left\{a_{iij}(t)-\sum_{k=1,k\neq i}^{m}\mid a_{kij}(t)\mid\right\}>0,$$

$$i=1,2,\ldots,m; j=1,2,\ldots,d; r=1,2,\cdots,s;$$

**(A2)** $\rho_r,\tau_j\in C([0,\infty);R),\rho_r(t)\le t,\tau_j(t)\le t$, and $\lim_{t\to\infty}\rho_r(t)=\lim_{t\to\infty}\tau_j(t)=\infty, r=1,2,\cdots,s; j=1,2,\ldots,d;$

**(A3)** $p_i\in C(\overline{G};[0,\infty))$, $p_i(t)=\min_{x\in\overline{\Omega}}p_i(x,t)$, $p(t)=\min_{1\le i\le m}\{p_i(t)\}$, $i=1,2,\ldots,m;$

**(A4)** $q_{ikh}\in C(\overline{G}\times[a,b];R), q_{iih}(x,t,\xi)>0$, and

$$q_{iih}(t,\xi)=\min_{x\in\overline{\Omega}}q_{iih}(x,t,\xi),\overline{q}_{ikh}(t,\xi)=\max_{x\in\overline{\Omega}}\mid q_{ikh}(x,t,\xi)\mid,$$

$$Q_h(t,\xi)=\min_{1\le i\le m}\left\{q_{iih}(t,\xi)-\sum_{k=1,k\neq i}^{m}\overline{q}_{kih}(t,\xi)\right\}\ge 0,$$

$$i=1,2,\ldots,m; k=1,2,\ldots,m; h=1,2,\ldots,l;$$

**(A5)** $g_h\in C([0,\infty)\times[a,b];R), g_h(t,\xi)\le t,\xi\in[a,b]$, and $g_h(t,\xi)$ is a nondecreasing function with respect to $t$ and $\xi$, respectively,

$$\lim_{t\to\infty}\min_{\xi\in[a,b]}\{g_h(t,\xi)\}=\infty, h=1,2,\ldots,l;$$

**(A6)** $\sigma\in C([a,b];R)$ and $\sigma(\xi)$ is nondecreasing in $\xi$.

Consider the following boundary conditions:

$$\frac{\partial u_i(x,t)}{\partial N} + f_i(x,t)u_i(x,t) = 0, (x,t) \in \partial\Omega \times [0,\infty), i = 1,2,\ldots,m, \tag{1.2}$$

where $N$ is the unit exterior normal vector to $\partial\Omega$ and $f_i(x,t)$ is a nonnegative continuous function on $\partial\Omega \times [0,\infty), i = 1,2,\ldots,m$, and

$$u_i(x,t) = 0, (x,t) \in \partial\Omega \times [0,\infty), i = 1,2,\ldots,m. \tag{1.3}$$

**Definition 1.** The vector function $u(x,t) = \{u_1(x,t), u_2(x,t), \ldots, u_m(x,t)\}^T$ is said to be a solution of the problem (1.1), (1.2) (or (1.1), (1.3)) if it satisfies (1.1) in $G = \Omega \times [0,\infty)$ and boundary condition (1.2) (or (1.3)).

**Definition 2.** The vector solution $u(x,t) = \{u_1(x,t), u_2(x,t), \ldots, u_m(x,t)\}^T$ of the problem (1.1),(1.2) (or (1.1),(1.3)) is said to be oscillatory in the domain $G = \Omega \times [0,\infty)$ if at least one of its nontrivial component is oscillatory in $G$. Otherwise, the vector solution $u(x,t)$ is said to be nonoscillatory.

## 2 Main Results

**Theorem 2.1.** *If the neutral differential inequality with continuous distributed deviating arguments*

$$\left(V(t) + \sum_{r=1}^{s} \mu_r(t)V(\rho_r(t))\right)'' + p(t)V(t) + \sum_{h=1}^{l} \int_a^b Q_h(t,\xi)V(g_h(t,\xi))d\sigma(\xi) \le 0 \tag{2.1}$$

*has no eventually positive solutions, then every solution of the problem (1.1),(1.2) is oscillatory in G.*

*Proof :* The proof of Theorem 2.1 is similar to that of Theorem 2.1 in [1] and we omit it.

**Theorem 2.2.** *Suppose that*

$$0 \le \sum_{r=1}^{s} \mu_r(t) \le 1. \tag{2.2}$$

*If there exist some* $h_0 \in \{1,2,\ldots,l\}$ *such that*

$$\int_{t_0}^{\infty} \int_a^b Q_{h_0}(s,\xi)\left[1 - \sum_{r=1}^{s} \mu_r(g_{h_0}(s,\xi))\right] d\sigma(\xi)ds = \infty, \ \ t_0 \ge 0. \tag{2.3}$$

*Then every solution of the problem (1.1), (1.2) is oscillatory in G.*

*Proof :* We prove that the inequality (2.1) has no eventually positive solutions if the conditions of Theorem 2.2 hold. Suppose that $V(t)$ is an eventually positive solution of the inequality (2.1), then there exists a number $t_1 \ge t_0$ such that $V(\rho_r(t)) > 0, V(g_h(t,\xi)) > 0, r = 1,2,\cdots,s; h = 1,2,\ldots,l$, for $t \ge t_1$. Thus we have

$$\left(V(t) + \sum_{r=1}^{s} \mu_r(t)V(\rho_r(t))\right)'' + \int_a^b Q_{h_0}(t,\xi)V(g_{h_0}(t,\xi))d\sigma(\xi) \le 0, \ t \ge t_1. \tag{2.4}$$

Let $Y(t)=V(t)+\sum_{r=1}^{s}\mu_r(t)V(\rho_r(t))$. We have $Y(t)>0$ and $Y''(t)<0$ for $t\geq t_1$. Hence $Y'(t)>0$ for $t\geq t_1$.

It is obvious that $Y(t)\geq V(t)$, thus

$$Y(t)\leq V(t)+\sum_{r=1}^{s}\mu_r(t)Y(\rho_r(t))\leq V(t)+\sum_{r=1}^{s}\mu_r(t)Y(t),\ t\geq t_1,$$

that is

$$[1-\sum_{r=1}^{s}\mu_r(t)]Y(t)\leq V(t). \tag{2.5}$$

Combining (2.4) and (2.5), we get

$$\begin{aligned} 0 &\geq Y''(t)+\int_a^b Q_{h_0}(t,\xi)V(g_{h_0}(t,\xi))d\sigma(\xi)\\ &\geq Y''(t)+\int_a^b Q_{h_0}(t,\xi)[1-\sum_{r=1}^{s}\mu_r(g_{h_0}(t,\xi))]Y(g_{h_0}(t,\xi))d\sigma(\xi),\ t\geq t_1. \end{aligned} \tag{2.6}$$

Noting that

$$Y(t)>0,\ Y'(t)>0,\ t\geq t_1,\ \lim_{t\to\infty}\min_{\xi\in[a,b]}g_{h_0}(t,\xi)=\infty,$$

we obtain that there exist $m>t_1, t_2\geq t_1$ such that

$$Y(m)>0,\ g_{h_0}(t,\xi)>m,\ t\geq t_2,\ \xi\in[a,b].$$

Therefore,

$$Y(g_{h_0}(t,\xi))\geq Y(m),\ t\geq t_2,\ \xi\in[a,b]. \tag{2.7}$$

Combining (2.6) and (2.7), we get

$$Y''(t)+Y(m)\int_a^b Q_{h_0}(t,\xi)[1-\sum_{r=1}^{s}\mu_r(g_{h_0}(t,\xi))]d\sigma(\xi)\leq 0,\ t\geq t_2. \tag{2.8}$$

Integrating (2.8) from $t_2$ to $t$, we have

$$Y'(t)-Y'(t_2)+Y(m)\int_{t_2}^{t}\int_a^b Q_{h_0}(s,\xi)[1-\sum_{r=1}^{s}\mu_r(g_{h_0}(s,\xi))]d\sigma(\xi)ds\leq 0,$$

that is

$$\int_{t_2}^{t}\int_a^b Q_{h_0}(s,\xi)[1-\sum_{r=1}^{s}\mu_r(g_{h_0}(s,\xi))]d\sigma(\xi)ds\leq \frac{Y'(t_2)-Y'(t)}{Y(m)}. \tag{2.9}$$

By taking $t\to\infty$, (2.9) leads to a contradiction with (2.3). The proof of Theorem 2.2 is complete.

**Theorem 2.3.** *Suppose that the condition (2.2) holds and there exist some* $h_0\in\{1,2,\dots,l\}$ *such that* $\frac{dg_{h_0}(t,a)}{dt}$ *exists. If there exists a function* $\psi\in C^1([t_0,\infty),[0,\infty))$, $t_0\geq 0$, *such that*

$$\int_{t_0}^{\infty}\left\{\psi(s)\int_a^b Q_{h_0}(s,\xi)\left[1-\sum_{r=1}^{s}\mu_r(g_{h_0}(s,\xi))\right]d\sigma(\xi)-\frac{\psi'^2(s)}{4\psi(s)g'_{h_0}(s,a)}\right\}ds=\infty, \tag{2.10}$$

*then every solution of the problem (1.1), (1.2) is oscillatory in G.*

*Proof :* We prove that the inequality (2.1) has no eventually positive solutions if the conditions of Theorem 2.3 hold. Suppose that $V(t)$ is an eventually positive solution of the inequality (2.1), as in the proof of Theorem 2.2, we have (2.6). Noting that $g_{h_0}(t,\xi)$ is nondecreasing in $\xi$, it is easy to see that $g_{h_0}(t,a) \le g_{h_0}(t,\xi), \xi \in [a,b]$, thus from (2.6) we have

$$Y''(t) + Y(g_{h_0}(t,a)) \int_a^b Q_{h_0}(t,\xi)\left[1 - \sum_{r=1}^{s} \mu_r(g_{h_0}(t,\xi))\right] d\sigma(\xi) \le 0,\ t \ge t_1. \tag{2.11}$$

Let

$$W(t) = \frac{\psi(t)Y'(t)}{Y(g_{h_0}(t,a))},\ t \ge t_1,$$

then we easily get $W(t) > 0, t \ge t_1$. From the condition of Theorem 2.3, there exists $(Y(g_{h_0}(t,a)))'$ $= Y'(g_{h_0}(t,a))g'_{h_0}(t,a)$. Using the fact that $Y''(t) \le 0, Y'(t) > 0, t \ge t_1$, and the condition (A5), we have $0 < Y'(t) \le Y'(g_{h_0}(t,a)), g'_{h_0}(t,a) \ge 0, t \ge t_1$. Therefore,

$$\begin{aligned}
W'(t) &= \frac{\psi'(t)Y'(t)}{Y(g_{h_0}(t,a))} + \frac{\psi(t)Y''(t)}{Y(g_{h_0}(t,a))} - \frac{\psi(t)Y'(t)Y'(g_{h_0}(t,a))g'_{h_0}(t,a)}{Y^2(g_{h_0}(t,a))} \\
&\le \frac{\psi'(t)Y'(t)}{Y(g_{h_0}(t,a))} + \frac{\psi(t)Y''(t)}{Y(g_{h_0}(t,a))} - \frac{\psi(t)Y'^2(t)g'_{h_0}(t,a)}{Y^2(g_{h_0}(t,a))} \\
&= \frac{\psi(t)Y''(t)}{Y(g_{h_0}(t,a))} + \frac{\psi'^2(t)}{4\psi(t)g'_{h_0}(t,a)} - \left[\frac{Y'(t)\sqrt{\psi(t)g'_{h_0}(t,a)}}{Y(g_{h_0}(t,a))} - \frac{\psi'(t)}{2\sqrt{\psi(t)g'_{h_0}(t,a)}}\right]^2 \\
&\le \frac{\psi(t)Y''(t)}{Y(g_{h_0}(t,a))} + \frac{\psi'^2(t)}{4\psi(t)g'_{h_0}(t,a)},\ t \ge t_1.
\end{aligned} \tag{2.12}$$

Combining (2.11) and (2.12), we obtain

$$W'(t) \le -\left\{\psi(t)\int_a^b Q_{h_0}(t,\xi)\left[1 - \sum_{r=1}^{s}\mu_r(g_{h_0}(t,\xi))\right] d\sigma(\xi) - \frac{\psi'^2(t)}{4\psi(t)g'_{h_0}(t,a)}\right\},\ t \ge t_1. \tag{2.13}$$

Integrating both sides of (2.13) from $t_1$ to $t$, we have

$$W(t) \le W(t_1) - \int_{t_1}^{t}\left\{\psi(s)\int_a^b Q_{h_0}(s,\xi)\left[1 - \sum_{r=1}^{s}\mu_r(g_{h_0}(s,\xi))\right] d\sigma(\xi) - \frac{\psi'^2(s)}{4\psi(s)g'_{h_0}(s,a)}\right\} ds. \tag{2.14}$$

Passing with $t \to \infty$ in (2.14) we have a contradiction with (2.10). The proof of Theorem 2.3 is complete.

**Theorem 2.4.** *Suppose that the condition (2.2) holds. If*

$$\int_{t_0}^{\infty} p(t)\left[1 - \sum_{r=1}^{s}\mu_r(t)\right] dt = \infty. \tag{2.15}$$

*Then every solution of the problem (1.1), (1.2) is oscillatory in G.*

*Proof :* Similar to the proof of Theorem 2.2, we have

$$Y^{''}(t)+p(t)V(t)\leq 0, t\geq t_1. \tag{2.16}$$

The remainder of the proof is similar to that of Theorem 2.2 and we omit it.

Next, we study the oscillation of the problem (1.1), (1.3).

It is well known that the smallest eigenvalue $\alpha_0$ of the Dirichlet problem

$$\begin{cases} \Delta\omega(x)+\alpha\omega(x)=0 \text{ in } \Omega, \\ \omega(x)=0 \text{ on } \partial\Omega, \end{cases}$$

is positive and the corresponding eigenfunction $\varphi(x)$ is positive in $\Omega$ (see [2]).

**Theorem 2.5.** *If the differential inequality*

$$\left(V(t)+\sum_{r=1}^{s}\mu_r(t)V(\rho_r(t))\right)^{''}+\alpha_0\sum_{j=1}^{d}A_j(t)V(\tau_j(t))$$

$$+(\alpha_0 a(t)+p(t))V(t)+\sum_{h=1}^{l}\int_a^b Q_h(t,\xi)V(g_h(t,\xi))d\sigma(\xi)\leq 0 \tag{2.17}$$

*has no eventually positive solutions, then every solution of the problem (1.1), (1.3) is oscillatory in G.*

*Proof :* The proof of Theorem 2.5 is similar to that of Theorem 3.1 in [1] and we omit it.

Using the above results, it is easy to obtain the following conclusions.

**Theorem 2.6.** *If all conditions of Theorem 2.2 hold, then every solution of the problem (1.1),(1.3) is oscillatory in G.*

**Theorem 2.7.** *If all conditions of Theorem 2.3 hold, then every solution of the problem (1.1),(1.3) is oscillatory in G.*

**Theorem 2.8.** *If all conditions of Theorem 2.4 hold, then every solution of the problem (1.1),(1.3) is oscillatory in G.*

**Theorem 2.9.** *Suppose that the condition (2.2) holds and there exist some* $j_0\in\{1,2,\dots,d\}$ *such that*

$$\int_{t_0}^{\infty}A_{j_0}(t)\left[1-\sum_{r=1}^{s}\mu_r(\tau_{j_0}(t))\right]dt=\infty,\ \ t_0\geq 0. \tag{2.18}$$

*Then every solution of the problem (1.1), (1.3) is oscillatory in G.*

**Theorem 2.10.** *Suppose that the condition (2.2) holds. If*

$$\int_{t_0}^{\infty}a(t)\left[1-\sum_{r=1}^{s}\mu_r(t)\right]dt=\infty, \tag{2.19}$$

*then every solution of the problem (1.1), (1.3) is oscillatory in G.*

## 3 Examples

In this section, we give two illustrative examples. Obviously, the results in [1] were failed in these examples.

**Example 3.1.** Consider the system of neutral hyperbolic differential equations

$$\begin{cases} \dfrac{\partial^2}{\partial t^2}\left[u_1(x,t)+\frac{1}{2}u_1(x,t-\pi)+\frac{1}{3}u_1(x,t-\frac{\pi}{2})\right]=3\Delta u_1(x,t) \\ \qquad +\frac{11}{3}\Delta u_1(x,t-\frac{3\pi}{2})+\Delta u_2(x,t-\frac{3\pi}{2})-\frac{1}{2}u_1(x,t) \\ \qquad -\displaystyle\int_{-\pi}^{-\frac{\pi}{2}} 3u_1(x,t+\xi)d\xi-\int_{-\pi}^{-\frac{\pi}{2}} u_2(x,t+\xi)d\xi, \\ \dfrac{\partial^2}{\partial t^2}\left[u_2(x,t)+\frac{1}{2}u_2(x,t-\pi)+\frac{1}{3}u_2(x,t-\frac{\pi}{2})\right]=\frac{1}{2}\Delta u_2(x,t) \\ \qquad +3\Delta u_1(x,t-\frac{3\pi}{2})+\frac{5}{3}\Delta u_2(x,t-\frac{3\pi}{2})-u_2(x,t) \\ \qquad -\displaystyle\int_{-\pi}^{-\frac{\pi}{2}} u_1(x,t+\xi)d\xi-\int_{-\pi}^{-\frac{\pi}{2}} 3u_2(x,t+\xi)d\xi, \\ \qquad (x,t)\in(0,\pi)\times[0,\infty), \end{cases} \tag{3.1}$$

with boundary condition

$$\frac{\partial}{\partial x}u_i(0,t)=\frac{\partial}{\partial x}u_i(\pi,t)=0, t\geq 0, i=1,2. \tag{3.2}$$

Here $n=1,\ m=2,\ s=2,\ d=1,\ l=1,\ \mu_1(t)=\frac{1}{2},\ \mu_2(t)=\frac{1}{3},\ \rho_1(t)=t-\pi, \rho_2(t)=t-\frac{\pi}{2},\ a_1(t)=3,\ a_{111}(t)=\frac{11}{3}, a_{121}(t)=1,\ \tau_1(t)=t-\frac{3\pi}{2},\ p_1(x,t)=\frac{1}{2},\ q_{111}(x,t,\xi)=3,\ q_{121}(x,t,\xi)=1,\ g_1(t,\xi)=t+\xi,\ a_2(t)=\frac{1}{2},\ a_{211}(t)=3,\ a_{221}(t)=\frac{5}{3},\ p_2(x,t)=1,\ q_{211}(x,t,\xi)=1,\ q_{221}(x,t,\xi)=,\ a=-\pi,\ b=-\frac{\pi}{2}$. It is easy to see that $Q_1(t,\xi)=2$, $\sum_{i=1}^{2}\mu_i(t)=\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$, and

$$\int_{t_0}^{\infty}\int_a^b Q_1(s,\xi)\left[1-\sum_{i=1}^{2}\mu_i(g_1(s,\xi))\right]d\xi ds=\int_{t_0}^{\infty}\int_{-\pi}^{-\pi/2}\frac{1}{3}d\xi ds=\infty.$$

Hence all the conditions of Theorem 2.2 are fulfilled. Then every solution of the problem (3.1), (3.2) is oscillatory in $(0,\pi)\times[0,\infty)$. In fact, $u_1(x,t)=\cos x\sin t, u_2(x,t)=\cos x\cos t$ is such a solution.

On the other hand, we easily see that $p(t)=\frac{1}{2}$, and

$$\int_{t_0}^{\infty} p(t)\left[1-\sum_{i=1}^{2}\mu_i(t)\right]dt=\int_{t_0}^{\infty}\frac{1}{12}dt=\infty,$$

so the conditions of Theorem 2.4 are fulfilled. Then using Theorem 2.4, we also obtain that every solution of the problem (3.1), (3.2) is oscillatory in $(0,\pi)\times[0,\infty)$.

**Example 3.2.** Consider the system of neutral hyperbolic equations

$$\begin{cases} \frac{\partial^2}{\partial t^2}\left[u_1(x,t)+\frac{1}{3}u_1(x,t-\pi)+\frac{1}{2}u_1(x,t-\frac{\pi}{2})\right]=3\Delta u_1(x,t) \\ \qquad +\frac{3}{2}\Delta u_1(x,t-\frac{3\pi}{2})+\Delta u_2(x,t-\frac{3\pi}{2})-\frac{2}{3}u_1(x,t) \\ \qquad -\int_{-\pi}^{-\frac{\pi}{2}}3u_1(x,t+\xi)d\xi-\int_{-\pi}^{-\frac{\pi}{2}}u_2(x,t+\xi)d\xi, \\ \frac{\partial^2}{\partial t^2}\left[u_2(x,t)+\frac{1}{3}u_2(x,t-\pi)+\frac{1}{2}u_2(x,t-\frac{\pi}{2})\right]=\frac{2}{3}\Delta u_2(x,t) \\ \qquad +\Delta u_1(x,t-\frac{3\pi}{2})+\frac{7}{2}\Delta u_2(x,t-\frac{3\pi}{2})-3u_2(x,t) \\ \qquad -\int_{-\pi}^{-\frac{\pi}{2}}u_1(x,t+\xi)d\xi-\int_{-\pi}^{-\frac{\pi}{2}}3u_2(x,t+\xi)d\xi, \\ \qquad (x,t)\in(0,\pi)\times[0,\infty), \end{cases} \tag{3.3}$$

with boundary condition

$$u_i(0,t)=u_i(\pi,t)=0, t\geq 0, i=1,2. \tag{3.4}$$

Here $n=1, m=2, s=2, d=1, l=1, \mu_1(t)=\frac{1}{3}, \mu_2(t)=\frac{1}{2}, \rho_1(t)=t-\pi, \rho_2(t)=t-\frac{\pi}{2}$, $a_1(t)=3$, $a_{111}(t)=\frac{3}{2}$, $a_{121}(t)=1$, $\tau_1(t)=t-\frac{3\pi}{2}$, $p_1(x,t)=\frac{2}{3}$, $q_{111}(x,t,\xi)=3$, $q_{121}(x,t,\xi)=1$, $g_1(t,\xi)=t+\xi$, $a_2(t)=\frac{2}{3}, a_{211}(t)=1, a_{221}(t)=\frac{7}{2}$, $p_2(x,t)=3, q_{211}(x,t,\xi)=1$, $q_{221}(x,t,\xi)=3$, $a=-\pi$, $b=-\frac{\pi}{2}$. It is easy to see that $Q_1(t,\xi)=2$, $\sum_{i=1}^{2}\mu_i(t)=\frac{5}{6}$. Let $\psi(t)=\sqrt{t}$. Then all the conditions of Theorem 2.7 are fulfilled. Thus all solutions of the problem (3.3), (3.4) are oscillatory in $(0,\pi)\times[0,\infty)$. In fact, such a solution is $u_1(x,t)=\sin x\cos t, u_2(x,t)=\sin x\sin t$.

On the other hand, we easily see that $a(t)=\frac{2}{3}, p(t)=\frac{2}{3}, A_1(t)=\frac{1}{2}$, thus the conditions of Theorem 2.8, Theorem 2.9 and Theorem 2.10 are fulfilled. Using these theorems we also obtain that every solution of the problem (3.3), (3.4) is oscillatory in $(0,\pi)\times[0,\infty)$.

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