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A note on a conjecture of Graffiti

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Abstract: In this paper, we will give a very simpleproof of a conjecture of Graffiti. (WOW Conjecture 584): Let T be a tree of order n with independence number α, then $\lambda_1 \leqslant 2+\alpha$, where λ_1 is the Laplacian spectralradius. (Xiao - dong Zhang, On the two conjectures of Graffiti, Linear Algebra and its Applications, described all extremal treesthat attain the maximal Laplacian spectral radius and used theresults to show conjectures.

Key words: Laplacian eigenvalue; tree; independence number; girth

Let $G=(V,E)$ be a simple graph on vertex set and edge set E. The matrix $L(G)=D(G)-A(G)$ is called the Laplacian matrix of graph G, where $D=\mathrm{diag}(d(u), u\in V)$ is the diagonal matrix of vertex degrees of G and $A(G)$ is the adjacency matrix of G. It, is easy to see $L(G)$ is singular, positive semidefinite. The eigenvalues of $L(G)$ are called the Laplacian eigenvalues and denoted by $\lambda_1\ \lambda_2 \lambda_n = 0$, Especially, we call λ_1 the Laplacian spectral radius.

Let G be a simple graph. A pendant vertex is a vertex of degree one. A subset S of V is called an independence set of G, if no two vertices of S are adjacent in G. The independence number of G, denoted by α or $\alpha(G)$, is the size of a maximum independence set of G. The girth of a graph with a cycle, denoted by g, is the length of its shortest cycle. A graph with no cycle has infinite girth. The terminology not defined here can be found in [2].

Fajtlowicz[3] raised the conjecture on the relationship between the Laplacian eigenvalues and the independence number of a graph, such as (WOW Conjecture 584), Xiao - dong Zhang, On the two conjectures of Graffiti, Linear Algebra and its Applications, described all extremal trees that attain the maximal Laplacian spectral radius and used the results to show conjectures. Let T be a tree of order n with independence number

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α, then λ_1 2 + α . Now we will give a simple proof of it.

Anderson and Morley[1] proved the following result, we will use it in the proof of main result.

Lemma Let G be a connected graph of order n, then $\lambda_1(G) \le \max\{d(u)+d(v) \mid uv \in E(G)\}$.

Theorem Let G be a connected graph of order n with independence number α, $g \ge 5$, then $\lambda_1(G) \leqslant 2+\alpha$.

Proof Let $e = uv$ be an arbitrary edge of G, as $g \ge 5$, there is no edge in $G[(N(v)-\{u\}) \cup (N(u)-\{v\})]$, and $\{N(v)-u\} \cap \{N(u)-v\} = $. So $\{N(v)-u\} \cup \{N(u)-v\}$ is an independence set of G, $\alpha \mid N(u)-v\mid + \mid N(v)-u\mid = d(u)+d(v)-2$, thus α $\max\{d(u)+d(v) \mid uv \in E\}-2$. By the Lemma, we get $\lambda_1(G) \leqslant 2+\alpha$. □

Corollary Let T be a tree of order n with independence number α, then $\lambda_1 \leqslant 2+\alpha$.

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