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PERIODICITY FOR A COOPERATIVE SYSTEM ON TIME SCALES

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Abstract

In this paper, by using the method of coincidence degree theory, we explore the existence of periodic solutions for a cooperative ecological model on time scales.

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Keywords: time scales, cooperative system, coincidence degree, continuation theorem, periodic solutions.

1 Introduction

Recently, the continuation theorem of coincidence degree theory has been widely applied to the existence problems of periodic solutions in differential equations and difference equations, such as [1] and [10]. However, the research methods and results are similar. We cannot refrain from asking whether there is a unified way to investigate these problems. The theory of calculus on time scales, which was initiated by Stefan Hilger [8], well solved these problems and unified the differential and difference analysis. In [2] and [3], Bohner and Fan systematically studied the existence of periodic solutions of dynamic equations on time scales of predator–prey type and competition type.

Motivated by [1] and [2], we consider the cooperative ecological system on time scales as follows:

$$\begin{cases} x_1^{\Delta}(t) = r_1(t)\left(1 - \frac{e^{x_1(t)}}{a_1(t)+b_1(t)e^{x_2(t)}} - c_1(t)e^{x_1(t)}\right), \\ x_2^{\Delta}(t) = r_2(t)\left(1 - \frac{e^{x_2(t)}}{a_2(t)+b_2(t)e^{x_1(t)}} - c_2(t)e^{x_2(t)}\right), \end{cases} \tag{1.1}$$

where a_i, b_i, c_i and $r_i (i = 1,2)$ are rd–continuous positive $\omega-$periodic functions on time scales $\mathbb{T}$. Set $u_1(t) = e^{x_1(t)}$, $u_2(t) = e^{x_2(t)}$, if $\mathbb{T} = \mathbb{R}$, then (1.1) can be reduced to the follow-

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ing form:

$$\begin{cases} \dot{u}_1(t) = r_1(t)u_1(t)\left(1 - \frac{u_1(t)}{a_1(t)+b_1(t)u_2(t)} - c_1(t)u_1(t)\right), \\ \dot{u}_2(t) = r_2(t)u_2(t)\left(1 - \frac{u_2(t)}{a_2(t)+b_2(t)u_1(t)} - c_2(t)u_2(t)\right), \end{cases} \tag{1.2}$$

which was constructed by May in [6]. x_i denotes the density of $i-$population. The existence of one population enlarges the carrying capacity of the other population.

If $\mathbb{T} = \mathbb{Z}$, system (1.1) is reformulated as

$$\begin{cases} u_1(k+1) = u_1(k)\exp\left\{r_1(k)\left(1 - \frac{u_1(k)}{a_1(k)+b_1(k)u_2(k)} - c_1(k)u_1(k)\right)\right\}, \\ u_2(k+1) = u_2(k)\exp\left\{r_2(k)\left(1 - \frac{u_2(k)}{a_2(k)+b_2(k)u_1(k)} - c_2(k)u_2(k)\right)\right\}, \end{cases} \tag{1.3}$$

in [1], Bai and Fan proved the existence of positive periodic solutions when a_i, b_i, c_i and $r_i (i = 1,2)$ are positive $\omega-$periodic sequences.

In this paper, we mainly explore the periodic solutions of (1.1) by the continuation theorem in coincidence degree theory. With the help of an integral inequality on time scales established in [9], we can find the sharp priori bounds and improve existence criteria for periodic solutions.

The paper is organized as follows. In the next section, we present some preliminary results about the calculus on time scales and the continuation theorem. In Section 3, the sufficient conditions for the existence of periodic solutions for (1.1) are obtained.

2 Preliminaries

For the convenience of reading, we first present some basic definitions and lemmas about time scales and the continuation theorem of the coincidence degree theory, more details can be found in [4], [5] and [7].

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of real numbers $\mathbb{R}$. Throughout this paper, we assume that the time scale $\mathbb{T}$ is unbounded above and below, such as $\mathbb{R}$, $\mathbb{Z}$, and $\bigcup_{k\in\mathbb{Z}}[2k, 2k+1]$.

Let $f : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}$. Then we define $f^{\Delta}(t)$ to be the number (provided it exists) with the property that given any $\varepsilon > 0$, there is a neighborhood U of t such that

$$\left| f(\sigma(t)) - f(s) - f^{\Delta}(t)(\sigma(t) - s) \right| \leq \varepsilon|\sigma(t) - s| \quad \text{for all } s \in U.$$

In this case, $f^{\Delta}(t)$ is called the delta (or Hilger) derivative of f at t. Moreover, f is said to be delta or Hilger differentiable on $\mathbb{T}$ if $f^{\Delta}(t)$ exists for all $t \in \mathbb{T}$. Obviously, if $\mathbb{T} = \mathbb{R}$, then $f^{\Delta}(t) = f'(t)$; if $\mathbb{T} = \mathbb{Z}$, then $f^{\Delta}(t) = f(t+1) - f(t) = \Delta f(t)$. Then we define

$$\int_r^s f(t)\Delta t = F(s) - F(r) \qquad \text{for } r, s \in \mathbb{T}.$$

Let $\omega > 0$, the time scale $\mathbb{T}$ is assumed to be $\omega-$periodic, that is $t \in \mathbb{T}$ implies $t + \omega \in \mathbb{T}$. The set of rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C_{rd}(\mathbb{T})$. We denote

$$k = \min\{\mathbb{R}^+ \cap \mathbb{T}\}, \;\; I_\omega = [k, k+\omega] \cap \mathbb{T}, \;\; \bar{g} = \frac{1}{\omega}\int_{I_\omega} g(s)\Delta s = \frac{1}{\omega}\int_k^{k+\omega} g(s)\Delta s,$$

where $g \in C_{rd}(\mathbb{T})$ is an $\omega-$periodic real function, i.e., $g(t+\omega) = g(t)$ for all $t \in \mathbb{T}$.

The following lemma from [9] will be used in the proof of our main results and can help to improve many existence criteria for periodic solutions.

Lemma 2.1. *[9] Let $t_1, t_2 \in I_\omega$ and $t \in \mathbb{T}$. If $g : \mathbb{T} \to \mathbb{R} \in C_{rd}(\mathbb{T})$ is $\omega-$periodic, then*

$$g(t) \leq g(t_1) + \frac{1}{2}\int_k^{k+\omega} |g^\Delta(s)|\Delta s \quad , \quad g(t) \geq g(t_2) - \frac{1}{2}\int_k^{k+\omega} |g^\Delta(s)|\Delta s,$$

and the constant factor $\frac{1}{2}$ is the best possible.

Now, we introduce some concepts and a useful result in [7].

Let X, Z be normed vector spaces, $L : \mathrm{Dom}L \subset X \to Z$ be a linear mapping, $N : X \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim\mathrm{Ker}L = \mathrm{codim}\mathrm{Im}L < +\infty$ and $\mathrm{Im}L$ is closed in Z. If L is a Fredholm mapping of index zero and there exist continuous projections $P : X \to X$ and $Q : Z \to Z$ such that $\mathrm{Im}P = \mathrm{Ker}L$, $\mathrm{Im}L = \mathrm{Ker}Q = \mathrm{Im}(I-Q)$, then it follows that $L|\mathrm{Dom}L \cap \mathrm{Ker}P : (I-P)X \to \mathrm{Im}L$ is invertible. We denote the inverse of that map by K_P. If Ω is an open bounded subset of X, the mapping N will be called $L-$compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_P(I-Q)N : \overline{\Omega} \to X$ is compact. Since $\mathrm{Im}Q$ is isomorphic to $\mathrm{Ker}L$, there exists an isomorphism $J : \mathrm{Im}Q \to \mathrm{Ker}L$.

Lemma 2.2. *(Continuation Theorem) Let L be a Fredholm mapping of index zero and N be $L-$compact on $\overline{\Omega}$.Suppose*
(a) for each $\lambda \in (0,1)$, every solution u of $Lu = \lambda Nu$ is such that $u \notin \partial\Omega$;
(b) $QNu \neq 0$ for each $u \in \partial\Omega \cap KerL$ and the Brouwer degree $deg\{JQN, \Omega \cap KerL, 0\} \neq 0$.
Then the operator equation $Lu = Nu$ has at least one solution lying in $DomL \cap \overline{\Omega}$.

3 Existence of Periodic Solutions

In this section, we will prove the theorem related to system (1.1). To explore the existence of periodic solutions of (1.1), we first embed our problem in the frame of coincidence degree theory. Define

$$\mathscr{L}^\omega = \{(u,v) \in C(\mathbb{T}, \mathbb{R}^2) : u(t+\omega) = u(t), v(t+\omega) = v(t) \quad \text{for all} \quad t \in \mathbb{T}\},$$

$$\| (u,v) \| = \max_{t \in I_\omega}\{u(t)\} + \max_{t \in I_\omega}\{v(t)\} \quad \text{for} \quad (u,v) \in \mathscr{L}^\omega.$$

It is easy to show that $\mathscr{L}^\omega$ is a Banach space when it is endowed with the above norm $\| \cdot \|$. Let

$$\mathscr{L}_0^\omega = \{(u,v) \in \mathscr{L}^\omega : \overline{u} = 0, \overline{v} = 0\},$$

$$\mathscr{L}_c^\omega = \{(u,v) \in \mathscr{L}^\omega : (u(t), v(t)) \equiv (h_1, h_2) \in \mathbb{R}^2 \quad \text{for} \quad t \in \mathbb{T}\}.$$

Thus, $\mathscr{L}_0^\omega$ and $\mathscr{L}_c^\omega$ are both closed linear subspaces of $\mathscr{L}^\omega$, $\mathscr{L}^\omega = \mathscr{L}_0^\omega \oplus \mathscr{L}_c^\omega$, and $\dim\mathscr{L}_c^\omega = 2$.

Theorem 3.1. *If r_i, a_i, b_i and $c_i, i = 1,2$, are positive rd–continuous $\omega-$periodic functions on time scales $\mathbb{T}$, then (1.1) has at least one $\omega-$periodic solution.*

Proof. Let $X = Z = \mathscr{L}^\omega$,

$$N\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} = \begin{bmatrix} r_1(t)\left(1 - \frac{e^{x_1(t)}}{a_1(t)+b_1(t)e^{x_2(t)}} - c_1(t)e^{x_1(t)}\right) \\ r_2(t)\left(1 - \frac{e^{x_2(t)}}{a_2(t)+b_2(t)e^{x_1(t)}} - c_2(t)e^{x_2(t)}\right) \end{bmatrix},$$

$$L\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1^\Delta \\ x_2^\Delta \end{bmatrix}, \qquad P\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = Q\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \overline{x_1} \\ \overline{x_2} \end{bmatrix}.$$

Then $\mathrm{Ker}L = \mathscr{L}_c^\omega$, $\mathrm{Im}L = \mathscr{L}_0^\omega$, and $\dim \mathrm{Ker}L = 2 = \mathrm{codim}\,\mathrm{Im}L$. Since $\mathrm{Im}L$ is closed in Z, then L is a Fredholm mapping of index zero. It is easy to find that P and Q are continuous projections such that $\mathrm{Im}P = \mathrm{Ker}L$ and $\mathrm{Im}L = \mathrm{Ker}Q = \mathrm{Im}(I-Q)$. Furthermore, the generalized inverse (to L) $K_P : \mathrm{Im}L \to \mathrm{Ker}P \cap \mathrm{Dom}L$ exists and is given by

$$K_P\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \int_k^t x_1(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t x_1(s)\Delta s\Delta t \\ \int_k^t x_2(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t x_2(s)\Delta s\Delta t \end{bmatrix}.$$

Thus

$$QN\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega}\int_k^{k+\omega} r_1(t)\left(1 - \frac{e^{x_1(t)}}{a_1(t)+b_1(t)e^{x_2(t)}} - c_1(t)e^{x_1(t)}\right)\Delta t \\ \frac{1}{\omega}\int_k^{k+\omega} r_2(t)\left(1 - \frac{e^{x_2(t)}}{a_2(t)+b_2(t)e^{x_1(t)}} - c_2(t)e^{x_2(t)}\right)\Delta t \end{bmatrix},$$

$$K_P(I-Q)N\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= \begin{bmatrix} \int_k^t x_1(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t x_1(s)\Delta s\Delta t - \left(t - k - \frac{1}{\omega}\int_k^{k+\omega}(t-k)\Delta t\right)\overline{x_1} \\ \int_k^t x_2(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t x_2(s)\Delta s\Delta t - \left(t - k - \frac{1}{\omega}\int_k^{k+\omega}(t-k)\Delta t\right)\overline{x_2} \end{bmatrix}.$$

Obviously, QN and $K_P(I-Q)N$ are continuous. According to Arzela–Ascoli theorem, it is easy to show that $K_P(I-Q)N(\overline{\Omega})$ is compact for any open bounded set $\Omega \subset X$ and $QN(\overline{\Omega})$ is bounded. Thus, N is $L-$compact on $\overline{\Omega}$.

Now, we are in the position to build up the suitable open bounded subset Ω for the application of the continuation theorem. For the operator equation $Lu = \lambda Nu$, where $\lambda \in (0,1)$, we have

$$\begin{cases} x_1^\Delta(t) = \lambda r_1(t)\left(1 - \frac{e^{x_1(t)}}{a_1(t)+b_1(t)e^{x_2(t)}} - c_1(t)e^{x_1(t)}\right), \\ x_2^\Delta(t) = \lambda r_2(t)\left(1 - \frac{e^{x_2(t)}}{a_2(t)+b_2(t)e^{x_1(t)}} - c_2(t)e^{x_2(t)}\right). \end{cases} \tag{3.1}$$

Assume that $(x_1,x_2)^T \in X$ is a solution of system (3.1) for a certain $\lambda \in (0,1)$. Integrating (3.1) on both sides from k to $k+\omega$, we get

$$\begin{cases} \overline{r_1}\omega = \int_k^{k+\omega}\left(\frac{r_1(t)e^{x_1(t)}}{a_1(t)+b_1(t)e^{x_2(t)}} + r_1(t)c_1(t)e^{x_1(t)}\right)\Delta t, \\ \overline{r_2}\omega = \int_k^{k+\omega}\left(\frac{r_2(t)e^{x_2(t)}}{a_2(t)+b_2(t)e^{x_1(t)}} + r_2(t)c_2(t)e^{x_2(t)}\right)\Delta t. \end{cases} \tag{3.2}$$

From (3.1) and (3.2), we have

$$\int_k^{k+\omega}|x_1^\Delta|\Delta t < \int_k^{k+\omega} r_1(t)\left(1 + \frac{e^{x_1(t)}}{a_1(t)+b_1(t)e^{x_2(t)}} + c_1(t)e^{x_1(t)}\right)\Delta t = 2\overline{r_1}\omega,$$

$$\int_k^{k+\omega} |x_2^\Delta|\Delta t < \int_k^{k+\omega} r_2(t)\left(1+\frac{e^{x_2(t)}}{a_2(t)+b_2(t)e^{x_1(t)}}+c_2(t)e^{x_2(t)}\right)\Delta t = 2\bar{r}_2\omega.$$

Since $(x_1(t), x_2(t))^T \in X$, there exist $\xi_i, \eta_i \in I_\omega, i = 1,2$, such that

$$x_i(\xi_i) = \min_{t\in I_\omega}\{x_i(t)\}, \tag{3.3}$$

$$x_i(\eta_i) = \max_{t\in I_\omega}\{x_i(t)\}. \tag{3.4}$$

From the first equation of (3.2) and (3.3), we get

$$\bar{r}_1\omega > e^{x_1(\xi_1)}\omega\overline{r_1c_1},$$

so we have

$$e^{x_1(\xi_1)} < \frac{\bar{r}_1}{\overline{r_1c_1}}.$$

From the second equation of (3.2) and (3.3), we can obtain

$$e^{x_2(\xi_2)} < \frac{\bar{r}_2}{\overline{r_2c_2}}.$$

Similarly, from the first equation of (3.2) and (3.4)

$$\begin{aligned}\bar{r}_1\omega &< e^{x_1(\eta_1)}\int_k^{k+\omega}\left(\frac{r_1(t)}{a_1(t)}+r_1(t)c_1(t)\right)\Delta t\\ &= e^{x_1(\eta_1)}\omega\overline{\frac{r_1}{a_1}+r_1c_1},\end{aligned}$$

thus,

$$e^{x_1(\eta_1)} > \frac{\bar{r}_1}{\overline{\frac{r_1}{a_1}+r_1c_1}}.$$

From the second equation of (3.2) and (3.4)

$$e^{x_2(\eta_2)} > \frac{\bar{r}_2}{\overline{\frac{r_2}{a_2}+r_2c_2}}.$$

From Lemma 2.1, we have

$$x_1(t) \le x_1(\xi_1)+\frac{1}{2}\int_k^{k+\omega}|x_1^\Delta(s)|\Delta s \le \ln\frac{\bar{r}_1}{\overline{r_1c_1}}+\bar{r}_1\omega, \tag{3.5}$$

$$x_2(t) \le x_2(\xi_2)+\frac{1}{2}\int_k^{k+\omega}|x_2^\Delta(s)|\Delta s \le \ln\frac{\bar{r}_2}{\overline{r_2c_2}}+\bar{r}_2\omega, \tag{3.6}$$

and

$$x_1(t) \ge x_1(\eta_1)-\frac{1}{2}\int_k^{k+\omega}|x_1^\Delta(s)|\Delta s \ge \ln\frac{\bar{r}_1}{\overline{\frac{r_1}{a_1}+r_1c_1}}-\bar{r}_1\omega, \tag{3.7}$$

$$x_2(t) \ge x_2(\eta_2)-\frac{1}{2}\int_k^{k+\omega}|x_2^\Delta(s)|\Delta s \ge \ln\frac{\bar{r}_2}{\overline{\frac{r_2}{a_2}+r_2c_2}}-\bar{r}_2\omega. \tag{3.8}$$

From (3.5) to (3.8), we have

$$\max_{t\in[k,k+\omega]}|x_1(t)| \le \max\left\{\left|\ln\frac{\bar{r}_1}{\overline{r_1c_1}}\right| + \bar{r}_1\omega, \left|\ln\frac{\bar{r}_1}{\overline{\frac{r_1}{a_1}+r_1c_1}}\right| + \bar{r}_1\omega\right\} := R_1,$$

$$\max_{t\in[k,k+\omega]}|x_2(t)| \le \max\left\{\left|\ln\frac{\bar{r}_2}{\overline{r_2c_2}}\right| + \bar{r}_2\omega, \left|\ln\frac{\bar{r}_2}{\overline{\frac{r_2}{a_2}+r_2c_2}}\right| + \bar{r}_2\omega\right\} := R_2.$$

Clearly, R_1 and R_2 are independent of λ. Let $M = R_1 + R_2 + R_0$, where R_0 is taken sufficiently large such that for the following algebraic equations

$$\begin{cases} \bar{r}_1 - \overline{r_1c_1}e^x - \frac{1}{\omega}\int_k^{k+\omega}\frac{r_1(t)e^x}{a_1(t)+b_1(t)e^y}\Delta t = 0, \\ \bar{r}_2 - \overline{r_2c_2}e^y - \frac{1}{\omega}\int_k^{k+\omega}\frac{r_2(t)e^y}{a_2(t)+b_2(t)e^x}\Delta t = 0, \end{cases} \tag{3.9}$$

every solution $(x^*,y^*)^T$ of (3.9) satisfies $\| (x^*,y^*)^T \| < M$. Now, we define $\Omega = \{(x_1,x_2)^T \in X, \| (x_1,x_2)^T \| < M\}$. Then it is clear that Ω verifies the requirement (a) of Lemma 2.2. If $(x_1,x_2)^T \in \partial\Omega\cap \mathrm{Ker}L = \partial\Omega\cap\mathbb{R}^2$, then $(x_1,x_2)^T$ is a constant vector in $\mathbb{R}^2$ with $\| (x_1,x_2)^T \| = |x_1|+|x_2| = M$. If (3.9) has at least a solution, then

$$QN\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \bar{r}_1 - \overline{r_1c_1}e^{x_1} - \frac{1}{\omega}\int_k^{k+\omega}\frac{r_1(t)e^{x_1}}{a_1(t)+b_1(t)e^{x_2}}\Delta t \\ \bar{r}_2 - \overline{r_2c_2}e^{x_2} - \frac{1}{\omega}\int_k^{k+\omega}\frac{r_2(t)e^{x_2}}{a_2(t)+b_2(t)e^{x_1}}\Delta t \end{bmatrix} \ne \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

if (3.9) has no solution, then

$$QN\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \ne \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Moreover, define

$$\phi(x_1,x_2,\mu) = \begin{bmatrix} \bar{r}_1 - \overline{r_1c_1}e^{x_1} \\ \bar{r}_2 - \overline{r_2c_2}e^{x_2} \end{bmatrix} + \mu\begin{bmatrix} -\frac{1}{\omega}\int_k^{k+\omega}\frac{r_1(t)e^{x_1}}{a_1(t)+b_1(t)e^{x_2}}\Delta t \\ -\frac{1}{\omega}\int_k^{k+\omega}\frac{r_2(t)e^{x_2}}{a_2(t)+b_2(t)e^{x_1}}\Delta t \end{bmatrix},$$

where $\mu\in[0,1]$ is a parameter. If $(x_1,x_2)^T \in \partial\Omega\cap\mathrm{Ker}L$, then $\phi(x_1,x_2,\mu)\ne 0$. In addition, we can easily see that the algebraic equation $\phi(x_1,x_2,0) = 0$ has a unique solution in $\mathbb{R}^2$. Thus the invariance of homotopy produces

$$\begin{aligned} \deg(JQN,\Omega\cap\mathrm{Ker}L,0) &= \deg(QN,\Omega\cap\mathrm{Ker}L,0) \\ &= \deg(\phi(x_1,x_2,1),\Omega\cap\mathrm{Ker}L,0) \\ &= \deg(\phi(x_1,x_2,0),\Omega\cap\mathrm{Ker}L,0) \\ &= 1. \end{aligned}$$

By now, we have verified that Ω fulfills all requirements of Lemma 2.2, therefore, system (1.1) has at least one $\omega-$periodic solution in $\mathrm{Dom}L\cap\bar{\Omega}$. The proof is complete. □

Remark 3.2. From Theorem 3.1, we can see that (1.1) has at least one $\omega-$periodic solution provided that the coefficients of (1.1) are rd–continuous $\omega-$periodic functions on time scales $\mathbb{T}$. Thus, the special cases (1.2) and (1.3) both have at least one $\omega-$periodic solution under certain conditions. This study shows that the results we obtained are more generalized than those in [1]. Moreover, the priori bounds for periodic solutions are more accurate than those in [1].

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