Demographic Research a free, expedited, online journal
of peer-reviewed research and commentary
in the population sciences published by the
Max Planck Institute for Demographic Research
Konrad-Zuse Str. 1, D-18057 Rostock · GERMANY
www.demographic-research.org

DEMOGRAPHIC RESEARCH
www.demographic-research.org

VOLUME 13, ARTICLE 1, PAGES 1-34
PUBLISHED 15 JULY 2005
http://www.demographic-research.org/Volumes/Vol13/1/

Research Article

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Placing the poor while keeping the rich in their place: separating strategies for optimally managing residential mobility and assimilation

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Abstract

A central objective of modern US housing policy is deconcentrating poverty through "housing mobility programs" that move poor families into middle class neighborhoods. Pursuing these policies too aggressively risks inducing middle class flight, but being too cautious squanders the opportunity to help more poor families. This paper presents a stylized dynamic optimization model that captures this tension. With base-case parameter values, cost considerations limit mobility programs before flight becomes excessive. However, for modest departures reflecting stronger flight tendencies and/or weaker destination neighborhoods, other outcomes emerge. In particular, we find state-dependence and multiple equilibria, including both de-populated and oversized outcomes. For certain sets of parameters there exists a Skiba point that separates initial conditions for which the optimal strategy leads to substantial flight and depopulation from those for which the optimal strategy retains or even expands the middle class population. These results suggest the value of estimating middle-class neighborhoods' "carrying capacity" for absorbing mobility program placements and further modeling of dynamic response.

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1. Introduction

This paper addresses the dynamic optimization problem faced by a social planner who wants to integrate a stream of poor families into an existing middle-class neighborhood without inducing “middle-class flight”. Attempting to place “too many” poor families too quickly could induce current residents to relocate and/or deter affluent residents of other communities from moving in, both of which could reduce the tax base of the communities to which poor families relocate. On the other hand, placing too few poor families squanders the opportunity to use the resources of the community to help assimilate poor families into the middle class.

The model is highly stylized but is inspired by a pressing practical problem. The United States has pockets of concentrated poverty in most of its major cities and some adjoining older suburbs. Upward social mobility from these neighborhoods is limited, creating a persistent “underclass” of the “truly disadvantaged” (Wilson 1987). Some of these neighborhoods contain publicly-subsidized housing developments. Over the last ten years the government has pursued an active policy of “de-concentrating” poverty by subsidizing movement of tenants from high-poverty, racially segregated communities into more affluent neighborhoods using “housing mobility programs” such as the nationwide Moving to Opportunity (MTO) experiment (U.S. Department of Housing and Urban Development 1999). The underlying premise, for which there is considerable empirical support, is that poor families can do better on a variety of social, health, education, and economic indicators if they have the opportunity to choose good-quality housing in more-affluent destination communities (Johnson, Ladd, and Ludwig 2001).

The model is formulated as an optimal dynamic control problem. This is a natural framework for reflecting the dynamic, endogenous response of current residents to an inflow of poor residents, but to the best of our knowledge this methodology has rarely been used to study housing issues and is not used very frequently in demographic research more generally. Note: city growth is not the only domain of interest to demographers that invites dynamic models with growth under externalities. Another example would be ecological models where both the flow of agricultural products and the stock of wild nature bring utility.

This dynamic approach generates interesting insights. For example, the model displays “tipping” – the phenomena studied extensively by Schelling (1973) of having multiple stable equilibria surrounded by neighborhoods of attraction and unstable equilibria, although in this case this characterization pertains to trajectories observed when the socially optimal policy is pursued, not the uncontrolled dynamics.

The source of tipping here is akin to that in a much earlier model of housing segregation by Schelling (1971), namely externalities imposed by neighbours. Schelling’s original model was written for racial segregation, but it can be applied to any social groups with little tolerance to each other. In this case, economic class. Unlike Schelling's and other traditional models, ours explicitly includes two types of externalities: (1) negative externalities generated by poor families and which are of private concern to the middle-class and (2) positive externalities generated by middle class families and which are of interest to the social planner at least in part because of their effect on the population dynamics. On the other hand, Schelling's model relied on a spatial concept of neighborhood, whereas we abstract from an explicit grid representation of neighborhood location.

As so often occurs, bringing new methods to a problem domain generates results that are interesting from the methodological perspective. In this case we find a one-state model with two so-called DNS thresholds and a "lens" that focuses candidate solution trajectories in a way that lets them pass through a singularity. Mathematical properties of the lens are explored by Caulkins et al. (2004), who primarily consider a simpler version of this model (one without assimilation) and for more arbitrary parameter values. Here we simply use those properties when establishing what are the optimal solutions; readers are referred to the earlier paper for derivation of those properties.

Paper structure. In Section 2 we elaborate on the policy context examined. Section 3 introduces the formal assumptions and formulation of the mathematical model, while Section 4 provides its qualitative analysis. In Section 5 we discuss what are realistic parameter values for the model. Section 6 describes the results of numerical simulations and comparative statics, that often reveal structural changes. It is a core part of the paper, since many policy issues are also discussed here, as well as bifurcation diagrams and other simulation results. The paper ends with conclusions and policy implications.

2. Policy context

The recent emphasis on integrating low-income families into middle-class neighborhoods represents a substantial shift in housing policy. Public housing in the US was originally largely a product of the Great Depression and immediate post World-War II era. The emphasis was on increasing the supply of affordable housing by building geographically concentrated projects run by public housing authorities (PHAs). Much of the country was racially segregated at the time, and PHAs were no different. As Popkin, Rosenbaum, and Meaden (1993, p. 179) note, "Over several generations, many public housing authorities established and perpetuated racially segregated and discriminatory systems for public housing and other housing assistance, in violation of fair housing laws."

A series of civil rights lawsuits begun in the 1960s and continuing into the 1980s compelled PHAs to de-segregate these housing projects. In a number of cities it was difficult to achieve integration by placing minorities in majority white projects and vice versa, e.g., because the vast majority of current residents and of people seeking public housing were minority or because members of one group refused offers of placement in facilities where they would be in the minority. Indeed in a series of Texas court cases known collectively as the Young case it was ruled that massive and mandatory transfer of tenants within public housing should not be seen as the primary remedy.

An alternative strategy was pioneered in the settlement to the Gautreaux class-action suit originally filed in Chicago in 1966. That settlement sought to remediate past discrimination by moving low-income African-Americans to white areas via “Section 8” certificates and vouchers that subsidized the rent tenants paid to private landlords in “scattered site housing” instead of placing them in other PHA projects.

Evaluations of the Gautreaux experiment found that participants who moved to suburban communities enjoyed better educational and labor market outcomes than did participants who moved to neighborhoods within the central city (Popkin, Rosenbaum, and Meaden 1993, Rosenbaum 1995). These results came to light at a time when a consensus was emerging in the social science literature that neighborhood attributes influence a variety of economic, health, and criminal behaviors, and that the proportion of neighborhood households with incomes below the poverty line is a particularly important attribute (e.g., Mayer and Jencks 1989, Massey, Gross, and Eggers 1991, Turner and Gould Ellen 1997).

In light of these developments, the Clinton Administration broke with the past policy of contesting the lawsuits and fundamentally changed the goal of federal housing policy from supplying housing units to expanding opportunities for tenants to live in high-quality, desegregated neighborhoods, both by converting traditional housing projects into mixed-income residential developments (so-called “HOPE VI” revitalizations) and by aggressively using vouchers to place tenants in privately-owned scattered site housing.

A leading example of this new policy is the Moving To Opportunity for Fair Housing (MTO) program launched by the US Department of Housing and Urban Development in 1994. Unlike the Gautreaux settlement, MTO was cast in terms of economic not racial integration. It employed means testing, not race, to determine eligibility, seeking to place poor tenants in middle-class neighborhoods (specifically, those in which fewer than 10% of families had incomes below the poverty line). Hence, in this paper we describe the problem in terms of placing poor families in middle-class neighborhoods, not ethnic minority families in white neighborhoods. This is not meant in any way to deny the long history of overt and tacit racial and ethnic discrimination related to housing opportunity (Galster 1988). Indeed, race and economic-status are not separable in defining disadvantage or opportunity in this context. Still, there are reasons for preferring to cast the problem in economic terms. First, in other countries and, indeed, sometimes in the US, integration pertains primarily to international immigrants who are not necessarily of a different race than the “domestic” population (e.g., Betts and Fairlie 2003). Second, even in the US, racial and ethnic composition and tensions are more complicated than black and white, with evidence of one minority group fleeing from another (Fairlie 2002) and middle-class minorities fleeing from poor minorities (Winsberg 1985, Murphy 1995). Finally, residential segregation by race has been falling – albeit very slowly – in the US since 1970, but economic segregation within race has been rising, particularly for minorities (Jargowsky 1996).

A central concern with government interventions designed to place poor families in middle-class neighborhoods is the possibility of adverse economic impact on the neighborhood, e.g., through declining property values (Galster, Tatian, and Smith 1999, Santiago, Galster, and Tatian 2001) or triggering a downward economic spiral if long-time residents move away ("middle-class flight" or "white flight" in the racial integration context). There is a large literature on "flight" that debates many things such as whether the primary driver is increased egress or normal turnover coupled with lack of replacement (c.f., Galster 1998, Gould Ellen 2000), but with some exceptions concerning particular issues (e.g., Freeman and Rohe 2000), the consensus is that flight still occurs and is an important issue (Clotfelter 2001).

Clearly if the principal policy objective is to integrate poor families into middle-class neighborhoods, that objective is undermined if there is a large net exodus of middle-class neighbors. Hence, we view the policy planner's objective with respect to a given neighborhood as two-fold: place there as many poor families as possible while simultaneously maximizing the number of middle-class residents. This dual-objective is broader than some in the literature, which focus exclusively on outcomes for the PHA/Section 8 tenants, although it still neglects outcomes for poor families "left behind" in the neighborhoods from which the MTO families come (cf., Johnson and Hurter 2000). This exclusion can be partially justified on grounds of tractability, lack of basis for estimating outcomes for those left behind, and the parochial interests of municipal policy makers when the movement from poor to middle-class neighborhoods crosses local jurisdictional boundaries.

Before proceeding, some caveats are in order. First, policy makers can counsel or direct families to particular jurisdictions but not allocate them directly, so the control variable is an abstraction of the overall counseling process. Second, we do not know for sure that de-concentrating poverty is a good policy. Initial evidence related to short-term MTO outcomes Johnson, Ladd, and Ludwig (2001) indicates that, compared to control groups, "Section 8-only" and "experimental" groups have enjoyed substantial benefits, but it is not clear whether the effects will persist or whether they could be replicated if the program were scaled up nationwide. Also, Galster (2002) notes that de-concentrating poverty only improves aggregate social welfare under certain moderately strong assumptions concerning individual responses to neighborhood poverty levels. After reviewing the literature, Mayer and Jencks (1989) observe that the empirical evidence is somewhat weak, outcomes can vary by context, and affluent neighbors can be an advantage in some respects and a disadvantage in other respects. Manski (1993) elaborates on why the econometric identification problems limit the inferences that can reliably be drawn from the empirical record. Furthermore, De Souza Briggs (2003) notes there is something of a choice between "cure" strategies that try to reduce segregation and "mitigate" strategies that seek to reduce the social costs of segregation without actually changing where people are willing and able to reside.

In this paper we take no position concerning whether integrating poor families into middle-class neighborhoods is the right overall strategy. Rather, given that overall policy directive, we explore the fundamental management question: how best could such a strategy be implemented?

3. The model

The model presented here was first introduced by Caulkins et al. (2004). It is highly stylized, and many considerations are suppressed in the interest of framing an essential dynamic of the problem in a novel way.

The key measure of the health of the neighborhood is taken to be the number of middle-class families who live there at time t, denoted by the state variable $X(t)$. The key policy variable is the rate at which poor families are placed in the neighborhood, denoted by the control variable $u(t)$.[7]

The number of middle-class families, X, varies over time due to three influences. First, there are the underlying natural or "uncontrolled" dynamics that would pertain even if there were no MTO policy intervention (i.e., $u = 0$). In many respects, housing markets operate like other economic markets, with price adjusting to balance supply and demand. For simplicity we imagine the neighborhood is an established, fully-developed area so the housing stock is fixed at a size that would under normal circumstances support some given population (without loss of generality normalized to be unity). If the resident population falls below that level, presumably local prices would decline, attracting immigration from other, comparable middle-class neighborhoods. Conversely, if the neighborhood population grew beyond its normal level ($X > 1$), residents would flow to other middle-class neighborhoods that were less congested. Realistically, the neighborhood's normal population density depends on the surrounding city's overall growth trajectory. If the city were booming, the neighborhood's normal population density would increase over time. Conversely if the population base were eroding. We abstract from such considerations and imagine that the normal population for this neighborhood is constant over time. There

[7] Note that the time argument t will mostly be omitted in what follows.

does not appear to be a definitive, standard model in the literature for describing these natural adjustment processes. Hence, for convenience we adopt the familiar logistic growth curve.

The second factor influencing changes in the stock of middle-class residents is "middle-class flight" (and/or "middle-class avoidance") induced by the placement of poor families in the neighborhood. Again there does not appear to be a standard functional form in the literature, in part because the reality is rather more complicated than what can be captured in a one-state model. For example, Galster (1998) finds that racial transition depends not only on racial composition in the immediate neighborhood, but also on proximity to other majority minority areas, attitudes, and affirmative marketing strategies. Likewise, a one-state model cannot reflect details of the spatial distribution (DeMarco and Galster 1993). Also, flight may be driven not only by immigration into the residential neighborhood but rather by immigration of children into the school district (e.g., Clotfelter 2001, Fairlie 2002). Perhaps not surprisingly some subgroups appear more likely to flee than others. E.g., Gould Ellen (2000) argues that homeowners are more likely to leave than are renters and that families with children are more likely to flee than families without children, particularly if the children attend public schools.

One central question is whether flight is driven more by the current inflow of poor immigrants or by their accumulation over time, perhaps relative to the size of the stock of middle-class families. On that point, there seems to be some reason to believe it is the current inflow. E.g., Gould Ellen (2000, p. 686) argues that "whites do not appear to care very much about the proportion of a neighborhood that is African-American, [but] whites do tend to avoid neighborhoods in which the proportion of families who are African-American is increasing (independent of the current size of the minority population)". Again we opt for the benefits of simplicity and assume that middle-class flight is simply proportional to the rate of inflow of poor families. This is akin to the finding of Betts and Fairlie (2003) in the context of native-born and immigrant populations that "For every four immigrants who arrive in public high schools, it is estimated that one native student switches to a private school."

The final factor influencing changes in the stock of middle-class residents is the rate at which incoming poor families are "assimilated". That such assimilation can occur is in some sense an underlying premise of the overall policy. The idea is not so much that poor families moving to middle-class neighborhoods will simply derive great short-term happiness from having affluent neighbors. Indeed, as Mayer and Jencks (1989) note, one school of thought emphasizes that affluent neighbors can provoke resentment among the poor over their relative deprivation. Rather, the hope is that immersion in a middle-class neighborhood will improve outcomes, including labor market participation and income, for the poor adults and educational outcomes for their children, which translate into social opportunity and higher incomes over time.

Presumably the number of poor people assimilating is increasing both in the number who are placed and, hence, are candidates for assimilation (i.e., is increasing in u) and also in the number of middle-class neighbors (X). Again, the literature does not offer guidance as to what functional forms might be preferred. At the suggestion of Rosser, we opt for a simple bi-linear relationship: assimilation is proportional to the product of u and X.

One potentially awkward aspect of this simple form is that if X is large enough, the proportionality constant γ times X may exceed 1, implying that the number of poor people being assimilated can exceed the number of poor people being placed there by the policy maker through the housing subsidy program. That is by no means implausible since poor people in the program might attract other people to enter the neighborhood who are not part of the formal program and, hence, do not have their flow enter the cost-function and are not so visible as to affect out-migration. E.g., residents might be very conscious of poor people placed by a formal public program, but their friends who just move in on their own may not be noticed and, hence, might not engender the same amount of middle-class flight. Nevertheless, it will be important to remember this interpretation when the analysis suggests the optimal solution has $X\gamma > 1$.

The other great simplification of assimilation in this model stems directly from having only a single state variable to represent the neighborhood's current population. As a result, newly placed poor families either assimilate immediately or depart. There are not additional states that explicitly model the gradual process by which a family remaining in the neighborhood moves up the socio-economic ladder over time. Elaborating on that dimension would be an important extension for further research.

Together these consideration suggest the dynamic state equation:

$$\dot{X} = aX(1 - X) - \beta u + \gamma X u, \tag{1}$$

where a, β and γ are positive constants governing, respectively, the speed with which the equilibrium population is approached, the extent of middle-class flight, and the rate of assimilation of poor families into the middle class. Note that families that are not assimilated play no further role in the state dynamics because there is not a second state variable to track poor people living in the neighborhood. Practically speaking, those that do not assimilate might well leave the community, particularly after their eligibility for tenant-based housing subsidies expires and they would have to pay full market rates for housing.

To complete the formulation we must specify an objective function. We adopt the familiar perspective of maximizing the infinite-horizon, discounted net social benefit. [8] The

[8] Note that we assume an infinite planning horizon for reasons of analytical tractability. For instance, this assumption is essential for the use of the powerful phase-plane analyses as carried out later in this paper. However, it is certainly also possible to deal with a finite horizon.

review of the policy context above justifies counting both placing poor families (u) and maintaining middle-class residents (X) as socially beneficial. Without loss of generality, we scale the benefit function so the coefficient on u is unity and let ρ denote the relative benefit per unit time of X compared with u.

Finally, we presume that the policy intervention itself carries some cost, including administrative costs, costs of counseling families concerning their initial placement and follow up, property value impacts, and deadweight losses associated with constraining choices. We presume for the standard diminishing returns arguments that these costs are convex. Again in the absence of evidence favoring one specific functional form over another, we opt for simplicity and use a quadratic form. The linear term is subsumed into the (scaled) coefficient on u, leaving only the squared term to appear independently in the objective function.

Hence, the dynamic optimization problem we wish to analyze can be written:

$$\max_{\{u(t)\geq 0\}} \int_0^\infty e^{-rt}(u(t) - cu^2(t) + \rho X(t))dt,$$
$$s.t. \quad \dot{X}(t) = aX(t)(1 - X(t)) - \beta u(t) + \gamma X(t)u(t), \tag{2}$$

where r is the time discount rate and c is the program cost coefficient.

It should be clear to the reader that this is a highly stylized model not only in its structural simplicity (e.g., using a single state variable to represent the neighborhood's resident population) but also in terms of the functional forms employed, none of which have been validated in any sense of the word. Hence, we focus on qualitative insights, not specific numerical results and view this entire paper as somewhat exploratory.

4. Qualitative analysis

4.1 Derivation of the canonical system

The analysis proceeds in the usual manner for an optimal dynamic control problem (cf., Chiang 1992, Feichtinger and Hartl 1986). The current value Hamiltonian, denoted by H,

$$H = (u - cu^2 + \rho X) + \lambda[aX(1 - X) - \beta u + \gamma Xu], \tag{3}$$

leads to the following set of first order conditions:

$$\dot{X} = aX(1 - X) - \beta u + \gamma Xu, \tag{4}$$
$$\dot{\lambda} = \lambda(r - a(1 - 2X) - \gamma u) - \rho, \tag{5}$$
$$H_u = 1 - 2cu - \lambda\beta + \lambda\gamma X = 0, \tag{6}$$

where the Hamiltonian maximizing condition (6) must hold for interior solutions. [9]

If we exclude λ, then our system can be written in the $X - u$ space:

$$\dot{X} = aX(1-X) - \beta u + \gamma X u, \tag{7}$$

$$\dot{u} = \frac{\gamma(1-2cu)}{2c(\beta - \gamma X)}[aX(1-X) - \beta u + \gamma X u] + \frac{1-2cu}{2c}[a - 2aX + \gamma u - r] + \frac{\rho}{2c}(\beta - \gamma X). \tag{8}$$

4.2 Isoclines and steady states

We start the analysis by locating the steady states of the canonical system. The first isocline, $\dot{X} = 0$, leads to the equation

$$u(X) = \frac{aX(1-X)}{\beta - \gamma X}. \tag{9}$$

It always has two roots (i.e., $X = 0$ and $X = 1$) and a vertical asymptote at $X = X^*, X^* \equiv \beta/\gamma$, separating two branches of this curve. The general shape depends on the value of the critical parameter X^*. If $X^* < 1$, both branches have positive slope, and while crossing X^* the value of u changes from $+\infty$ to $-\infty$. If $X^* > 1$, the left branch has an inverse-U shape, while the right branch is U-shaped.

The second isocline, $\dot{u} = 0$, has a more complicated form, but some qualitative insights can still be gained. First of all, it has a singularity in the point $X = X^*$. Hence, the vertical line $X = \beta/\gamma$ marks the border between two zones, where the vector field is continuous. This singularity is broken only in one point, which belongs to the isocline itself. The intersection between the isocline $\dot{u} = 0$ and the singular line $X = X^*$ takes place at $(X = \beta/\gamma \equiv X^*, u = 1/(2c) \equiv u^*)$. This point (which we will call the **critical point**) has an important role, and its properties will be elaborated below.

The points of intersection between the isoclines give us candidates for equilibria. Note that at all points satisfying $\dot{X} = 0$, the expression for $\dot{u} = 0$ simplifies considerably, so we can solve for

$$X(u) = \frac{(1-2cu)(a - r + \gamma u) + \rho\beta}{2a(1-2cu) + \rho\gamma}, \tag{10}$$

which can be studied together with the equation $u(X)$ derived above. In the asymptotical case $c \to 0$, $X(u) = c_1 + c_2 u + O(c)$, where $c_2 = \gamma/(2a + \rho\gamma) > 0$. Such a positively

[9] Since $H_{uu} = -2c < 0$, even the second order condition for an interior maximum is satisfied.

sloped curve in many cases will have at least two intersections with $u(X)$. Thus, generically we have multiple equilibria. Having $c > 0$ makes the curve $X(u)$ nonlinear, and this can increase the number of equilibria. Further analysis can be done using numerical methods. For analytical results and the special case of no-assimilation (i.e., $\gamma = 0$) see Caulkins et al. (2004).

4.3 Critical line and lens

Before proceeding with the analysis of the case $\gamma > 0$, we need to address one mathematical oddity. Typically when solving dynamical systems, if there is more than one saddle point, then those saddle points are separated by some unstable equilibria. However, it turns out that for many parameter values, we find for $X > 0$ three saddle points but just one unstable equilibrium. Obviously in a one-state model, one unstable equilibrium can't separate all pairs of saddle points. It turns out that in this model the critical point behaves in a rather interesting and unusual way. It is not a steady state, but it acts like an unstable equilibrium in the sense of separating successive saddle point equilibria.

The properties of this critical point are explored by Caulkins et al. (2004) who show that a finite measure of trajectories can pass between two semiplanes separated by a singular border. They converge in a lens formed by the critical point, and then diverge. If $X^* < 1$, these trajectories pass through the lens from left to right; if $X^* > 1$ they move from right to left. These observations solve the paradox about the coexistence of one unstable node with three saddles and helps us to determine which dynamic trajectories are candidates for optimal solutions.

4.4 Limitations of the qualitative analysis

Except for the special case $\gamma = 0$ the underlying problem is too hard to address analytically. That special case of no assimilation is investigated by Caulkins et al. (2004). Here we pursue the more realistic and complicated case for $\gamma > 0$ numerically. To do so, it is important to give some thought to appropriate ranges for the parameter values, which is the content of the following section.

5. Choice of parameters

5.1 Housing market adjustment speed coefficient, a

The uncontrolled dynamics are $\dot{X} = aX(1 - X)$. That means parameter a can be interpreted as the half-life of decay of vacancies when the neighborhood is near its uncontrolled equilibrium. That is, how long, on average, does it take for a house to sell in a healthy middle-class neighborhood (X close to 1)?

Over the last 15 years, the average for a new home has been between 3.6 and 6.9 months (U.S. Census Bureau 2004a). The average of those averages is 4.89, suggesting $a = (12/4.89) * ln(2) = 1.7$, which we round up to a base case of 2.

However, the American Housing Survey 2001 (AHS) (U.S. Census Bureau 2004b) indicates that there is considerable variation in time spent on the market, and lower values of parameter a can yield qualitatively different results, so we also explore the consequences of introducing housing mobility programs to neighborhoods with weaker real estate markets. The AHS data indicate that 13.4% of units were on the market for more than 2 years. The data are reported in categories, so we do not know exactly what vacancy period is the 90th percentile. It is clearly above 24 months and might be on the order of 36 months ($a = 0.231$). For a round number we take $a = 0.2$ (vacancy period 41.6 months) because that is our base case value ($a = 2$) divided by 10.

5.2 Assimilation coefficient, γ

The parameter γ reflects the "success" or "assimilation" rate for persons who participate in housing mobility programs, i.e. the proportion of families initially placed in low-poverty or low-percent minority (what we call "middle-class") neighborhood who stay for an extended period of time.

The MTO Interim Impacts Evaluation (U.S. Department of Housing and Urban Development 2003) reports that 35% of all "experimental group" families who successfully found rental housing between 1994 and 1998 were recorded in 2002 as living in Census tracts with Census 2000 poverty rates of 20% or less. Also for MTO, Shroder (2001) reports that 64% of treatment group started on welfare. Thirteen quarters later it fell to 34%, suggesting an assimilation rates of 1 - $34/64 = 47\%$.

DeLuca and Rosenbaum (2003) concluded that for the Gautreaux program, after an average of 17 years post-move, about 57% of suburban movers remain in the suburbs. Also, about 37% of participants who moved to neighborhoods that were 15% black or less initially remain in neighborhoods that are 30% black or less. This would imply a success rate for the Gautreaux Program of between 37% and 57%.

Given these three figures ($37\%, 47\%, 57\%$), we set $\gamma = 45\%$ in our base case.

5.3 Flight coefficient, β

Betts and Fairlie (2003) find that one native-born person moved out of the school district for every four immigrants entering. That suggests that $\beta = 0.25$ in their context. Flight from lower-class could be stronger than flight from immigrants, suggesting somewhat larger values of β in our context.

Gould Ellen (2000, p. 124) reports that "The probability of the typical white home-owner moving is between 2.0 and 3.5 percentage points higher when the black population has grown by 10 percentage points over the decade." For a neighborhood around the uncontrolled steady state, an increase of 10% over the decade means $\gamma u = 0.01$. If that inflow leads to a 0.02 to 0.035 increase in the per capita outflow rate, that means $\beta u = 0.02$ - 0.035 and, hence, $\beta = 2$ - 3.5γ. Given $\gamma = 0.45$, that implies β in the range of 0.9 - 1.575.

In light of this, we take $\beta = 0.5$ as our base case, but also consider sensitivity excursions with smaller ($\beta = 0.25$) and larger ($\beta = 1.0$ or 1.5) values.

5.4 Objective function coefficient for X, ρ

Recall that ρ is the value per middle class family per year relative to the value per low-income family placed, so first we need the value per low-income family placed. The ONDCP (2002, Table 13) is a federal government agency that offers an estimate of the monetary value of saving a high-risk youth, based on the work by Cohen (1998). Interpolating for a 5% discount rate, the values would be \$ 1.1 - $1.6M$.

Shroder (2001) reports that for kids in MTO treatment vs. control: (1) 21% vs. 35% had trouble with teachers, (2) 21% vs. 32% were disobedient at home, (3) 5% vs. 19% were "mean or cruel to others", and (4) 16% vs. 28% were "unhappy, sad, or depressed". These short-term findings might suggest that about 12% of kids will be "saved", but gains can erode over time. The MTO experiment is too recent for there to be long-run results, but Caulkins et al. (2002) found that, in the context of drug prevention, long-term gains were about one-fifth of short-term gains in percentage terms. So assuming an average of 2.5 kids per family, the social benefit per family placed might be something on the order of $\$1.1M$ - $1.6M * 12\% * 2.5 * 20\% = \$66,000$ - $\$96,000$, or an average of about $\$80,000$.

The average cost per family placed is roughly the \$70M Congress appropriated divided by 2414 households = $\$29,000$ per household placed, or a net benefit of $\$80,000 - \$29,000$ or about $\$50,000$.

When X declines in this model, it is the result of middle class flight, not middle class death, so it is relevant to focus on the local municipality's marginal loss of local tax revenue. Personal income in the US in 1995 was about \$4.3B (counting wages, salary, other labor income, and proprietor's income, but not rental, dividend, or interest income because many states tax only earned income), or about $\$43,000$ per household. Assuming a 2.5% municipal income tax, that is roughly \$1,000 per year in foregone tax revenues, suggesting values of ρ of $\$1,000/\$50,000 = 0.02$.

5.5 Quadratic cost coefficient, c

There is currently no empirical basis for estimating parameter c. The inflow u in the five MTO cities was about 0.24 per $1,000$ current residents or $u = 0.0002$. That is so small that the cu^2 term is negligible and, indeed, there is no discernable relationship between costs per person placed and placement intensity across the five cities.

Hence, what value of parameter c makes sense is best thought of by reference to the instantaneous part of the objective function pertaining to the control: $u - cu^2$. This says that, leaving aside the ρX term, the instantaneous satisfaction the policy maker derives is maximized when $u = 1/(2c)$ and is driven to zero if $u = 1/c$. In other words, when focusing only on short-run considerations, the convex program costs make the preferred rate of poverty de-concentration in the short run $u = 1/(2c)$ and by the time $u = 1/c$ they would offset all of the poverty de-concentration benefits. A value of $c = 20$ implies that these rates are placing one poor family per 40 middle class families per year and one poor family per 20 middle class families per year, respectively. At one level these seem about right. The first might be a good target; the second might be overly aggressive. However, those judgments are probably tempered by long-run considerations including flight and assimilation. Focusing only on the short-run considerations driven by the convexity of the program cost structure, the optimal and maximum desirable placement rates might be higher, so we also consider examples below with lower values of the parameter c, specifically $c = 2$. Given the lack of empirical basis, we view $c = 20$ and $c = 2$ as both being in some sense base case values, rather than arbitrarily anointing $c = 20$ as the best guess and viewing $c = 2$ as merely a sensitivity excursion.

5.6 Discount rate, r

The annual social planner discount rate is customarily between 3% and 7%. We take as our base value $r = 0.05$.

5.7 Summary of parameter values

Table 1 summarizes the parameter values used in numerical experiments. In this paper we maintain the values of $r = 0.05$, $\rho = 0.02$ and $\gamma = 0.45$, but allow parameters a, c and β to vary, when performing comparative static analysis. Many other parameter combinations were also investigated, but they do not augment significantly those represented here or, in the case of varying ρ, in Caulkins et al. (2004).

Table 1: Summary of estimates for model parameters

Parameter	Base Case
r, discount rate	0.05
a, housing market adjustment speed	2
β, flight coefficient	0.5
γ, assimilation coefficient	0.45
ρ, objective function coefficient on middle class state	0.02
c, program cost coefficient	2 and 20

6. Results of numerical simulations

6.1 Economic meaning of different equilibria

Numerical simulations reveal the existence of up to three saddle point equilibria of different types. To understand them intuitively, it is useful to contrast them with what happens when there is no inflow of poor families. In that case, the model reduces to the logistic equation $\dot{X} = aX(1-X)$, which has unstable and stable steady states at $X = 0$ and $X = 1$, respectively. The second dominates because for any positive initial state ($X(0) > 0$), the trajectories always converge to this natural or uncontrolled size of the neighborhood ($X = 1$).

With a mobility program ($u > 0$), two other factors in addition to own growth ($aX(1-X)$) influence the dynamics of X, namely flight due to immigration ($-\beta u$) and assimilation of immigrants ($\gamma X u$).[10] The relative and absolute magnitudes of these three flows differ in the three types of saddle point equilibria.

In the first type of equilibrium, the state X is substantially below the natural level 1, undermining assimilation. In such equilibria, it is growth via the logistic term (in-migration of families attracted by a relatively abundant housing stock) that is primarily responsible for off-setting mobility program-induced middle-class flight. In the case when the equilibrium X is small enough that the natural logistic growth term ($aX(1-X)$) is small, then u must be quite small as well. This is a relatively unhealthy community, but it could still be optimal to pursue a policy that creates such a community if the decision maker is relatively short-sighted (r large) and values highly program placements relative to other considerations (ρ and c small). Such equilibria might be called "small X, modest u equilibria."

Another type of equilibrium with $u > 0$ is close to the uncontrolled equilibrium (X near 1). At this point the natural population growth is small (logistic term is close

[10] The functional form of the assimilation term was proposed to the authors by Barkley Rosser (personal communication).

to 0), and the typically modest inflow of poor families (u small to moderate) generates assimilation and flight to roughly equal degrees, leaving the middle-class population very near its normal level in the absence of a control (X close to 1). In such an equilibrium, the poverty de-concentration program does not dramatically alter the character of the neighborhood, and we might call it a "low u, average X equilibrium."

In the third type of equilibrium the neighborhood is densely populated ($X > 1$), so not only program-induced middle-class flight but also the natural population dynamics tend to reduce X. Assimilation is the only factor tending to increase X and preserve the high population density. Due to the assumption that assimilation is proportional to the level of X, a high proportion of the fairly large inflow of poor families, u, is assimilated. However, even the middle-class residents are relatively transient, leaving at high rates. Such equilibria might be called "high X, high u" equilibria, and be thought of as akin to the transition neighborhoods in New York City that assimilate foreign immigrants.

Depending on the parameter values, 1, 2, or all 3 equilibria may exist as saddle point solutions to the canonical system. In some sense the policy questions in this model boil down to: for any given initial neighborhood, under what conditions is it optimal to approach each of these three types of neighborhoods and through what immigration "trajectory" u?

6.2 Basecase results

With base case parameter values, there are saddles corresponding to all three equilibria. There is a vortex between the first two, and the lens separates the second and third. However, the middle saddle, with $X = 0.9994$, dominates. That means, regardless of the initial state, it is always optimal to converge to a standard middle class neighborhood with mobility programs run at such a modest rate that they have minimal impact on the neighborhood.[11]

That the third saddle can never be a long-run optimum follows directly from the properties of the lens sketched above. Since the flow lines all pass from right to left, all trajectories starting with large X pass through the lens from right to left and must approach a long-run equilibrium that is to the left of the lens.

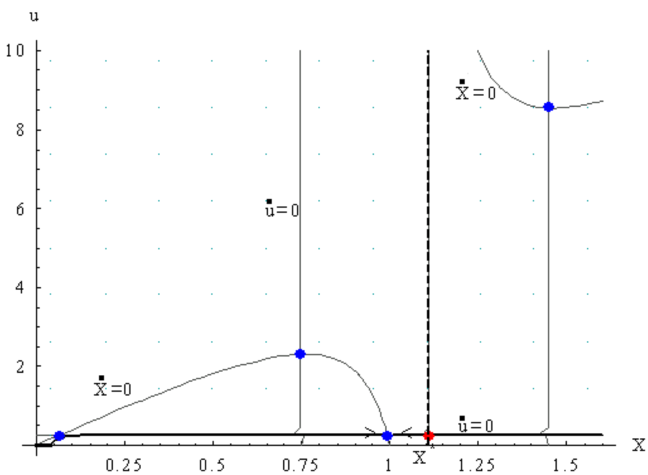
That the small X equilibrium can never be optimal requires a numerical proof analogous to that given in Caulkins et al. (2004). Intuitively, the essence of the proof is that the trajectories approaching the smallest saddle point all involve using mobility programs at levels with $u > 1/2c$, which does not make economic sense. (With such high u, the short-term costs of the mobility program undercut even its short-run benefits. Furthermore, there is rapid erosion of the middle-class population due to flight, and the shadow

[11] Later we will refer to this case as Fig. 0, but we do not present it in the paper.

value of the middle class population is always positive.)

If c were smaller, then $1/2c$ would be larger, so given the uncertainty concerning the quadratic cost coefficient c (see above), this suggests examining results for a smaller $c = 2$, not $c = 20$. The resulting phase portrait (Fig. 1) is topologically identical. (It, rather than the portrait with $c = 20$ is shown because it is easier to read.) The long-run equilibrium shifts slightly from $X = 0.9994$ to 0.99333, but otherwise there is little difference.[12]

Figure 1: Parameter values: $\mathbf{a = 2}, \beta = \mathbf{0.5}, \mathbf{c = 2}, \gamma = \mathbf{0.45}, \mathbf{r = 0.05}, \rho = \mathbf{0.02}$

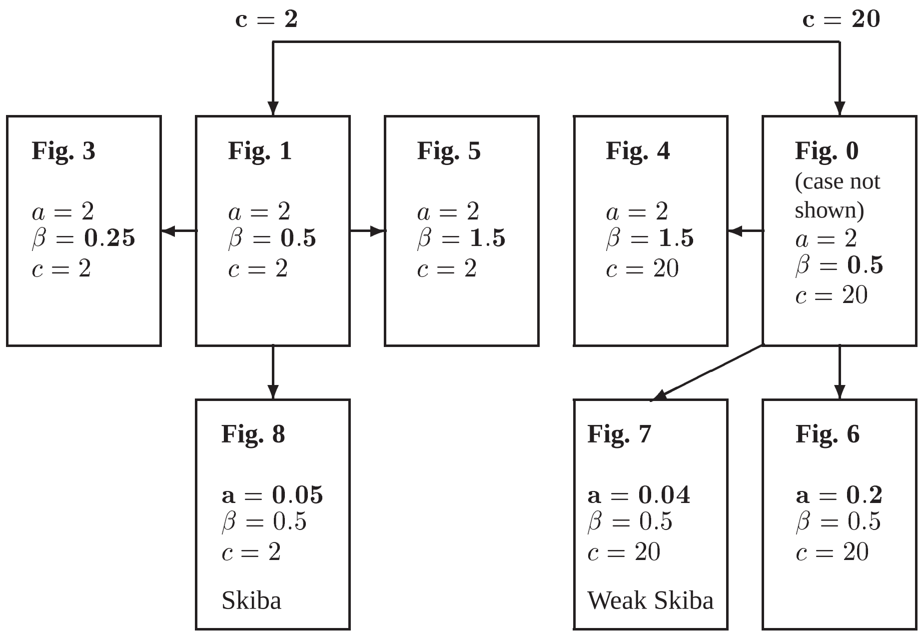


Note: The saddle $\mathbf{X = 0.993}$ is optimal. Saddle paths from both left and right are shown; they almost coincide with $\mathbf{u} = \frac{\mathbf{1}}{\mathbf{2c}}$. The steady states representing small and large cities are suboptimal.

In the following subsections we deal with sensitivity analyses for different parameters. For a better understanding of how the different cases discussed are interrelated, we provide a “road-map” in Fig. 2. This “road-map” shows interrelations between all considered cases, marked as corresponding figures with particular parameter values.

[12] Clearly, the corresponding u grows approximately by factor 10, from $u = 0.0249$ to $u = 0.249$, always staying slightly below $1/(2c)$.

Figure 2: “Road-map” showing the link between different cases investigated

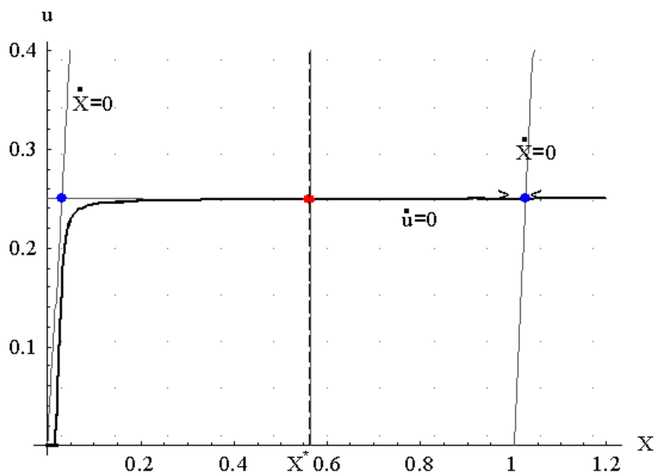


Note: In all cases, $\gamma = \mathbf{0.45}, \mathbf{r} = \mathbf{0.05}, \rho = \mathbf{0.02}$. We start from two baseline cases: Fig. 0 ($\mathbf{c} = \mathbf{20}$) and Fig. 1 ($\mathbf{c} = \mathbf{2}$). In both cases, $\mathbf{a} = \mathbf{2}$ and $\beta = \mathbf{0.5}$. From there we continue by changing parameter β, which also can take the values $\mathbf{0.25}$ (Fig. 3) and $\mathbf{1.5}$ (Figs. 4 and 5). Note that Figs. 4 and 5 differ only in the value of $\mathbf{c}$. Three figures at the bottom line have different values for parameter $\mathbf{a}$. In the cases of very low *a* (Figs. 7 and 8) we have Skiba points.

6.3 Sensitivity with respect to flight parameter β

Mathematically, decreasing the amount of middle-class flight by reducing β from 0.5 to 0.25 changes the picture considerably. Now the lens separates the first two saddle points, passing the flow from left to right. The small X equilibrium cannot be optimal due to a theorem that says that the objective function value for a given level of the state variable is increasing in the distance to the $\dot{X} = 0$ isocline (see, e.g., Feichtinger and Hartl 1986, p. 118), because the trajectory converging to the small X steady state is closer to this isocline than is the path leading to the equilibrium close to 1. The vortex and the far-right saddle disappear, leaving the saddle near $X = 1$ as the only true candidate for a long-run equilibrium. That is, this reduction in the flight parameter only reinforces the strength of the middle-type saddle. Substantively, the only consequence is that this greater strength allows the mobility programs to increase the population to slightly more than its uncontrolled steady state. In particular, we obtain $u = 0.2505$ and X shifts from 0.9933 to 1.025 (Fig. 3).

Figure 3: Isoclines and saddle paths for the case $\mathbf{a = 2}, \beta = \mathbf{0.25}, \mathbf{c = 2}, \gamma = \mathbf{0.45}, \mathbf{r = 0.05}, \rho = \mathbf{0.02}$



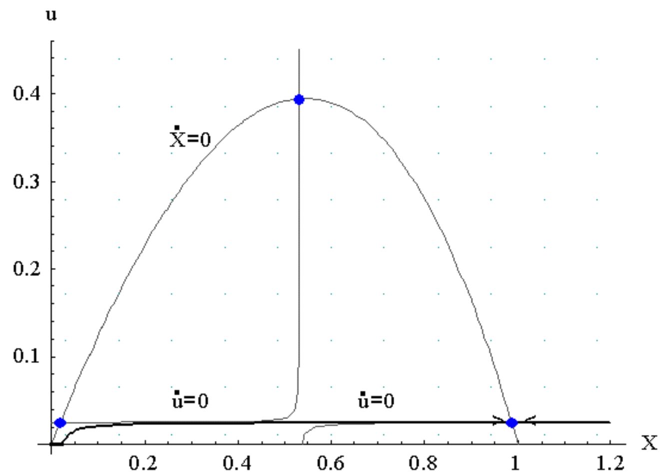
Note: The critical line is shown as dashed. There are 2 saddles, and the right (close to $\mathbf{X = 1}$) is optimal.

Increasing the amount of middle-class flight by increasing β from 0.5 to 1.0 with $c = 20$ not only does not change the solution much, merely shifting the long-run equilibrium from $(X = 0.9994, u = 0.0249)$ to $(X = 0.9930, u = 0.0248)$, it also leaves the topological structure intact.[13] Even increasing β to 1.5 (Fig. 4) just pushes the long-run equilibrium (middle-size city) down a little further, to $(X = 0.986, u = 0.0247)$.

However, if the mobility program is cheaper ($c = 2$) and flight is greater ($\beta = 1.5$), the combined effects move the long-run equilibrium more substantially, down to X near 0.834 instead of close to 1.0 (Fig. 5).

[13]Increasing β makes the u at the unstable node smaller and at the same time leads to an increase of the X value for the equilibrium, which corresponds to an oversized city.

Figure 4: Parameters: $\mathbf{a = 2, \beta = 1.5, c = 20, \gamma = 0.45, r = 0.05, \rho = 0.02}$

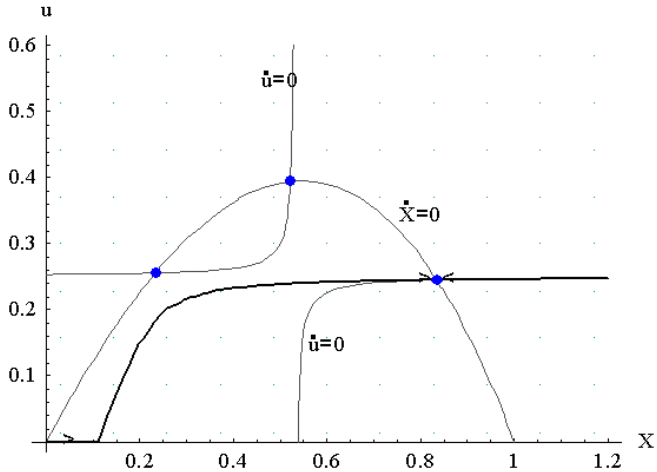


Note: The middle-size city ($\mathbf{X = 0.986}$) always dominates. Small ($\mathbf{X = 0.02}$) and oversized city ($\mathbf{X = 6.1}$, not shown) are never optimal. The fourth equilibrium ($\mathbf{X = 0.53}$) is a vortex.

As one can see from Fig. 5, the vortex has dropped substantially (lower value of u), approaching the levels of those for the saddle points. While at this point the equilibrium with X near 1 continues to dominate, that dominance could be threatened by further parameter changes, such as a decline in the objective function coefficient ρ.

To summarize, given base-case values of the other parameters or even with mobility program costs reduced (smaller c), varying the extent of flight (parameter β) seems to affect only the optimal intensity of the mobility program. (The greater the proclivity toward flight, the less aggressively the program should be pursued.) It does not alter the general strategy of preserving the essential character of the neighborhood. We next examine a parameter whose variation can lead to more fundamental changes in the policy prescription.

Figure 5: **Parameters:** $\mathbf{a = 2, \beta = 1.5, c = 2, \gamma = 0.45, r = 0.05, \rho = 0.02}$



Note: The middle-size city still dominates. Small and large city (not shown) are suboptimal. The saddle paths to the small city ($\mathbf{X = 0.23}$), consisting of a spiral from the vortex ($\mathbf{X = 0.52}$) and a path starting at ($\mathbf{X = 0.11, u = 0}$) are not shown. If the initial city size is below $\mathbf{0.12}$, it is optimal to have a boundary solution ($\mathbf{u = 0}$) until $\mathbf{X = 0.12}$, and to then follow the saddle path growing to ($\mathbf{X = 0.834}$, $\mathbf{u = 0.245}$).

6.4 Sensitivity with respect to parameter a

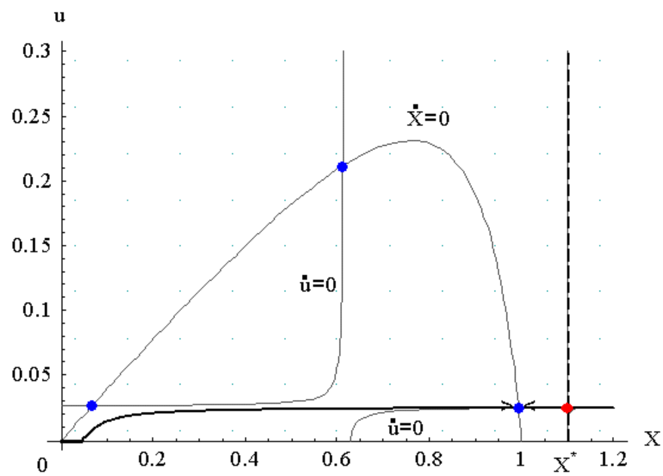
The housing market adjustment speed parameter (a) turns out to play a key role in the structure of the optimal housing mobility policy. When a is rather large, as in our base case, the middle equilibrium with X close to 1 is strong because the uncontrolled component of the dynamics ($\dot{X} = aX(1 - X)$) is powerful. One might say that our system displays homeostasis [14], and parameter a measures the strength of homeostasis. Just as a strong virus attack can overwhelm a weakened immune system, we can expect that a neighborhood with small (weakened) a can be moved out of its natural equilibrium.

We have already considered several cases for large $a = 2$, shown in Figs. 1, 3-5. Sometimes there are 4 equilibria, sometimes 2, but there is no policy impact: the middle-size neighborhood is always located near the unperturbed value $X = 1$ and it is always optimal. Small reductions in the value of a do not change this property. Even substantial reduction of a, from 2 to 0.2 (see Fig. 6) still preserve 4 equilibria: 3 saddles, corresponding to low population ($X = 0.067$), normal size ($X = 0.993$) and oversized ($X = 1.36$,

[14] This term is often used in biological sciences to describe the systems that are able to return to their natural steady state after being perturbed by external forces.

not shown), and one vortex at $X = 0.61$. Besides this complex structure, the middle-size city still remains an optimal solution.

Figure 6: Parameters: $\mathbf{a = 0.2, \beta = 0.5, c = 20, \gamma = 0.45, r = 0.05, \rho = 0.02}$



Note: Again, the middle-size city ($\mathbf{X = 0.993}$) is optimal, despite the existence of small ($\mathbf{X = 0.067}$) and large ($\mathbf{X = 1.359}$, not shown) cities. The vertical dashed line shows the critical set, with the lens at $\mathbf{u = 0.025}$. The horizontal dashed line represents the boundary solution with $\mathbf{u = 0}$, which is a part of the saddle path to the middle-sized city.

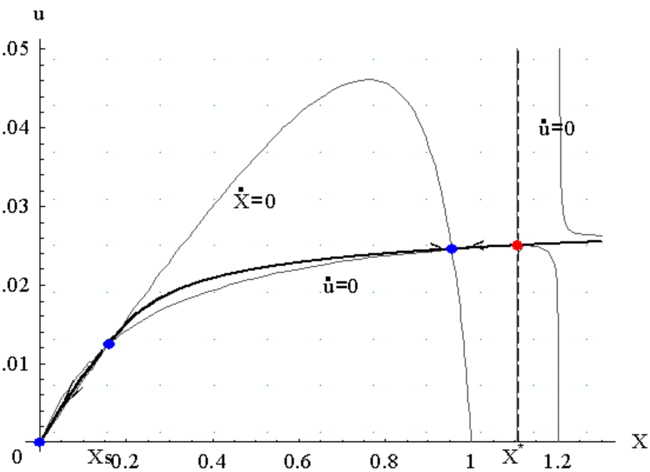
Although lower values of a might be viewed as extreme cases, we should not ignore them for two reasons. First, all of our parameter estimates involve judgments; changing some others can increase the minimum value of a such that different behaviors emerge. Second, for whatever reasons, some neighborhoods might have "weakened homeostasis" and correspondingly low a. For such values of a, we found several topologically different cases, where an optimal policy destroys the uniqueness of middle-size neighborhood as an optimal solution and sometimes even eliminates it completely.

Consider first the case of $a = 0.04$ with high program cost $c = 20$ (see Fig. 7). Here we still have 4 equilibria, and 3 of them are saddles, representing small, medium size and large city. The main difference is that the unstable steady state becomes a node now, and its location represents a weak Skiba point.[15] If we are initially located in the unstable node $(X = 0.16, u = 0.012)$, we are indifferent between either going to the low equilibrium (which represents complete destruction of the city, $(X = 0, u = 0)$ for our

[15] For some details of Skiba points and their classification see, e.g., Caulkins et al. (2004).

parameter set) or to move to middle size city, $(X = 0.95, u = 0.024)$. The oversized city $(X = 1.20, u = 0.023)$ is never optimal because the lens at $X = 1.11$ passes all trajectories from the right to the left, to converge to the middle-size neighborhood. Only initially small cities, with $X(0) < 0.16$, will eventually converge to the state with $X = 0$. [16]

Figure 7: The case of $\mathbf{a = 0.04, \beta = 0.5, c = 20, \gamma = 0.45, r = 0.05, \rho = 0.02}$

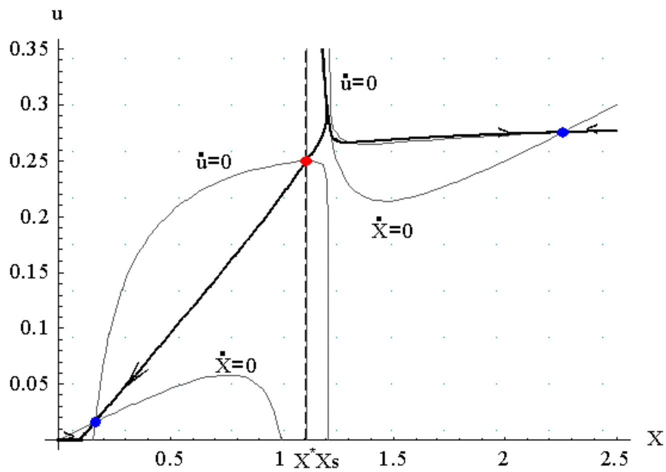


Note: The unstable node $\mathbf{Xs = 0.0158}$ is a weak Skiba threshold. Below it, the convergence is to the disappearing city with $\mathbf{X = 0}$, and above it is to the middle-size city with $\mathbf{X = 0.954}$. Trajectories starting at $\mathbf{X > X^* = 1.11}$ (including the saddle path to the middle-size city), pass through the lens at the critical line.

The results are even more dramatic when we reduce a to this low level and also use the lower placement cost $c = 2$. In general, reduction in c leads to higher levels of u in equilibrium and stronger influence on the system. Fig. 8 shows one such case with $a = 0.05$ for $c = 2$. In this case only two saddles are left: small neighborhood, with $X = 0.16, u = 0.016$, and oversized neighborhood, with $X = 2.26, u = 0.27$. Since now both solutions play an important role, it is useful to characterize them. In the oversized city the population stock exceeds the normal level by more than a factor of two, and the equilibrium flow of migrants (some of whom assimilate) is very high. At the same time, while the flow of migrants into the small neighborhood remains low, it does not have a capacity to reach more or less normal size. There exists a threshold (strong Skiba point) at

[16] For slightly different parameter values the small population equilibrium may increase to be on the order of 0.1.

Figure 8: A Skiba point $\mathbf{Xs}$ (between 2 saddles, separated by the critical line $\mathbf{X}^*$) occurs for the parameter values $\mathbf{a} = \mathbf{0.05}, \beta = \mathbf{0.5}, \mathbf{c} = \mathbf{2}, \gamma = \mathbf{0.45}$, $\mathbf{r} = \mathbf{0.05}, \rho = \mathbf{0.02}$. If $\mathbf{X}(\mathbf{0}) < \mathbf{Xs} = \mathbf{1.21}$, there is convergence to the left saddle (small city) with $\mathbf{X} = \mathbf{0.16}, \mathbf{u} = \mathbf{0.016}$, while for $\mathbf{X}(\mathbf{0}) > \mathbf{Xs}$ the convergence is to the oversized city $\mathbf{X} = \mathbf{2.26}, \mathbf{u} = \mathbf{0.27}$.

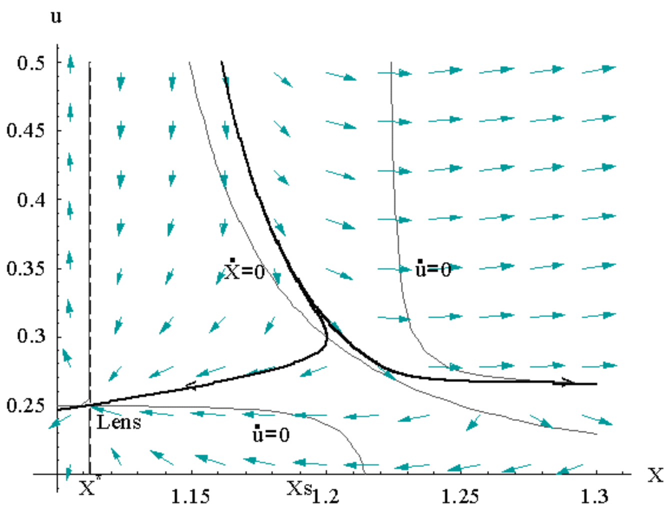


X_S, close to 1.2. If we start at this Skiba threshold, there exist two trajectories, converging to left and right saddles and having identical value of the objective. In other words, at X_S we are indifferent in selecting a path that converges either to heavily undersized or heavily oversized cities.

This case is also interesting mathematically, as it has not previously been described in the dynamic optimization literature. There exists a substantial literature about Skiba points [17], but in one-state models a Skiba point typically emerges near a vortex that separates two saddles. In our case there are just two saddles, separated by a critical line $X = X^* = 1.11$. The lens allows the trajectories to pass from the right to the left. But the direction of the field in the right neighbourhood of $X = X^*$ is such, that two trajectories can coexist only for $1.11 < X < 1.2$ (see Fig. 9). This means that for $X(0) < 1.11$ there is always a convergence to the low population equilibrium, while for $X > 1.2$ there is always a convergence to the high population equilibrium. Hence, there should exist a threshold as the border of these sets, and this can indeed be proven numerically (See Fig. 10 with the details of Skiba point and its neighbourhood).

[17] For a recent and extensive survey on these thresholds, see Deissenberg et al. (2004).

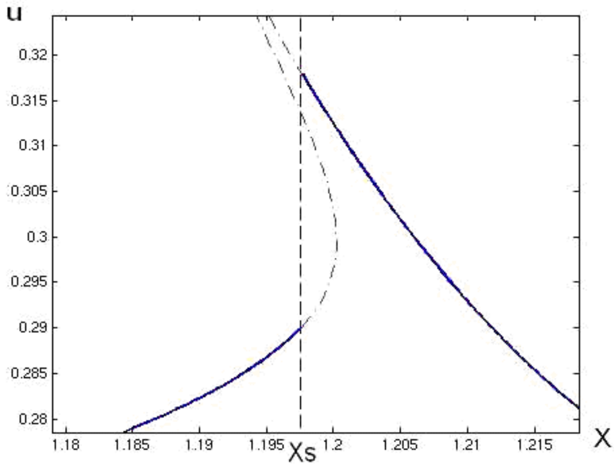
Figure 9: Detail of Fig. 8, showing the initial parts of the saddle paths around the Skiba point Xs



Note: While for $\mathbf{u} > \mathbf{0.32}$ the difference between these trajectories is not visible in this illustration, for topological reasons of course they never intersect. Isoclines, field direction and the lens are also shown.

There exists a third topologically different type of solution with 3 saddles and a vortex. It has been described by Caulkins et al. (2004) but only occurs here if we change simultaneously several parameters. As in the previous case, the strong Skiba has a threshold $X = X_S$ (typically $X_S < 1$, but opposite cases are also discovered), and we converge to the low population equilibrium if $X(0) < X_S$, and to the high population equilibrium in the opposite case. While the middle size equilibrium does exist as a candidate equilibrium in this case, it is never optimal to converge to it.

Figure 10: The exact location of the Skiba point in Fig. 8 is $\mathbf{Xs} = \mathbf{1.1975}$



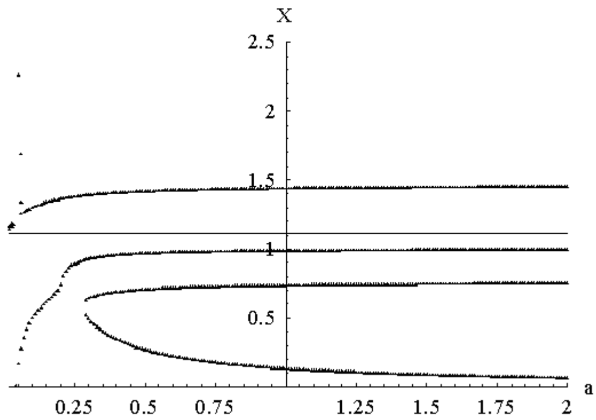
Note: The initial policy $\mathbf{u} = \mathbf{0.29}$ leads to the low saddle, while the initial policy $\mathbf{u} = \mathbf{0.318}$ leads to the high saddle.

6.5 Bifurcation diagram in $a - X$ space

It is interesting to examine a bifurcation diagram showing how the topological structure of the solutions varies with a (See Fig. 11). We see that when a is decreased from its basecase value of 2 down to 0.3, there is no topological change, i.e. we continue to have four equilibria. For smaller values of a, specifically for $0.06 < a < 0.29$, we have just two saddles, and the results for $a = 0.2$ are as described above. For still smaller a ($a < 0.04$), the small X equilibrium becomes negative (infeasible).

One can examine a similar bifurcation diagram (not shown here) over a for smaller values of β, such that $\beta < \gamma$ and the critical point is less than $X = 1$. Here the pattern changes radically: we have only two saddles for all $a > 0.07$. Note that while for lower a (about 0.1) the lower saddle is closer to 1, for a higher a the upper saddle is closer to one. In a tiny interval $0.04 < a < 0.07$ we have 4 equilibria and all of them are positive.

Figure 11: Bifurcation diagram in $\mathbf{a-X}$ space for parameter values: $\beta = \mathbf{0.5}, \mathbf{c} = \mathbf{2}, \gamma = \mathbf{0.45}, \mathbf{r} = \mathbf{0.05}, \rho = \mathbf{0.02}$



Note: Some phase diagrams have been shown already: for $\mathbf{a} = \mathbf{2}$ see Fig. 1, for $\mathbf{a} = \mathbf{0.05}$ see Fig. 8. Since parameter c leads mostly to a change in $\mathbf{u}$ and has less influence on $\mathbf{X}$, Fig. 6, drawn for $\mathbf{a} = \mathbf{0.2}$ and $\mathbf{c} = \mathbf{20}$, topologically corresponds to $\mathbf{a} = \mathbf{0.2}$ in this diagram.

6.6 Summary of sensitivity with respect to interactions between a and c

Reviewing the results above, we observe an interesting interaction between variation (specifically reductions from basecase values) in parameters a and c. When a is at its (high) basecase value, then variation in c does not alter the basic policy. Even when mobility programs are cheap (c is small), the optimal strategy is always to have the neighborhood approach a situation very near its uncontrolled or “natural” state.

However, when a is small, then the strategy depends strongly on the specific value of c. When c is at its (high) basecase value, we get a weak Skiba separating the small and medium size equilibria. The mobility program should not be used in a way that alters the basic character of the neighborhood, but if the neighborhood is initially depopulated, the mobility program will be pursued in such a way that the neighborhood never recovers.

When a is small and c is small, we get a radically different prescription. Regardless of the initial state of the neighborhood, it should never approach its natural long-run equilibrium, *even if it starts out at that size!* Instead, if the initial population is low, it will remain low. If the initial population is not low, then so many people should be placed in the neighborhood that it eventually grows beyond its natural size, becoming densely populated with newly assimilated immigrants who do not remain long (are fairly transient).

7. Conclusions and policy implications

We analyzed a highly stylized model of how poverty deconcentration programs influence population dynamics in the destination neighborhood. The model tracks the stock of middle-class residents and the flow of poor families entering the neighborhood. It considers the effects of both negative externalities that incoming poor families place on current residents (including flight) and positive externalities from middle-class neighborhoods on incoming poor families (through assimilation and directly in the social planner's objective function). The model is formulated as a dynamic optimization problem faced by a governmental entity that can control the rate at which the program places poor residents and wishes to do so in a manner that maximizes the discounted weighted sum of net benefits over time.

This model is inspired by policy debates in the US regarding poverty deconcentration through housing mobility initiatives. Typically these discussions, if translated into mathematics, would have the character of static concave maximization with a unique interior optimum. Recent work has proposed non-monotonic objective functions, emphasized the inadequacy of analyses that focus solely on outcomes for the poor families, and developed policy prescriptions based on static, multi-objective optimization models. However, no prior research known to us has explicitly addressed the dynamic nature of housing mobility policy design.

For base case parameter values we get convergence to a unique equilibrium in which the middle-class population is very close to its uncontrolled or natural level. This result appears to be fairly robust with respect to parameters governing mobility program cost and the extent of middle-class flight induced per poor family placed in the neighborhood.

However, if the neighborhood's underlying population dynamics are not very resilient, in the sense that it can take a long time for population to adjust when it is either above or below the uncontrolled or natural size, then other outcomes may be possible, or indeed optimal. Somewhat similar results can pertain for short-sighted decision makers.

One alternative structure obtained via a "weak" Skiba point might be summarized, "keep the neighborhood in its current state, even if that initial state is de-populated relative to its natural uncontrolled state." In particular, if the neighborhood is already weakened by under population, then paradoxically creating a new population inflow can prevent the neighborhood from growing because the induced middle-class flight exceeds the assimilation of program participants, in part because the scarcity of current middle-class neighbors undermines that assimilation.

If in addition to weak underlying population dynamics it is also the case that program costs are low, then yet another structure can emerge via a "strong" Skiba threshold. In that case the Skiba threshold separates lower initial population levels for which it is optimal to keep the neighborhood under-populated from higher initial population levels for which the optimal strategy leads to a "super-populated" neighborhood with population densities above those in the uncontrolled steady state. These outcomes involve high-inflow and high assimilation of poor families into the middle class, but also relatively rapid outflow of middle-class residents in response to both poor families entering and general population pressure. The resulting transient, high-density neighborhoods might be thought of as akin to those in New York City that traditionally absorbed large numbers of foreign immigrants.

Methodologically one of the most interesting results was finding a critical point that acts as a lens to focus trajectories in state-control space in a manner that lets them pass through a singularity separating two continuous semi-planes. This point seems to be able to separate saddles the way nodes and vortices often do. Because of the continuity of flow through that point, its existence can help reveal what the optimal solution strategy is and how that strategy does and does not depend on various parameters.

Substantively these results have three principal implications. First, inasmuch as the specific quantitative not just qualitative results can be trusted (which is subject to question given how stylized the model is), it appears that placement rates far in excess of those pursued by the Moving To Opportunity program may be both optimal and unlikely to generate prohibitive levels of middle class flight, at least with basecase parameters. Second, dynamic modeling of population flows related to housing mobility programs can yield interesting, indeed surprising, results and merits further investigation. Third, the likelihood of surprising or structurally different results seems to depend particularly on the dynamic resilience of host neighborhoods, program costs, and the modeling of flight and assimilation. So those topics merit further investigation, particularly from a dynamic perspective. We would highlight in particular the benefits of a refined model with a larger state space to model explicitly the process of upward social mobility over time.

Explicitly specifying functional forms and constraining the state space to dimensions that permit explicit dynamic optimization might inevitably involve a high degree of abstraction, but the modeling suggests the benefits of realistic quantification of a few attributes whose importance exists independent of a dynamic optimization framework. Notably, how large is the stock of poor families that are eligible for mobility programs relative to the "carrying capacity" of neighborhoods in which they might be placed? In addition, how quickly do each of these stocks grow? Growth for the former pertains to some combination of the rates of upward and downward social mobility combined with the natural reproductive rate for poor families. Growth rates for the latter pertain to how quickly newly placed residents are assimilated, and how flight depends on the rate and accumulation of placed families.

At this point, it is not even clear whether in order of magnitude terms the absorptive capacity of middle class neighborhoods is large or small compared to the number of poor families. If it is small, the housing mobility programs, no matter how desirable or costeffective, must inevitably be a relative minor complement to core housing programs that help poor families where they are now located. If it is large, then residential mobility programs have the potential to be the primary strategy for meeting housing policy objectives. The current model represents a small step toward trying to frame and answer such fundamental questions.

8. Acknowledgements

This research was partly financed by the Austrian National Bank (ÖNB) under grant No. 9414. We thank the participants of a Workshop on Skiba Problems at the Vienna University of Technology (13 May 2003) for their comments, especially Barkley Rosser. We also thank Claudia Knoll and Doris Zuba for error-checking.

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