Adaptive Fuzzy Control for a Class of MIMO Nonlinear Systems with Unknown Dead-zones

ZHANG Tian-Ping[1] YI Yang[1]

Abstract A design scheme of adaptive fuzzy controller for a class of uncertain MIMO nonlinear systems with unknown dead-zones and a triangular control structure is proposed in this paper. The design is based on the principle of sliding mode control and the property of Nussbaum function. The approach does not require a priori knowledge of the signs of the control gains and the upper bounds and lower bounds of dead-zone parameters to be known a priori. By introducing the integral-type Lyapunov function and adopting the adaptive compensation term of the upper bound of the optimal approximation error and the dead-zone disturbance, the closed-loop control system is proved to be semi-globally stable in the sense that all signals involved are bounded, with tracking errors converging to zero.

Key words Dead-zone, fuzzy control, adaptive control, sliding mode control, Nussbaum function

1 Introduction

Dead-zone is one of the most important non-smooth nonlinearities in many industrial processes, which can severely limit system performance. Thus the study of dead-zone models involved has been of great interest to control researchers for a long time[1~5]. An immediate method for the control of dead-zone is to construct an adaptive dead-zone inverse[1,2]. Continuous and discrete-time adaptive dead-zone inverses for linear systems with unmeasurable dead-zone outputs were built, respectively. The work done by [3] continued the above research and an asymptotically adaptive cancellation of an unknown dead-zone achieved under the condition that the output of a dead-zone was measurable. In [4], giving a matching condition to the reference model, an adaptive controller with an adaptive dead-zone inverse has been introduced. In addition, a new adaptive control approach was studied without constructing the inverse of the dead-zone[5].

In recent years, the analytical study of adaptive nonlinear control systems using universal function approximators has received much attention[6~13]. Typically, these methods use fuzzy logic systems and neural networks as approximation models for the unknown nonlinearities. In [8], the approach was able to avoid the requirement of the upper

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1. Department of Computer, College of Information Engineering, Yangzhou University, Yangzhou 225009, P. R. China 2. Research Institute of Automation, Southeast University, Nanjing 210096, P. R. China
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bound of the first time derivative of the control gain by using integral Lyapunov function. The problem of adaptive control for a class of MIMO nonlinear systems with a triangular structure in control inputs was discussed in [9]. Furthermore, two design schemes of adaptive controller for SISO/MIMO uncertain nonlinear systems were proposed in [11, 12], and ensured tracking error converged to zero. In [13], the problem of direct adaptive fuzzy control was studied for a class of MIMO interconnected systems with known gain signs and without dead-zone inputs. Nussbaum function was introduced in [14]. The Nussbaum gain technique for coping with the unknown sign of the function control gain was employed in [15].

In this paper, we consider a class of uncertain MIMO nonlinear systems with both unknown dead-zones and unknown gain signs. Based on the intuitive concept and the piece-wise description of a dead-zone model, a robust adaptive sliding mode control is introduced. Compared with the above mentioned references, our controller design approach has four features: 1) The approach does not require the borders of dead-zone model parameters to be known; 2) Based on the approximation capability of the fuzzy logic systems and the property of Nussbaum function, the signs of the function control gains may be unknown; 3) The adaptive compensation term of the optimal approximation error is adopted to minimize the influence of modeling error and parameter estimation error and ensures that the tracking errors converge to zero; and 4) Jittering can be eliminated by smoothing out the discontinuous sign function.

2 Problem statement and basic assumptions

Consider a class of uncertain MIMO nonlinear systems with dead-zones and a triangular control structure in the following form.

$$\left\{\begin{array}{rcl} \dot{x}_{1j} &=& x_{1,j+1},\ j=1,\cdots,n_1-1 \\ \dot{x}_{1n_1} &=& f_1(\boldsymbol{x})+b_{11}(\boldsymbol{x}_1)w_1(t) \\ \dot{x}_{ij} &=& x_{i,j+1}, j=1,\cdots,n_i-1 \\ \dot{x}_{in_i} &=& f_i(\boldsymbol{x})+b_{i1}(\boldsymbol{x})w_1(t)+b_{i2}(\boldsymbol{x})w_2(t) \\ && +\cdots+b_{i,i-1}(\boldsymbol{x})w_{i-1}(t) \\ && +b_{ii}(\boldsymbol{x}_1,\boldsymbol{x}_2,\cdots,\boldsymbol{x}_i)w_i(t), i=2,\cdots,m \\ y_1 &=& x_{11},\cdots,y_m=x_{m1} \end{array}\right. \tag{1}$$

where $\boldsymbol{x} = \left(\boldsymbol{x}_1^{\rm T},\boldsymbol{x}_2^{\rm T},\cdots,\boldsymbol{x}_m^{\rm T}\right)^{\rm T} \in {\rm R}^n$ is the state vector, $\boldsymbol{x}_i = (x_{i1},\cdots,x_{in_i})^{\rm T}$, $n = \sum\limits_{i=1}^{m} n_i$; $w_i(t) \in {\rm R}$ is the output of the dead-zone model with the input $v_i(t) \in {\rm R}$, $y_i \in {\rm R}$ denotes the system output; $f_1(\boldsymbol{x}),\cdots,f_m(\boldsymbol{x})$ are the unknown continuous functions; $b_{11}(\boldsymbol{x}_1), b_{21}(\boldsymbol{x}), b_{22}(\bar{\boldsymbol{x}}_2),\cdots,b_{m1}(\boldsymbol{x}),\cdots,b_{m,m-1}(\boldsymbol{x})$, $b_{mm}(\bar{\boldsymbol{x}}_m)$ are the unknown control gains, $\bar{\boldsymbol{x}}_i = (\boldsymbol{x}_1^{\rm T},\boldsymbol{x}_2^{\rm T},\cdots,\boldsymbol{x}_i^{\rm T})^{\rm T}$.

The dead-zone model with input $v_i(t)$ and output $w_i(t)$ is described as follows.

$$\begin{aligned} w_i(t) &= D_i(v_i(t)) \\ &= \left\{\begin{array}{ll} k_{ir}(v_i(t)-b_{ir}), & \text{for } v_i(t)\geq b_{ir} \\ 0, & \text{for } b_{il}<v_i(t)<b_{ir} \\ k_{il}(v_i(t)-b_{il}), & \text{for } v_i(\mathrm{t})\leq b_{il} \end{array}\right. \end{aligned} \tag{2}$$

The key features of dead-zone in the control problem investigated in this paper are

Assumption 1. The dead-zone output $w_i(t)$ is not available for measurement.

Assumption 2. The dead-zone slopes in positive and negative regions are the same, $k_{ir} = k_{il} = k_i$.

Assumption 3. The dead-zone parameters b_{ir}, b_{il}, k_i are unknown bounded constants, but their signs are known, *i.e.* $b_{ir} > 0, b_{il} < 0$ and $k_i > 0$.

Based on the above features, we can redefine dead-zone model as

$$w_i(t) = D_i(v_i(t)) = k_i v_i(t) + d_i(v_i(t)) \tag{3}$$

where

$$d_i(v_i(t)) = \left\{\begin{array}{ll} -k_i b_{ir}, & \text{for } v_i(t)\geq b_{ir} \\ -k_i v_i(t), & \text{for } b_{il}<v_i(t)<b_{ir} \\ -k_i b_{il}, & \text{for } v_i(t)\leq b_{il} \end{array}\right. \tag{4}$$

and $|d_i(v_i(t))| \leq p_i^*$, p_i^* is an unknown positive constant.

The control objective is to force the system output y_i to follow the specified desired trajectory y_{id}. Define x_{id}, e_i and the filtered tracking error s_i as

$$\begin{aligned} \boldsymbol{x}_{id} &= (y_{id},\dot{y}_{id},\cdots,y_{id}^{(n_i-1)})^{\rm T} \\ \mathbf{e}_i &= \boldsymbol{x}_i-\boldsymbol{x}_{id} = (e_{i1},e_{i2},\cdots,e_{in_i})^{\rm T} \\ s_i &= (\frac{d}{dt}+\lambda_i)^{n_i-1}e_{i1} = \sum_{j=1}^{n_i-1} c_{ij}e_{ij}+e_{in_i} \end{aligned} \tag{5}$$

where $c_{ij} = C_{n_i-1}^{j-1}\lambda_i^{n_i-j}, j = 1,\cdots,n_i-1, \lambda_i > 0$ is a positive constant, specified by the designer.

From (1), (3), and (5), we obtain

$$\left\{\begin{array}{rl} \dot{s}_1 = & f_1(\boldsymbol{x})+\gamma_1+b_{11}(\boldsymbol{x}_1)k_1v_1(t)+b_{11}(\boldsymbol{x}_1)d_1(v_1(t)) \\ \dot{s}_i = & f_i(\boldsymbol{x})+\sum\limits_{j=1}^{i-1} b_{ij}(\boldsymbol{x})D_j(v_j)+\gamma_i+b_{ii}(\bar{\boldsymbol{x}}_i)k_iv_i(t) \\ & +b_{ii}(\bar{\boldsymbol{x}}_i)d_i(v_i(t)),\quad i=2,\cdots,m \end{array}\right. \tag{6}$$

where $\gamma_i = \sum\limits_{j=1}^{n_i-1} c_{ij}e_{i,j+1} - y_{id}^{(n_i)}$.

In order to design a stable adaptive fuzzy control, we make the following assumptions.

Assumption 4. $0 < b_{i0} \leq |b_{ii}(\bar{\boldsymbol{x}}_i)| \ \leq b_{i1}$.

Assumption 5. $(\bar{\boldsymbol{x}}_{id}^{\rm T}, y_{id}^{(n_i)})^{\rm T} \in \Omega_{id} \subset R^{n_i+1}$, where b_{i0} and b_{i1} are known constants; Ω_{id} is a known bounded compact set; $i = 1,\cdots,m$.

In order to cope with the unknown control gain sign, the Nussbaum gain technique is employed in this paper. A function $N(\varsigma)$ is called a Nussbaum -type function if it has the following properties.

1) $\lim\limits_{s\to+\infty}\sup\frac{1}{s}\int_0^s N(\varsigma){\rm d}\varsigma = +\infty$; and

2) $\lim\limits_{s\to+\infty}\inf \frac{1}{s}\int_0^s N(\varsigma)\mathrm{d}\varsigma = -\infty$

For clarity, the even Nussbaum function $N(\varsigma) = e^{\varsigma^2}\cos((\pi/2)\varsigma)$ is used throughout the paper.

Lemma 1[15]**.** Let $V(\cdot), \varsigma(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(t) \geq 0, \forall t \in [0, t_f)$, and $N(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds.

$$V(t) \leq c_0 + \int_0^t (gN(\varsigma)+1)\dot{\varsigma}\mathrm{d}\tau, \qquad \forall t \in [0, t_f) \quad (7)$$

where g is a nonzero constant and c_0 represents some suitable constant, then $V(t), \varsigma(t), \int_0^t (gN(\varsigma)+1)\dot{\varsigma}\mathrm{d}\tau$ must be bounded on $[0, t_f)$.

3 Adaptive fuzzy controller design and stability analysis

3.1 Discontinuous adaptive control law and stability analysis

Let

$$\begin{aligned} h_i(\boldsymbol{z}_i) \;=\; & \frac{f_i(\boldsymbol{x}) + \sum\limits_{j=1}^{i-1} b_{ij}(\boldsymbol{x})D_j(v_j)}{|b_{ii}(\bar{\boldsymbol{x}}_i)|} \\ & + \frac{1}{s_i}\int_0^{s_i}\Big\{\sigma\Big[\sum_{j=1}^{i-1}\sum_{k=1}^{n_j}\frac{\partial\left|b_{ii}^{-1}(\bar{\boldsymbol{x}}_i^+, \sigma+\beta_i)\right|}{\partial x_{jk}}x_{j,k+1} \\ & + \sum_{k=1}^{n_i-1}\frac{\partial\left|b_{ii}^{-1}(\bar{\boldsymbol{x}}_i^+, \sigma+\beta_i)\right|}{\partial x_{ik}}x_{i,k+1}\Big] \\ & + \frac{\gamma_i}{|b_{ii}(\bar{x}_i^+, \sigma+\beta_i)|}\Big\}\mathrm{d}\sigma \end{aligned} \quad (8)$$

where $\boldsymbol{z}_i = (\boldsymbol{x}^{\mathrm{T}}, s_i, \gamma_i, \beta_i, v_1, \cdots, v_{i-1})^{\mathrm{T}}$, $\beta_i = y_{id}^{(n_i-1)} - \sum\limits_{j=1}^{n_i-1} c_{ij}e_{ij}$, $x_{j,n_j+1} = \dot{x}_{jn_j}$, $j = 1, \cdots, i-1$. Define the compact sets Ω_{z_1} and Ω_{z_i} as follows.

$$\begin{aligned} \Omega_{z_1} \;=\; & \Big\{(\boldsymbol{x}^{\mathrm{T}}, s_1, \gamma_1, \beta_1)^{\mathrm{T}} |\ \boldsymbol{x}_j \in \Omega_{\mu_j}, \\ & \qquad j = 1, \cdots, m,\ \bar{\boldsymbol{x}}_{id} \in \Omega_{id}\Big\} \\ \Omega_{z_i} \;=\; & \Big\{(\boldsymbol{x}^{\mathrm{T}}, s_i, \gamma_i, \beta_i, v_1, \cdots, v_{i-1})^{\mathrm{T}} |\ \boldsymbol{x}_j \in \Omega_{\mu_j}, \\ & \qquad j = 1, \cdots, m,\ \bar{\boldsymbol{x}}_{kd} \in \Omega_{kd},\ k = 1, \cdots, i\Big\} \end{aligned}$$

where Ω_{μ_i} will be defined later in Theorem 1, $i = 1, \cdots, m$. Let $h_i(\boldsymbol{z}_i, \boldsymbol{\theta}_i)$ be the approximation of the second type of fuzzy systems on the compact set Ω_{z_i} to $h_i(\boldsymbol{z}_i)$, *i.e.*

$$h_i(\boldsymbol{z}_i, \boldsymbol{\theta}_i) = \frac{\sum\limits_{l=1}^{M_i} y_i^l \prod\limits_{j=1}^{n+i+2}\exp(-\frac{(z_{ij}-a_{ij}^l)^2}{(b_{ij}^l)^2+b_0})}{\sum\limits_{l=1}^{M_i}[\prod\limits_{j=1}^{n+i+2}\exp(-\frac{(z_{ij}-a_{ij}^l)^2}{(b_{ij}^l)^2+b_0})]} \quad (9)$$

$\boldsymbol{\theta}_i = (y_i^1, \cdots, y_i^{M_i}, b_{1i}^1, \cdots, b_{n+i+2,i}^1, \cdots, b_{1i}^{M_i}, \cdots, b_{n+i+2,i}^{M_i}, a_{1i}^1, \cdots, a_{n+i+2,\ i}^1, \cdots, a_{1i}^{M_i}, \cdots, a_{n+i+2,\ i}^{M_i})^{\mathrm{T}}$ is the adjustable parameter vector. M_i is the number of fuzzy rules, b_0 is a positive constant specified by the designer. Let

$$\begin{aligned} \Omega_i \;&=\; \{\boldsymbol{\theta}_i |\ \|\boldsymbol{\theta}_i\| \leq M_{\theta_i}\} \\ \boldsymbol{\theta}_i^* \;&=\; \arg\min_{\boldsymbol{\theta}_i\in\Omega_i}[\sup_{\boldsymbol{z}_i\in\Omega_{z_i}}|h_i(\boldsymbol{z}_i, \boldsymbol{\theta}_i) - h_i(\boldsymbol{z}_i)|] \end{aligned} \quad (10)$$

where M_{θ_i} is a positive constant, $\hat{\boldsymbol{\theta}}_i(t) \in \Omega_i$ is the estimate of $\boldsymbol{\theta}_i^*$ at time t. Using the Taylor series expansion of $h_i(\boldsymbol{z}_i, \boldsymbol{\theta}_i^*)$ around $\hat{\theta}_i(t)$, we have

$$\begin{aligned} & h_i(\boldsymbol{z}_i, \boldsymbol{\theta}_i^*) - h_i(\boldsymbol{z}_i, \hat{\boldsymbol{\theta}}_i) \\ =\; & (\boldsymbol{\theta}_i^* - \hat{\boldsymbol{\theta}}_i)^{\mathrm{T}}\frac{\partial h_i(\boldsymbol{z}_i, \hat{\boldsymbol{\theta}}_i)}{\partial\hat{\boldsymbol{\theta}}_i} + O(\|\tilde{\boldsymbol{\theta}}_i\|^2) \end{aligned} \quad (11)$$

where $\tilde{\boldsymbol{\theta}}_i(t) = \boldsymbol{\theta}_i^* - \hat{\boldsymbol{\theta}}_i$. Let

$$\delta_i = \max_{\boldsymbol{z}_i\in\Omega_{z_i}, \hat{\boldsymbol{\theta}}_i\in\Omega_i}[|O(\|\tilde{\boldsymbol{\theta}}_i\|^2) + h_i(\boldsymbol{z}_i) - h_i(\boldsymbol{z}_i, \boldsymbol{\theta}_i^*)|] \quad (12)$$

then $\delta_i (i = 1, \cdots, m)$ are the unknown bounded constants.

Considering the following control law.

$$\begin{aligned} v_i(t) \;&=\; N(\varsigma_i)[k_{i0}s_i + h_i(\boldsymbol{z}_i, \hat{\boldsymbol{\theta}}_i) + \hat{\varepsilon}_i\mathrm{sgn}(s_i)] \quad (13) \\ \dot{\varsigma}_i \;&=\; k_{i0}s_i^2 + h_i(\boldsymbol{z}_i, \hat{\boldsymbol{\theta}}_i)s_i + \hat{\varepsilon}_i\,|s_i| \quad (14) \end{aligned}$$

where $N(\varsigma_i) = e^{\varsigma_i^2}\cos((\pi/2)\varsigma_i)$, k_{i0} is a positive constant, $\hat{\boldsymbol{\theta}}_i, \hat{\varepsilon}_i$ are the estimates of θ_i^* and $\varepsilon_i = \delta_i + p_i^*$ at time t.

Adopt the following adaptive laws.

$$\dot{\hat{\boldsymbol{\theta}}}_i = \begin{cases} \eta_{i1}s_i\frac{\partial h_i(\boldsymbol{z}_i, \hat{\boldsymbol{\theta}}_i)}{\partial\hat{\theta}_i}, \text{ if } \|\hat{\boldsymbol{\theta}}_i\| < M_{\theta_i} \text{ or } \|\hat{\boldsymbol{\theta}}_i\| = M_{\theta_i} \\ \qquad\qquad \text{and } s_i\hat{\boldsymbol{\theta}}_i^{\mathrm{T}}\frac{\partial h_i(\boldsymbol{z}_i, \hat{\boldsymbol{\theta}}_i)}{\partial\hat{\boldsymbol{\theta}}_i} \leq 0 \\ \eta_{i1}s_i\frac{\partial h_i(\boldsymbol{z}_i, \hat{\boldsymbol{\theta}}_i)}{\partial\hat{\boldsymbol{\theta}}_i} - \eta_{i1}s_i\frac{\hat{\boldsymbol{\theta}}_i\hat{\boldsymbol{\theta}}_i^{\mathrm{T}}}{\|\hat{\boldsymbol{\theta}}_i\|^2}\frac{\partial h_i(\boldsymbol{z}_i, \hat{\boldsymbol{\theta}}_i)}{\partial\hat{\boldsymbol{\theta}}_i}, \\ \quad \text{if } \|\hat{\boldsymbol{\theta}}_i\| = M_{\boldsymbol{\theta}_i} \text{ and } s_i\hat{\boldsymbol{\theta}}_i^{\mathrm{T}}\frac{\partial h_i(\boldsymbol{z}_i, \hat{\boldsymbol{\theta}}_i)}{\partial\hat{\boldsymbol{\theta}}_i} > 0 \end{cases} \quad (15)$$

$$\dot{\hat{\varepsilon}}_i = \eta_{i2}|s_i| \quad (16)$$

where η_{i1} and η_{i2} are strictly positive constants which determine the adaptive rate.

Define a smooth scalar function as follows.

$$V_{si} = \int_0^{s_i}\frac{\sigma}{|b_{ii}(\bar{\boldsymbol{x}}_i^+, \sigma+\beta_i)|}\mathrm{d}\sigma \quad (17)$$

where $\bar{\boldsymbol{x}}_i^+ = (\boldsymbol{x}_1^{\mathrm{T}}, \cdots, \boldsymbol{x}_{i-1}^{\mathrm{T}}, x_{i1}, x_{i2}, \cdots, x_{i,n_i-1})^{\mathrm{T}}$.

By second mean value theorem for Integrals, V_{si} can be rewritten as $V_{si} = s_i^2/2|b_{ii}(\bar{\boldsymbol{x}}_i^+, \lambda_{is}s_i+\beta_i)|$ with $\lambda_{is} \in (0, 1)$. Because $0 < b_{i0} \leq |b_{ii}(\bar{\boldsymbol{x}}_i)|$, it is shown that V_{si} is positive definitive with respect to s_i. Differentiating $V_{si}(t)$ with respect to time t, applying (6), Assumptions 2 and 3, we obtain

$$\begin{aligned} \dot{V}_{si} \;=\; & \frac{s_i}{|b_{ii}(\bar{\boldsymbol{x}}_i)|}\dot{s}_i + \int_0^{s_i}\sigma\Big[\sum_{j=1}^{i-1}\sum_{k=1}^{n_i}\frac{\partial|b_{ii}^{-1}(\bar{\boldsymbol{x}}_i^+, \sigma+\beta_i)|}{\partial x_{jk}} \\ & \times x_{j,k+1} + \sum_{k=1}^{n_i-1}\frac{\partial|b_{ii}^{-1}(\bar{\boldsymbol{x}}_i^+, \sigma+\beta_i)|}{\partial x_{ik}}x_{i,k+1}\Big]\mathrm{d}\sigma \\ & - \frac{\gamma_i s_i}{|b_{ii}(\bar{\boldsymbol{x}}_i)|} + \gamma_i\int_0^{s_i}|b_{ii}^{-1}(\bar{\boldsymbol{x}}_i^+, \sigma+\beta_i)|\mathrm{d}\sigma \\ \leq\; & s_i\frac{b_{ii}(\bar{\boldsymbol{x}}_i)}{|b_{ii}(\bar{\boldsymbol{x}}_i)|}k_iv_i + s_ih_i(\boldsymbol{z}_i) + |s_i|p_i^* \end{aligned} \quad (18)$$

Theorem 1. Consider the MIMO nonlinear systems (1) with the control law defined by (13) and (14). Let the parameter vector $\hat{\boldsymbol{\theta}}_i, \hat{\varepsilon}_i$ be adjusted by the adaptation laws determined by (15) and (16), and let Assumptions 1–5 be true. Then,

1) if $\hat{\boldsymbol{\theta}}_i(0) \in \Omega_i$, then $\|\hat{\boldsymbol{\theta}}_i(t)\| \le M_{\theta_i}, \forall t \ge 0$;

2) the overall closed-loop control system is semi-globally stable in the sense that all the closed-loop signals are bounded, and the state vector $\boldsymbol{x}_i \in \Omega_{\mu_i} = \{\boldsymbol{x}_i(t) |\ |e_{ij}(t)| \le 2^{j-1}\lambda_i^{j-n_i}\mu_i,\ j = 1, \cdots, n_i,\ \bar{\boldsymbol{x}}_{id} \in \Omega_{id}\}$, $t \ge T_i$;

3) $\lim\limits_{t\to\infty} s_i = 0$, *i.e.* $\lim\limits_{t\to\infty} e_{i1} = 0$,

where $\mu_i = \sqrt{2b_{i1}\sup_{t\ge 0} V_i(t)}$, $T_i = (n_i - 1)/\lambda_i$.

Proof. 1) Let $V_{i0}(t) = (1/2)\hat{\boldsymbol{\theta}}_i^T\hat{\boldsymbol{\theta}}_i$. Similar to the analysis in [7], we can know that if $\hat{\boldsymbol{\theta}}_i(0) \in \Omega_i$, then $\|\hat{\boldsymbol{\theta}}_i(t)\| \le M_{\theta_i}$, $\forall t \ge 0$.

2) Define the Lyapunov function candidate

$$V_i(t) = V_{si}(t) + \frac{1}{2\eta_{i1}}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_i + \frac{1}{2\eta_{i2}}\tilde{\varepsilon}_i^2 \tag{19}$$

Differentiating $V_i(t)$ with respect to time t, we have

$$\dot{V}_i(t) = \dot{V}_{si}(t) + \frac{1}{\eta_{i1}}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}(-\dot{\hat{\boldsymbol{\theta}}}_i) + \frac{1}{\eta_{i2}}\tilde{\varepsilon}_i(-\dot{\hat{\varepsilon}}_i)$$

Substituting (18) into the above equation, we obtain

$$\begin{aligned}\dot{V}_i(t) \quad &\le \quad \frac{b_{ii}(\bar{\boldsymbol{x}}_i)}{|b_{ii}(\bar{\boldsymbol{x}}_i)|}k_i N(\varsigma_i)\dot{\varsigma}_i + \dot{\varsigma}_i - k_{i0}s_i^2 \\ &\quad + I_i s_i \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\frac{\hat{\boldsymbol{\theta}}_i\hat{\boldsymbol{\theta}}_i^{\mathrm{T}}}{\|\hat{\boldsymbol{\theta}}_i\|^2}\frac{\partial h_i(\boldsymbol{z}_i, \hat{\boldsymbol{\theta}}_i)}{\partial\hat{\boldsymbol{\theta}}_i}\end{aligned} \tag{20}$$

where $I_i = 0(1)$, if the first (second) condition of (15) is true. If the second condition of (15) is true, then $s_i\hat{\boldsymbol{\theta}}_i^{\mathrm{T}}\frac{\partial h_i(\boldsymbol{z}_i,\hat{\boldsymbol{\theta}}_i)}{\partial\hat{\boldsymbol{\theta}}_i} > 0$, and $\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\hat{\boldsymbol{\theta}}_i = \frac{1}{2}[\|\boldsymbol{\theta}_i^*\|^2 - \|\hat{\boldsymbol{\theta}}_i\|^2 - \|\boldsymbol{\theta}_i^* - \hat{\boldsymbol{\theta}}_i\|^2] < 0$. Therefore, we have

$$\dot{V}_i(t) \le -k_{i0}s_i^2 + \frac{b_{ii}(\bar{\boldsymbol{x}}_i)}{|b_{ii}(\bar{\boldsymbol{x}}_i)|}k_i N(\varsigma_i)\dot{\varsigma}_i + \dot{\varsigma}_i \tag{21}$$

Integrating (21) over $[0, t]$, we have

$$\begin{aligned}V_i(t) \quad &\le \quad V_i(t) + \int_0^t k_{i0}s_i^2 d\tau \\ &\le \quad V_i(0) + \int_0^t \left(\frac{b_{ii}(\bar{\boldsymbol{x}}_i)}{|b_{ii}(\bar{\boldsymbol{x}}_i)|}k_i N(\varsigma_i)\dot{\varsigma}_i + \dot{\varsigma}_i\right)d\tau\end{aligned} \tag{22}$$

According to Lemma 1, we have $V_i(t), \int_0^t \frac{b_{ii}(\bar{\boldsymbol{x}}_i)}{|b_{ii}(\bar{\boldsymbol{x}}_i)|}N(\varsigma_i)\dot{\varsigma}_i + \dot{\varsigma}_i)d\tau, \varsigma_i(t)$ are bounded in $[0, t_f)$. Similar to the discussion in [15], we know that the above conclusion is true for $t_f = +\infty$. Therefore, $s_i, \tilde{\varepsilon}_i \in L_\infty$. According to (19) and Assumption 4, we have that $s_i^2 \le 2b_{i1}V_{si}(t) \le 2b_{i1}V_i(t) \le \mu_i^2$. Similar to the discussion in [8], the conclusion is true.

3) From (22), it is easy to show that $\int_0^{+\infty} s_i^2 dt$ exists. Using (5), (6) and (13), we have that $\dot{s}_i \in L_\infty$. Therefore, using the Barbalat′s Lemma, we have $\lim\limits_{t\to+\infty} s_i = 0$. □

3.2 Continuous adaptive control law and stability analysis

Define

$$s_{i\Delta} = s_i - \varphi_i \mathrm{sat}(s_i(t)/\varphi_i) \tag{23}$$

where saturation function $\mathrm{sat}(y)$ is defined as: $\mathrm{sat}(y) = y$, if $|y| \le 1$; $\mathrm{sat}(y) = \mathrm{sgn}(y)$, if $|y| > 1$, φ_i is a positive constant representing the width of the boundary. Let

$$\begin{aligned}h_i(z_i) \quad = \quad & \frac{f_i(\boldsymbol{x}) + \sum\limits_{j=1}^{i-1} b_{ij}(\boldsymbol{x})D_j(v_j)}{|b_{ii}(\bar{\boldsymbol{x}}_i)|} \\ & + \frac{1}{s_{i\Delta}}\int_0^{s_{i\Delta}} \Big\{\sigma\Big[\sum_{j=1}^{i-1}\sum_{k=1}^{n_j}\frac{\partial|b_{ii}^{-1}(\bar{\boldsymbol{x}}_i^+, \sigma + \beta_i)|}{\partial\boldsymbol{x}_{jk}} \\ & \times\boldsymbol{x}_{j,k+1} + \sum_{k=1}^{n_i-1}\frac{\partial\left|b_{ii}^{-1}(\bar{\boldsymbol{x}}_i^+, \sigma + \beta_i)\right|}{\partial\boldsymbol{x}_{ik}}\boldsymbol{x}_{i,k+1}\Big] \\ & + \frac{\gamma_i}{|b_{ii}(\bar{\boldsymbol{x}}_i^+, \sigma + \beta_i)|}\Big\}\mathrm{d}\sigma\end{aligned}$$

where $\beta_i = y_{id}^{(n_i-1)} - \sum\limits_{j=1}^{n_i-1} c_{ij}e_{ij} + \varphi_i\mathrm{sat}(s_i(t)/\varphi_i)$, $\boldsymbol{z}_i = (\boldsymbol{x}^{\mathrm{T}}, s_{i\Delta}, \gamma_i, \beta_i, v_1, \cdots, v_{i-1})^{\mathrm{T}}$. Therefore, $s_{i\Delta} + \beta_i = s_i + y_{id}^{(n_i-1)} - \sum\limits_{j=1}^{n_i-1} c_{ij}e_{ij} = \boldsymbol{x}_{in_i}$.

Considering the following control law.

$$v_i(t) \quad = \quad N(\varsigma_i)[k_{i0}s_{i\Delta} + h_i(\boldsymbol{z}_i, \hat{\boldsymbol{\theta}}_i) + \hat{\varepsilon}_i\mathrm{sat}(s_i/\varphi_i)] \tag{24}$$

$$\dot{\varsigma}_i \quad = \quad k_{i0}s_{i\Delta}^2 + h_i(\boldsymbol{z}_i, \hat{\boldsymbol{\theta}}_i)s_{i\Delta} + \hat{\varepsilon}_i s_{i\Delta}\mathrm{sat}(s_i/\varphi_i) \tag{25}$$

Adopt the following adaptive laws.

$$\dot{\hat{\boldsymbol{\theta}}}_i \quad = \quad \begin{cases}\eta_{i1}s_{i\Delta}\frac{\partial h_i(\boldsymbol{z}_i,\hat{\boldsymbol{\theta}}_i)}{\partial\hat{\boldsymbol{\theta}}_i}, \text{ if } \|\hat{\boldsymbol{\theta}}_i\| < M_{\boldsymbol{\theta}_i} \text{ or } \|\hat{\boldsymbol{\theta}}_i\| = M_{\boldsymbol{\theta}_i} \\ \qquad \text{and } s_{i\Delta}\hat{\boldsymbol{\theta}}_i^{\mathrm{T}}\frac{\partial h_i(\boldsymbol{z}_i,\hat{\boldsymbol{\theta}}_i)}{\partial\hat{\boldsymbol{\theta}}_i} \le 0 \\ \eta_{i1}s_{i\Delta}\frac{\partial h_i(\boldsymbol{z}_i,\hat{\boldsymbol{\theta}}_i)}{\partial\hat{\boldsymbol{\theta}}_i} - \eta_{i1}s_{i\Delta}\frac{\hat{\boldsymbol{\theta}}_i\hat{\boldsymbol{\theta}}_i^{\mathrm{T}}}{\|\hat{\boldsymbol{\theta}}_i\|^2}\frac{\partial h_i(\boldsymbol{z}_i,\hat{\boldsymbol{\theta}}_i)}{\partial\hat{\boldsymbol{\theta}}_i}, \\ \quad \text{if } \|\hat{\boldsymbol{\theta}}_i\| = M_{\boldsymbol{\theta}_i} \text{ and } s_{i\Delta}\hat{\boldsymbol{\theta}}_i^{\mathrm{T}}\frac{\partial h_i(\boldsymbol{z}_i,\hat{\boldsymbol{\theta}}_i)}{\partial\hat{\boldsymbol{\theta}}_i} > 0\end{cases} \tag{26}$$

$$\dot{\hat{\varepsilon}}_i \quad = \quad \eta_{i2}|s_{i\Delta}| \tag{27}$$

where η_{i1} and η_{i2} are strictly positive constants which determine the adaptive rate.

Theorem 2. Consider the MIMO nonlinear systems (1) with the control law defined by (24) and (25). Let the parameter vector $\hat{\boldsymbol{\theta}}_i, \hat{\varepsilon}_i$ be adjusted by the adaptation laws determined by (26) and (27), and let Assumptions 1–5 be true. Then,

1) if $\hat{\boldsymbol{\theta}}_i(0) \in \Omega_i$, then $\|\hat{\boldsymbol{\theta}}_i(t)\| \le M_{\boldsymbol{\theta}_i}, \forall t \ge 0$;

2) the overall closed-loop control system is semi-globally stable in the sense that all the closed–loop signals are bounded, and the state vector $\boldsymbol{x}_i \in \Omega_{\mu_i} = \{\boldsymbol{x}_i(t) |\ |e_{ij}(t)| \le 2^{j-1}\lambda_i^{j-n_i}\mu_i,\ j = 1, \cdots, n_i, \bar{\boldsymbol{x}}_{id} \in \Omega_{id}\}$, $t \ge T_i$;

3) $\lim\limits_{t\to\infty} s_{i\Delta} = 0$,

where $\mu_i = \sqrt{\varphi_i + 2b_{i1}\sup_{t\ge 0} V_{i\Delta}(t)}$, $T_i = (n - 1)/\lambda_i$.

Proof. 1) Similar to the proof in [7], it is easy to show that the conclusion 1) is true.

2) Define the Lyapunov function candidate

$$V_{i\Delta}(t) = \int_0^{s_{i\Delta}} \frac{\sigma}{\left|b_{ii}(\bar{\boldsymbol{x}}_i^+, \sigma + \beta_i)\right|}\mathrm{d}\sigma + \frac{1}{2\eta_{i1}}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_i + \frac{1}{2\eta_{i2}}\tilde{\varepsilon}_i^2 \tag{28}$$

If $|s_i| \le \varphi_i$, then $\dot{V}_{i\Delta} = 0$; if $|s_i| > \varphi_i$, then $s_{i\Delta} = s_i(t) - \varphi_i\mathrm{sgn}(s_i)$ and $\dot{s}_{i\Delta}(t) = \dot{s}_i(t)$. Similar to the calculations in

(18) and (20), and using (24)-(27), we have

$$\dot{V}_{i\Delta}(t) \quad \leq \quad -k_{i0}s_{i\Delta}^2 + \frac{b_{ii}(\bar{\boldsymbol{x}}_i)}{|b_{ii}(\bar{\boldsymbol{x}}_i)|}k_iN(\varsigma_i)\dot{\varsigma}_i + \dot{\varsigma}_i \tag{29}$$

Therefore,

$$V_{i\Delta}(t) \leq V_{i\Delta}(0) + \int_0^t (\frac{b_{ii}(\bar{\boldsymbol{x}}_i)}{|b_{ii}(\bar{\boldsymbol{x}}_i)|}k_iN(\varsigma_i)\dot{\varsigma}_i + \dot{\varsigma}_i)\mathrm{d}\tau \tag{30}$$

Similar to the proof of Theorem 1, it is easy to show that the conclusion ii is true.

3) Similar to the proof of Theorem 1, it is easy to show that $\lim\limits_{t\to\infty} s_{i\Delta} = 0$.

In order to save space, simulation results are omitted. □

4 Conclusions

Two adaptive fuzzy control schemes have been presented for a class of uncertain MIMO nonlinear systems with unknown dead-zones and a triangular control structure. Based on the intuitive concept and piece-wise description of dead-zone model and the principle of sliding mode control, the developed controller can guarantee that all signals involved are semi-globally uniformly ultimately bounded and the tracking errors asymptotically converge to zero. By smoothing out the discontinuous sign function, the jittering in practice can be eliminated.

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ZHANG Tian-Ping Received his Ph.D. degree in control theory and applications from Southeast University in 1996. He is now a professor in College of Information Engineering at Yangzhou University. His research interests include adaptive control, fuzzy control, intelligent control, and nonlinear control. Corresponding author of this paper. E-mail: tpzhang@yzu.edu.cn

YI Yang Received his master degree in computer applied technology from Yangzhou University in 2005. He is now a Ph. D. candidate at Southeast University. His research interests include intelligent computation and intelligent control.
E-mail: yiyangcontrol@163.com