On the Existrence and Iterative Approximations of Solutions to a Class of Multivalued Quasi-Complementarity Problems

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Abstract: We introduce and study a class of multivalued quasi-complementarity problems and construct new iterative algorithms which include many existing algorithms as special cases for solving the complementarity problems. Moreover, we prove the existence of solutions to this class of quasi-complementarity problems and the convergence of iterative sequences generated by the algorithms.

Key words: multivalued quasi-complementarity problem; existence; iterative approximation; convergence

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1 Introduction

The complementarity theory introduced by Lemke[4] and Cottle and Dantzig[5] in the early 1960s and later developed by others has become an important branch of the mathematical sciences with a wide range of applications; see, for example, [1～18] and the references therein.

In recent years, the technique of change of variables introduced mainly by Van Bokhoven has been used to develop some new iterative mehtods for solving the complementarity problems; see [8, 9, 11, 12]. In particular, by this technique of change of variables, Noor and Al-said[13] established an equivalence between the generalized complementarity problems and the Wiener-Hopf equations, and used this equivalence to construct a number of iterative algorithms for solving the generalized complementarity problems.

In this paper, motivated and inspired by Li and Ding[17] and Noor[18], we introduce and study a class of multivalued quasi-complementarity problems which includes the known classes of complementarity problems as special cases. Using the technique of Li and Ding[17] and Noor[18], we also discuss the existence of solutions of this class of quasi-complementarity problems and the convergence of iterative sequences generated by the algorithms. Our results improve and extend the corresponding results obtained previously by some

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authors including Noor[8,18], Chang and Huang[10], and Li and Ding[17].

2 Formulation and basic results

Let H be a real Hilbert space whose inner product and norm are denoted by $\langle\cdot,\cdot\rangle$ and $\|\cdot\|$, respectively. Let

$$K^* = \{u \in H; \langle u, v\rangle \geq 0, \text{ for all } v \in K\},$$

be a polar cone of a closed and convex cone K in H, let D be a nonempty subset in H, and let 2^H be the family of all nonempty subsets in H.

Given nonlinear single-valued operators $g, m: D \to H$ and $M\langle\cdot,\cdot\rangle: H \times H \to H$, and multivalued operators $K, T, V: D \to 2^H$, we consider the problem of finding $x^* \in D$, $u^* \in T(x^*)$ and $v^* \in V(x^*)$ such that

$$g(x^*) \in K(x^*),\ M(u^*, v^*) \in K^*(x^*), \text{ and } \langle M(u^*, v^*), g(x^*) - m(x^*)\rangle = 0, \tag{2.1}$$

where $K(x) = m(x) + K$, and $K^*(x) = (m(x) + K)^*$.

The problem of the type (2.1) is known as a multivalued quasi-complementarity problem. Note that if $M(u, v) = u + v$, and $V \equiv A$, then problem (2.1) is equivalent to finding $x^* \in D$, $u^* \in T(x^*)$ and $v^* \in A(x^*)$ such that

$$g(x^*) \in K(x^*),\quad u^* + v^{\in} K^*(x^*),\quad \text{and } \langle u^* + v^*, g(x^*) - m(x^*)\rangle = 0, \tag{2.2}$$

which is known as a generalized strongly nonlinear quasi-complementarity problem. Problem (2.2) is mainly due to Li and Ding[17]. Furthermore, if $m(x) = 0$, and $g \equiv I$ the identity operator, then problem (2.1) is equivalent to finding $x^* \in K$, $u^* \in T(x^*)$, and $v^* \in V(x^*)$ such that

$$M(u^*, v^*) \in K^*,\quad \text{and } \langle M(u^*, v^*), x^*\rangle = 0, \tag{2.3}$$

which is called the multivalued complementarity problem introduced and considered by NOOR[18].

Remark 2.1 For appropriate and suitable choices of the operators $g, m, M(\cdot,\cdot), T, V$, a nonempty subset D and a convex set K, a number of known classes of complementarity problems and variational inequalities can be obtained as special cases studied previously by many authors; see [1～18].

We shall need the following concepts and lemmas in the sequel.

Definition 2.1 Let D be a nonempty subset of H, and $g: D \to H$ be a single-valued operator, and $T: D \to 2^H$ be a multivalued operator. For all $x_1, x_2 \in D$, the operator $M(\cdot,\cdot): H \times H \to H$ is said to be g-strongly monotone and Lipschitz continuous with respect to the first argument, if there exist constants $\alpha > 0$, $\beta > 0$ such that

$$\langle g(x_1) - g(x_2), M(u_1, \cdot) - M(u_2, \cdot)\rangle \geq \alpha \|g(x_1) - g(x_2)\|^2$$

for all $u_1 \in T(x_1)$, $u_2 \in T(x_2)$, and

$$\|M(w_1, \cdot) - M(w_2, \cdot)\| \leq \beta \|w_1 - w_2\| \quad \text{for all } w_1, w_2 \in H.$$

Definition 2.2 Let D be a nonempty subset of H, and $g: D \to H$ be a single-valued operator. The multivalued operator $V: D \to C(H)$ is said to be g-H-Lipschitz continuous if there exists a constant $\eta > 0$ such that

$$H(V(u), V(v)) \leq \eta \|g(u) - g(v)\| \quad \text{for all } u, v \in D,$$

where $C(H)$ is the family of all nonempty compact subsets of H, and $H(\cdot,\cdot)$ is the Hausdorff metric on $C(H)$.

Remark 2.2 If $g \equiv I$ the identity operator, then the above Definitions 2.1 and 2.2 reduce to Definitions 2.1 and 2.2 of Noor[18], respectively.

Lemma 2.1[14] If $K(x) = m(x) + K$, then $K^*(x) = m^*(x) \cap K^*$.

Lemma 2.2 If K is a nonempty closed convex subset of H and $z \in H$ is a given point, then $u \in K$ satisfies the inequality

$$\langle u - z, v - u\rangle \geq 0 \quad \text{for all } v \in K.$$

if and only if $u = P_K z$, where P_K is the projection of H onto K. Furthermore, P_k is non-expansive.

Lemma 2.3 If $K(u) = m(u) + K$, and K is a nonempty colsed convex subset of H, then for all u, $v \in H$, we have $P_{K(u)} v = m(u) + P_k(v - m(u))$.

3 Iterative algorithms

Theorem 3.1 Let D be a nonempty subset of H, K be a closed convex cone of H, and $m: D \to K$ be a single-valued operator. Let $K \subset g(D)$, and $K(x) = m(x) + K$ for each $x \in D$. Then $x^* \in D$, $u^* \in T(x^*)$ and $v^* \in V(x^*)$ are a solution of problem (2.1) if and only if $x^* \in D$, $u^* \in T(x^*)$ and $v^* \in V(x^*)$ are a solution of the multivalued quasi-variational inequality, i.e., the problem of finding $x^* \in D$, $u^* \in T(x^*)$ and $v^* \in V(x^*)$ such that

$$g(x^*) \in K(x^*), \quad \text{and } \langle M(u^*, v^*), g(x) - g(x^*)\rangle \geq 0 \quad \text{for all } g(x) \in K(x^*). \tag{3.1}$$

Theroem 3.2 Under the assumption of Theorem 3.1, $x^* \in D$, $u^* \in T(x^*)$ and $v^* \in V(x^*)$ are a solution of the multivalued quasivariational inequality (3.1) if and only if $x^* \in D$, $u^* \in T(x^*)$ and $v^* \in V(x^*)$ are a solution of the operator equation, i.e.,

$$g(x^*) = m(x^*) + P_K[g(x^*) - \rho M(u^*, v^*) - m(x^*)], \tag{3.2}$$

where $\rho > 0$ is a constant.

Remark 3.1 The proofs of Theorems 3.1 and 3.2 are similar to those of Li and Ding's Theorems 3.1 and 3.2[17]. Thus, we omit them.

Algorithm 3.1 Let $D \subset H$ be a nonempty subset, and let $g: D \to H$, $m: D \to K$, $M\langle\cdot,\cdot\rangle: H \times H \to H$, $T: D \to C(H)$, and $V: D \to C(H)$, where $K \subset g(D)$ the range of g, and K be a closed convex cone in H. For any given $x_0 \in D$, we take $u_0 \in T(x_0)$, $v_0 \in V(x_0)$, and

$$w_1 = m(x_0) + P_K[g(x_0) - \rho M(u_0, v_0) - m(x_0)],$$

Since $m(x_0) \in \subset g(D)$, there exists an $x_1 \in D$ such that $g(x_1) = w_1$. Since

$$u_0 \in T(x_0) \in C(H) \quad \text{and} \quad v_0 \in V(x_0) \in C(H),$$

by [16] there exist a $u_1 \in T(x_1)$ and a $v_1 \in V(x_1)$ such that

$$\| u_0 - u_1 \| \leq H(T(x_0), T(x_1)) \quad \text{and} \quad \| vM0 - v_1 \| \leq H(V(x_0), v(x_1)).$$

Let

$$w_2 = m(x_1) + P_K[g(x_1) - \rho M(u_1, v_1) - m(x_1)].$$

Then there exists an $x_2 \in D$ such that $g(x_2) = w_2$. By induction, we can obtain three sequences $\{x_n\}$, $\{u_n\}$ and $\{v_n\}$:

$$u_n \in T(x_n), \quad \| u_n - u_{n+1} \| \leq H(T(x_n), T(x_{n+1})),$$

$$v_n \in V(x_n), \quad \| v_n - v_{n+1} \| \leq H(V(x_n), V(x_{n+1})),$$

$$g(x_{n+1}) = m(x_n) + P_K[g(x_n) - \rho M(u_n, v_n) - m(x_n)], \quad n = 0, 1, 2, \cdots,$$

where $\rho > 0$ is a constant.

Algorithm 3.2 Let $D \subset H$ be a nonempty subset, $K \subset H$ be a closed convex cone, and $g: D \to H$, $m: D \to K$, $M\langle\cdot,\cdot\rangle: H \times H \to H$, $T: D \to C(H)$, and $V: D \to C(H)$, where the range $g(D)$ is convex and satisfies $K \subset g(D)$. Suppose that the sequences $\{\alpha_n\}$ and $\{\beta_n\}$ satisfy the conditions: (i) $0 \le \alpha_n, \beta_n \le 1$ for all $n \ge 0$; and (ii) $\sum_{n=0}^{\infty} \alpha_n$ diverges. For any given $x_0 \in D$, we take $\hat{u} \in T(x_0)$, $\hat{v}_0 \in V(x_0)$, and

$$\hat{w}_1 = (1 - \beta_0) g(x_0) + \beta_0 [m(x_0) + P_K(g(x_0) - \rho M(\hat{u}_0, \hat{v}_0) - m(x_0))].$$

Then there exists a $y_0 \in D$ such that $g(y_0) = \hat{w}_1$. Since $\hat{u}_0 \in T(x_0) \in C(H)$ and $\hat{v}_0 \in V(x_0) \in C(H)$, by [16], there exist a $u_0 \in T(y_0)$ and a $v_0 \in V(y_0)$ such that

$$\|\hat{u}_0 - u_0\| \le H(T(x_0), T(y_0)), \text{ and } \|\hat{v}_0 - v_0\| \le H(V(x_0), V(y_0)).$$

Let

$$w_1 = (1 - \alpha_0) g(x_0) + \alpha_0 [m(y_0) + P_K(g(y_0) - \rho M(u_0, v_0) - m(y_0))].$$

Then there exists an $x_1 \in D$ such that $g(x_1) = w_1$. By induction, we can obtain three sequences $\{u_n\}$, $\{v_n\}$ and $\{x_n\}$:

$$\hat{u} \in T(x_n),\ u_n \in T(y_n),\ \|\hat{u} - u_n\| \le H(T(x_n), T(y_n)),$$

$$\hat{v} \in V(x_n),\ v_n \in V(y_n),\ \|\hat{v} - v_n\| \le H(V(x_n), V(y_n)),$$

$$g(y_n) = (1 - \beta_n) g(x_n) + \beta_n [m(x_n) + P_K(g(x_n) - \rho M(\hat{u}_n, \hat{v}_n) - m(x_n))],$$

$$g(x_{n+1}) = (1 - \alpha_n) g(x_n) + \alpha_n [m(y_n) + P_K(g(x_n) - \rho M(u_n, v_n) - m(y_n))],$$

for $n = 0, 1, 2, \cdots$, where $\rho > 0$ is a constant.

Remark 3.2 For appropriate and suitable choices of the operators $g, m, M(\cdot,\cdot), T, V$, a nonempty subset D, and a convex subset K, a number of known algorithms for solving complementarity problems and variational inequalities can be obtained as special cases studied previously by some authors including Noor[8,18], Chang and Huang[10], and L I and Ding[17].

4 Existence and Convergence

Theorem 4.1 Let the operators $g, m, M(\cdot,\cdot), T, V$ be the same as those in Algorithm 3.1. Let $M(\cdot,\cdot)$ be g-strongly monotone with constant $\alpha > 0$ and Lipschitz continuous with constant $\beta > 0$ with respect to the first argument. Let $M(\cdot,\cdot)$ be Lipschitz continuous with constant $\gamma > 0$ with respect to the second argument. Let T be g-H-Lipschitz continuous with constant $\xi > 0$ and V be g-H-Lipschitz continuous with constant $\eta > 0$. Let m be g-Lipschitz continuous with constant $\mu > 0$. If

$$\left|\rho - \frac{\alpha - (1 - 2\mu)\gamma\eta}{\beta^2\xi^2 - \gamma^2\eta^2}\right| < \frac{\sqrt{[\alpha - (1 - 2\mu)\gamma\eta]^2 - 4(\beta^2\xi^2 - \gamma^2\eta^2)(\mu - \mu^2)}}{\beta^2\xi^2 - \gamma^2\eta^2}, \tag{4.1}$$

$$\alpha > (1 - 2\mu)\gamma\eta + 2\sqrt{(\beta^2\xi^2 - \gamma^2\eta^2)(\mu - \mu^2)},\ \rho\gamma\eta < 1 - 2\mu, \text{ and } \gamma\eta < \beta\xi, \tag{4.2}$$

then there exist a $x^* \in D$, a $u^* \in T(x^*)$, and a $v^* \in V(x^*)$, which are a solution of the multivalued quasi-complementarity problem (2.1). Furthermore, the sequences $\{g(x_n)\}$, $\{u_n\}$ and $\{v_n\}$ converge strongly to $g(x^*)$, u^* and v^*, respectively, where the sequences $\{x_n\}$, $\{u_n\}$ and $\{v_n\}$ are generated by Algorithm 3.1.

Proof From Algorithm 3.1 and Lemma 2.2, it follows that

$\|w_{n+1} - w_n\| =$

$\|g(x_{n+1}) - g(x_n)\| =$

$$\|m(x_n) + P_K[g(x_n) - \rho M(u_n, v_n) - m(x_n)] - m(x_{n-1}) - P_K[g(x_{n-1}) - \rho M(u_{n-1}, v_{n-1}) - m(x_{n-1})\| \leq$$
$$\|m(x_n) - m(x_{n-1})\| + \|g(x_n) - \rho M(u_n, v_n) - m(x_n) - g(x_{n-1}) + \rho M(u_{n-1}, v_{n-1}) + m(x_{n-1})\| \leq$$
$$2\|m(x_n) - m(x_{n-1})\| + \|g(x_n) - g(x_{n-1}) - \rho\{M(u_n, v_n) - M(u_{n-1}, v_{n-1})\}\| \leq$$
$$2\|m(x_n) - m(x_{n-1})\| + \|g(x_n) - g(x_{n-1}) - \rho\{M(u_n, v_n) - M(u_{n-1}, v_n)\}\| + \rho\|M(u_{n-1}, v_n) - M(u_{n-1}, v_{n-1})\| \tag{4.3}$$

Since $M(\cdot,\cdot): H \to H$ is g-strongly monotone and Lipschitz continuous with respect to the first argument, and T is g-H-Lipschitz continuous, we obtain

$$g(x_n) - g(x_{n-1}) - \rho\{M(u_n, v_n) - M(u_{n-1}, v_n)\}\|^2 =$$
$$\|g(x_n) - g(x_{n-1})\|^2 - 2\rho\langle M(u_n, v_n) - M(u_{n-1}, v_n), g(x_n) - g(x_{n-1})\rangle + \rho^2\|M(u_n, v_n) - M(u_{n-1}, v_n)\|^2 \leq$$
$$\|g(x_n) - g(x_{n-1})\|^2 - 2\rho\alpha\|g(x_n) - g(x_{n-1})\|^2 + \rho^2\beta^2\|u_n - u_{n-1}\|^2 \leq$$
$$\|g(x_n) - g(x_{n-1})\|^2 - 2\rho\alpha\|g(x_n) - g(x_{n-1})\|^2 + \rho^2\beta^2\{H(T(x_n), T(x_{n-1}))\} \leq$$
$$(1 - 2\rho\alpha + \rho^2\beta^2\xi^2)\|g(x_n) - g(x_{n-1})\|^2. \tag{4.4}$$

Using the Lipschitz continuity of $M(\cdot,\cdot)$ with respect to the second argument and the g-H-Lipschitz continuity of $V: D \to C(H)$, we have

$$\|M(u_{n-1}, v_n) - M(u_{n-1}, v_{n-1})\| \leq \gamma\|v_n - v_{n-1}\| \leq \gamma\{H(V(x_n), V(x_{n-1}))\} \leq \gamma\eta\|g(x_n) - g(x_{n-1})\|. \tag{4.5}$$

Using the g-Lipschitz continuity of $m: D \to K$, and combining equations (4.3), (4.4) and (4.5), we have

$$\|w_{n+1} - w_n\| = \|g(x_{n+1}) - g(x_n)\| \leq$$
$$\{2\mu + \sqrt{1 - 2\rho\alpha + \rho^2\beta^2\xi^2)} + \rho\gamma\eta\}\|g(x_n) - g(x_{n-1})\| \leq$$
$$\{2\mu + \sqrt{1 - 2\rho\alpha + \rho^2\beta^2\xi^2)} + \rho\gamma\eta\}\|w_n - w_{n-1}\| =$$
$$\theta\|w_n - w_{n-1}\|, \tag{4.6}$$

where $\theta = 2\mu + \sqrt{1 - 2\rho\alpha + \rho^2\beta^2\xi^2)} + \rho\gamma\eta < 1$ (using (4.1) and (4.2)). hence from (4.6) we know that $\{w_n\}$ is a Cauchy sequence in K. Let $w_n \to w \in K$ as $n \to \infty$. Using $K \subset g(D)$, we imply that there exists an $x^* \in D$ such that $w = g(x^*)$. Thus, $\{g(x_n)\}$ converges strongly to $g(x^*) \in K$.

Since

$$\|v_n - v_{n-1}\| \leq H(V(x_n), V(x_{n-1})) \leq \eta\|g(x_n) - g(x_{n-1}\| = \eta\|w_n - w_{n-1}\|,$$

$\{v_n\}$ is a Cauchy sequence in H. Similarly, $\{u_n\}$ is also a Cauchy sequence in H. So, $\{u_n\}$ and $\{v_n\}$ converge strongly to u^* and v^* in H, respectively. Let

$$w' = m(x^*) + P_K[g(x^*) - \rho M(u^*, v^*) - m(x^*)].$$

Observe that

$$\|w_{n+1} - w'\| = \|g(x_{n+1}) - w'\| =$$
$$\|m(x_n) + P_K[g(x_n) - \rho M(u_n, v_n) - m(x_n)] - m(x^*) - P_K[g(x^*) - \rho M(u^*, v^*) - m(x^*)]\| \leq$$
$$2\|m(x_n) - m(x^*)\| + \|g(x_n) - g(x^*) - \rho\{M(u_n, v_n) - M(u^*, v_n)\}\| +$$

$$\rho \| M(u^*, v_n) - M(u^*, v^*) \| \leq$$
$$\theta \| g(x_n) - g(x^*) \| = \theta \| w_n - w' \|.$$

It can be easily seen that $\{w_n\}$ converges strongly to w', and hence that $w = w'$, i.e.,

$$g(x^*) = m(x^*) + P_K[g(x^*) - \rho M(u^*, v^*) - m(x^*)] \in K(x^*). \quad (4.7)$$

Next, we prove that $u^* \in T(x^*)$ and $v^* \in V(x^*)$.

Note that

$$d(u^*, T(x^*)) \leq \| u^* - u_n \| + d(u_n, T(x^*)) \leq$$
$$\| u^* - u_n \| + H(T(x_n), T(x^*)) \quad (\text{by using [16]}) \leq$$
$$\| u^* - u_n \| + \xi \| g(x_n) - g(x^*) \|,$$

where $d(u^*, T(x^*)) = \inf\{ \| u^* - u \|; u \in T(x^*)\}$. It can be readily seen that $d(u^*, T(x^*)) = 0$, which leads to $u^* \in T(x^*)$. Similarly, we also have $v^* \in V(x^*)$. Finally, from Theorems 3.1 and 3.2, and equation (4.7) it follows that $x^* \in D$, $u^* \in T(x^*)$ and $v^* \in V(x^*)$ are a solution of the multivalued quasi-complementarity problem (2.1). Furthermore, the sequences $\{g(x_n)\}$, $\{u_n\}$ and $\{v_n\}$ converge strongly to $g(x^*)$, u^* and v^*, respectively.

Remark 4.1 If in Theorem 4.1, $m \equiv 0$ with constant $\mu = 0$, T is single-valued and g-strongly monotone with constant $\alpha > 0$, and $M(\cdot,\cdot)$ is defined as $M(u, v) = u + v$, then Theorem 4.1 reduces to Theorem3.1 of Chang and Huang[10]. Furthermore, if in Theorem 4.1, $D = H$, $g \equiv I$ the identity operator, and $m \equiv 0$ with constant $\mu = 0$, then Theorem 4.1 reduces to Theorem 3.2 of NOOR[18]. In addition, it can be easily seen that Theorem 4.1 extends Theorem 4.1 of Li and Ding[17] to the multivalued quasi-complementarity problem (2.1) with the operator $M(\cdot,\cdot)$.

Theorem 4.2 Let the operators $g, m, M(\cdot,\cdot), T, V$ be the same as those in Algorithm 3.2. Let $M(\cdot,\cdot)$ be g-strongly monotone with constant $\alpha > 0$ and Lipschitz continuous with constant $\beta > 0$ with respect to the first argument. Let $M(\cdot,\cdot)$ be Lipschitz continuous with constant $\gamma > 0$ with respect to the second argument. Let T be g-H-Lipschitz continuous with constant $\xi > 0$ and V be g-H-Lipschitz continuous with constant $\eta > 0$. Let m be g-Lipschitz continuous with constant $\mu > 0$. Assume that the conditions (4.1) and (4.2) in Theorem 4.1 hold. If $x^* \in D$, $u^* \in T(x^*)$ and $v^* \in V(x^*)$ are a solution of the multivalued quasi-complementarity problem (2.1), where both $T(x^*)$ and $V(x^*)$ are singletons. Then the sequences $\{g(x_n)\}$, $\{u_n\}$ and $\{v_n\}$ converge strongly to $g(x^*)$, u^* and v^*, respectively, where the sequences $\{x_n\}$, $\{u_n\}$ and $\{v_n\}$ are generated by Algorithm 3.2.

Remark 4.2 Because the proof of Theorem 4.2 is similar to that of Theorem 4.1, we omit it.

References:

[1] COTTLE R W, GIANNESSI F, LIONS J L. Variational inequalities and complementarity problems: Theorey and Applications[M]. New York: John Wiley and Sons, 1980.

[2] COTTLE R W, PANG J S, STONE R. The linear complementarity problem[M]. New York: Academic Press, 1992.

[3] GIANNESSI F, MAUGERI A. Variational inequalities and network equilibrium problems[M]. New York: Plenum Press, 1995.

[4] LEMKE C E. Bimatrix equilibrium points and mathematical programming[J]. Management Sci, 1965, 11: 681-

689.

[5] COTTLE R W, DANTZIG G B. Complementarity pivot theory of mathematical programming[J]. Linear Algebra Appl, 1968, 1:163-185.

[6] ISAC G. Complementarity problems[J]. Lecture Notes in Mathematics, Springer-Verlag, Berlin, 1992, 1528.

[7] NOOR M A. General nonlinear complementarity problems[J]. Analysis, Geometry, and Groups: A Riemann Legacy Volume, Edited by H. M. Srivastava and T. M. Rassias, Hadronic Press, Palm Harbor, Florida, 1993: 337-371.

[8] NOOR M A. Fixed-point approach for complementarity problems[J]. J Math Anal Appl, 1998, 133:437-448.

[9] NOOR M A, ZARAE S. An iterative scheme for complementarity problems[J \ 〗. Engineering Anal, 1986, 3:240-243.

[10] CHANG S S, HUNAG N J. Generalized multivalued implicit complementarity problem in Hilbert space[J]. Math Japanica, 1991, 36(6):1093-1100.

[11] AHMAD K, KAZMI K R, REHMAN N. Fixed-point technique for implicit complementarity problem in Hilbert lattice[J]. J Optim Theory Appl, 1997, 93:67-72.

[12] NOOR M A, AL-Said E A. An iterative technique for generalized strongly nonlinear complementarity problems[J]. Appl Math Letters, 1998, 11.

[13] NOOR M A, AL-Said E A. Change of variable method for generalized complementarity problems[J]. J Optim Theory Appl, 1999, 100:389-395.

[14] CHANG S S, HUANG N J. Generalized strongly nonlinear quasi-complementarity problems in Hilbert spaces[J]. J Math Anal Appl, 1991, 158:194-202.

[15] ZENG L C. Completely generalized strongly nonlinear quasi-complementarity problems in Hilbert spaces[J]. J Math Anal Appl, 1995, 193:706-714.

[16] NADLER Jr S B. Multivalued contraction mappings[J]. Pacific J Math, 1969, 30:475-488.

[17] LI H M, DING X P. Generalized strongly nonlinear quasi-complementarity problems[J]. Appl Math Mech, 1994, 15(4):79-93.

[18] NOOR M A. On a class of multivalued variational inequalities[J]. J Appl Math Stochastic Anal, 1998, 11(1):79-93.

关于一类多值拟补问题解的存在性与迭代逼近

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摘要：引入与研究了一类多值拟补问题，并构造了新的迭代算法．这些新算法概括了用于解补问题的许多现有的算法成特例．而且，还证明了此类拟补问题解的存在性与由新算法生成的迭代序列的收敛性．

关键词：多值拟补问题；存在性；迭代逼近；收敛性