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多重指数势的 phantom 宇宙学

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摘 要:利用相空间方法研究具有 N 个标量场的多重指数势的 phantom 动力学模型. 特别地,作者详细研究了 $N = 2$ 的 phantom 宇宙学,指出在给定势的条件下允许存在 big rip 吸引子.

关键词:相空间;phantom; big rip 吸引子

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0 引 言

宇宙微波背景各向异性[1~2],红移和超新星的光度距离的联系[3~4],以及对星系分布的宇宙学观测表明我们的宇宙在空间上是平坦的,大约 2 / 3 的能量密度来自于暗能量. 暗能量主要候选者之一是由标量场描写的 quintessence 模型[5],几年来已被大量研究. 李新洲等人提出了具有 $O(N)$ 对称性的多标量模型[6],并详细讨论了它的动力学机制[7]. 态方程参数 $w = p / \rho$ 是一个重要的宇宙学参数,它与宇宙膨胀速率的变化密切相关. 目前的观测值为 $-1.38 < w < -0.82$[7],而且 WMAP,SDSS 和 SNeIa 等观测值倾向于 w 的取值小于 -1. 所有具有正则拉格朗日量的系统都有 $w \geqslant -1$. 为了实现 $w < -1$,方案之一是引进负动能,这样的标量场被称作为 phantom. 具有 $O(N)$ 对称性的多标量 phantom 场已为文献[8]讨论,单标量场的双指数势 phantom 模型已由孙昌波[9]等人讨论过,而本文讨论不具有对称性的多标量 phantom,并指出在给定势的条件下允许存在 big rip 吸引子.

1 多标量场 phantom 宇宙动力学

具有 m 重指数势的 N 个 phantom 场的拉格朗日量为

$$L = \sqrt{-g}\left(R - \frac{1}{2}\dot{\vec{\phi}} \cdot \dot{\vec{\phi}} - V(\vec{\phi})\right). \tag{1}$$

其中 R 是曲率标量,phantom 场 $\vec{\phi}$ 的分量场为 $\phi_I, I = 1, \cdots, N$; 多重指数势利用矩阵 $(\alpha_{iI}), i = 1, \cdots, m$,记作

$$V(\vec{\phi}) = \sum_{i=1}^{m} \Lambda_i e^{-\vec{\alpha}_i \cdot \vec{\phi}}. \tag{2}$$

由于 phantom 场的动能项具有 $O(N)$ 对称性,而多重势(2)不具有 $O(N)$ 对称性,通过 ϕ_I 场的 $O(N)$ 转

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动,可以将 (α_{iI}) 标准化.

当前的宇宙观测支持宇宙是平坦的,因此时空度规选取空间平坦的 Robertson – Walker 度规

$$ds^2 = dt^2 - a(t)^2(dx_1^2 + dx_2^2 + dx_3^2) \tag{3}$$

相应的运动方程和爱因斯坦方程为:

$$\begin{cases} \ddot{\phi}_I + 3H\dot{\phi}_I - \dfrac{\partial V}{\partial \phi_I} = 0 \\ H^2 = \dfrac{\kappa^2}{3}\rho_\phi \equiv -\dfrac{1}{12}(\dot{\vec{\phi}} \cdot \dot{\vec{\phi}}) + \dfrac{1}{6}V. \\ \dot{H} = -\dfrac{\kappa^2}{2}(-\dot{\vec{\phi}} \cdot \dot{\vec{\phi}}) \equiv \dfrac{1}{4}(\dot{\vec{\phi}} \cdot \dot{\vec{\phi}}) \end{cases} \tag{4}$$

适当选取单位,使 $\kappa^2 = 8\pi G \equiv \frac{1}{2}$,上点表示对时间的导数,中点表示 N 矢量的标积. $\rho_\phi = -\frac{1}{2}\dot{\vec{\phi}} \cdot \dot{\vec{\phi}} + V(\phi)$, $p_\phi = -\frac{1}{2}\dot{\vec{\phi}} \cdot \dot{\vec{\phi}} - V(\phi)$ 分别是 ϕ 场的能量密度和压强. H 是 Hubble 参数.

在一般的 N 和 m 值情形下寻找解析解是困难的,可以通过相空间的方法来分析模型的动力学性质. 引进下列无量纲变量

$$x_I = \frac{\dot{\phi}_2}{\sqrt{12}\,H}, \qquad y_i = \sqrt{\frac{\Lambda_i\, e^{-\vec{\alpha}_i \cdot \vec{\phi}}}{6H^2}}. \tag{5}$$

动力学方程组(4)约化为

$$\frac{\dot{x}_I}{H} = -3y^2 x_I - \sqrt{3}\sum_{i=1}^{m} \alpha_{iI} y_i^2, \tag{6}$$

$$\frac{\dot{H}}{H^2} = 3x^2, \tag{7}$$

$$\frac{\dot{y}_i}{H} = -\sqrt{3}(\sqrt{3}x^2 + \vec{\alpha}_i \cdot \vec{x})y_i. \tag{8}$$

其中

$$x^2 = \sum_{i=1}^{N} x_I^2, \qquad y^2 = \sum_{i=1}^{m} y_i^2. \tag{9}$$

由于本文仅讨论膨胀宇宙,宇宙标度因子 a 是时间的增函数,所以可用 $\ln a$ 来替代宇宙时 t 作为动力学系统的自变量,令

$$P = \ln a, \tag{10}$$

(6)和(8)式约化成自治系统

$$x'_I = -3y^2 x_I - \sqrt{3}\sum_{i=1}^{m} \alpha_{iI} y_i^2, \tag{11}$$

$$y'_i = -\sqrt{3}(\sqrt{3}x^2 + \vec{\alpha}_i \cdot \vec{x})y_i. \tag{12}$$

加上一个限制方程

$$\Omega_\phi = y^2 - x^2 = 1. \tag{13}$$

标量场的状态方程参数在新变量下可以写成一个简单的形式

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{x^2 + y^2}{x^2 - y^2}, \tag{14}$$

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由方程(11)和(12)可得知临界点方程:

$$\sqrt{3}y^2x_l + \sum_{i=1}^{m}\alpha_{il}y_i^2 = 0, \tag{15}$$

$$(\sqrt{3}x^2 + \vec{\alpha}_i \cdot \vec{x})y_i = 0. \tag{16}$$

2 $N = 2$, $m = 2$ 情形的临界点

多重指数势可从基本物理理论中导出,例如可从弦/M理论中导出. 11 维 M 理论的低能理论可以导出 $N = 2$, $m = 2$ 的指数势. 利用(2)式的形式,对应于额外维维数 $d = 1,2,\cdots,7$,势可以写作

$$\vec{\alpha}_1 = \left(3\sqrt{\frac{d}{d+2}}, \sqrt{\frac{14-2d}{d+2}}\right),\ \vec{\alpha}_2 = \left(\sqrt{\frac{d+2}{2}}, 0\right). \tag{17}$$

将(17)式代入临界点方程,则有

$$\begin{cases} -3y^2x_1 - \sqrt{3}(\alpha_{11}y_1^2 + \alpha_{21}y_2^2) = 0 \\ -3y^2x_2 - \sqrt{3}(\alpha_{12}y_1^2 + \alpha_{22}y_2^2) = 0 \\ -\sqrt{3}(\sqrt{3}x^2 + \alpha_{11}x_1 + \alpha_{12}x_2)y_1 = 0 \\ -\sqrt{3}(\sqrt{3}x^2 + \alpha_{21}x_1 + \alpha_{22}x_2)y_2 = 0 \end{cases}. \tag{18}$$

由方程组(18)和限制条件(13)求得临界点,记为 $(x_{1c}, x_{2c}, y_{1c}, y_{2c})$

$$\text{(i)}\ \left(-\frac{\alpha_{11}}{\sqrt{3}}, -\frac{\alpha_{12}}{\sqrt{3}}, \frac{\sqrt{3+\alpha_{11}^2+\alpha_{12}^2}}{\sqrt{3}}, 0\right), \tag{19}$$

$$\text{(ii)}\ \left(-\frac{\alpha_{21}}{\sqrt{3}}, -\frac{\alpha_{22}}{\sqrt{3}}, 0, \frac{\sqrt{3+\alpha_{21}^2+\alpha_{22}^2}}{\sqrt{3}}\right). \tag{20}$$

为了将临界点进行分类,将变量写成临界点加上一个扰动量

$$x_1 = x_{1c} + \mu,\ x_2 = x_{2c} + \nu,\ y_1 = y_{1c} + \rho,\ y_2 = y_{2c} + \sigma \tag{21}$$

其中 μ, ν, ρ, σ 是在临界点附近变量的扰动. 把上述表达式代入方程(11)和(12),得到线性化扰动方程.

$$\begin{cases} \mu' = (-3 - \alpha_{11}^2 - \alpha_{12}^2)\mu \\ \nu' = (-3 - \alpha_{11}^2 - \alpha_{12}^2)\nu \\ \rho' = \alpha_{11}\sqrt{3+\alpha_{11}^2+\alpha_{12}^2}\,\mu + \alpha_{12}\sqrt{3+\alpha_{11}^2+\alpha_{12}^2}\,\nu \\ \sigma' = (-\alpha_{11}^2 - \alpha_{12}^2 + \alpha_{11}\alpha_{21} + \alpha_{12}\alpha_{22})\sigma \end{cases}. \tag{22}$$

上式已忽略了高阶小项. 扰动方程的系数组成了 4×4 的矩阵,其本征值决定了临界点的类型和稳定性. 该矩阵的本征值为

$$(0,\ -3-\alpha_{11}^2-\alpha_{12}^2,\ -3-\alpha_{11}^2-\alpha_{12}^2,\ -\alpha_{11}^2-\alpha_{12}^2+\alpha_{11}\alpha_{21}+\alpha_{12}\alpha_{22}). \tag{23}$$

将(17)式代入(23)式,可知参数值满足 $-\alpha_{11}^2-\alpha_{12}^2+\alpha_{11}\alpha_{21}+\alpha_{12}\alpha_{22} < 0$,这表明临界点(i)(19)式是系统的动力学吸引子. 容易算出态方程参数

$$w = \frac{x^2+y^2}{x^2-y^2} = -1 - \frac{2}{3}(\alpha_{11}^2 + \alpha_{12}^2). \tag{24}$$

在 $\alpha_{11} = \alpha_{12} = 0$ 情形下,上述吸引子是后期 de Sitter 吸引子,否则为 big rip 吸引子.

采用上述同样的方法,可知临界点(ii)(20)式不是系统的动力学吸引子.

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3 $N = 2$, $m = 3$ 情形的临界点

在 $N = 2$, $m = 3$ 的一般情形下,方程(11)和(12)变成如下形式:

$$\begin{cases} x'_1 = -3y^2x_1 - \sqrt{3}(\alpha_{11}y_1^2 + \alpha_{21}y_2^2 + \alpha_{31}y_3^2) \\ x'_2 = -3y^2x_2 - \sqrt{3}(\alpha_{12}y_1^2 + \alpha_{22}y_2^2 + \alpha_{32}y_3^2) \\ y'_1 = -\sqrt{3}(\sqrt{3}x^2 + \alpha_{11}x_1 + \alpha_{12}x_2)y_1 \\ y'_2 = -\sqrt{3}(\sqrt{3}x^2 + \alpha_{21}x_1 + \alpha_{22}x_2)y_2 \\ y'_3 = -\sqrt{3}(\sqrt{3}x^2 + \alpha_{31}x_1 + \alpha_{32}x_2)y_3 \end{cases} \tag{25}$$

其临界点为 $(x_{1c}, x_{2c}, y_{1c}, y_{2c}, y_{3c})$

(iii) $\left(-\frac{\alpha_{11}}{\sqrt{3}}, -\frac{\alpha_{12}}{\sqrt{3}}, \frac{\sqrt{3+\alpha_{11}^2+\alpha_{12}^2}}{\sqrt{3}}, 0, 0\right)$.

(iv) $\left(-\frac{\alpha_{21}}{\sqrt{3}}, -\frac{\alpha_{22}}{\sqrt{3}}, 0, \frac{\sqrt{3+\alpha_{21}^2+\alpha_{22}^2}}{\sqrt{3}}, 0\right)$.

(v) $\left(-\frac{\alpha_{31}}{\sqrt{3}}, -\frac{\alpha_{32}}{\sqrt{3}}, 0, 0, \frac{\sqrt{3+\alpha_{31}^2+\alpha_{32}^2}}{\sqrt{3}}\right)$.

在临界点附近把变量写成

$$x_1 = x_{1c} + \mu,\ x_2 = x_{2c} + \nu,\ y_1 = y_{1c} + \rho,\ y_2 = y_{2c} + \sigma,\ y_3 = y_{3c} + \lambda,$$

代入方程(25),得到扰动方程. 扰动方程的系数组成了 5×5 的矩阵,其本征值决定了临界点的类型和稳定性.

临界点(iii)相应的本征值为

$(0, -3-\alpha_{11}^2-\alpha_{12}^2, -3-\alpha_{11}^2-\alpha_{12}^2, -\alpha_{11}^2-\alpha_{12}^2+\alpha_{11}\alpha_{21}+\alpha_{12}\alpha_{22}, -\alpha_{11}^2-\alpha_{12}^2+\alpha_{11}\alpha_{31}+\alpha_{12}\alpha_{32})$.

如果

$$-\alpha_{11}^2-\alpha_{12}^2+\alpha_{11}\alpha_{21}+\alpha_{12}\alpha_{22} < 0 \text{ 且 } -\alpha_{11}^2-\alpha_{12}^2+\alpha_{11}\alpha_{31}+\alpha_{12}\alpha_{32} < 0,$$

则此临界点为动力学吸引子. 又因为

$$w = \frac{x^2+y^2}{x^2-y^2} = -1-\frac{2}{3}(\alpha_{11}^2+\alpha_{12}^2) < -1, \tag{26}$$

所以临界点(iii)是 big rip 吸引子.

临界点(iv)相应的本征值为

$(0, -3-\alpha_{21}^2-\alpha_{22}^2, -3-\alpha_{21}^2-\alpha_{22}^2, -\alpha_{21}^2-\alpha_{22}^2+\alpha_{11}\alpha_{21}+\alpha_{12}\alpha_{22}, -\alpha_{21}^2-\alpha_{22}^2+\alpha_{21}\alpha_{31}+\alpha_{22}\alpha_{32})$.

如果

$$-\alpha_{21}^2-\alpha_{22}^2+\alpha_{11}\alpha_{21}+\alpha_{12}\alpha_{22} < 0 \text{ 且 } -\alpha_{21}^2-\alpha_{22}^2+\alpha_{21}\alpha_{31}+\alpha_{22}\alpha_{32} < 0,$$

则此临界点为动力学 big rip 吸引子.

临界点(v)的本征值为

$(0, -3-\alpha_{31}^2-\alpha_{32}^2, -3-\alpha_{31}^2-\alpha_{32}^2, -\alpha_{31}^2-\alpha_{32}^2+\alpha_{11}\alpha_{31}+\alpha_{12}\alpha_{32}, -\alpha_{31}^2-\alpha_{32}^2+\alpha_{21}\alpha_{31}+\alpha_{22}\alpha_{32})$.

如果

$$-\alpha_{31}^2-\alpha_{32}^2+\alpha_{11}\alpha_{31}+\alpha_{12}\alpha_{32} < 0 \text{ 且 } -\alpha_{31}^2-\alpha_{32}^2+\alpha_{21}\alpha_{31}+\alpha_{22}\alpha_{32} < 0,$$

则此临界点为动力学 big rip 吸引子.

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参考文献:

[1] BENNETT C L. The microwave anisotropy probe mission[J]. Astrophys J,2003,583:1.

[2] NETTERFIELD C B. A measurement by boomerang of multiple peaks in the angular power spectrum of the cosmic microwave background[J]. Astrophys J,2002,571:604.

[3] ADAM G RIESS. Observational evidence from supernovae for an accelerating universe and a cosmological constant[J]. Astron J,1998,116:1009.

[4] PERLMUTTER S. Measurements of omega and lambda from 42 high redshift supernovae[J]. Astrophys J,1999,517:565.

[5] RATRA B,PEEBLES P J. Cosmological consequences of a rolling homogeneous scalar field[J]. Phys Rev,1988,D37: 3406 .

[6] LI X Z,HAO J G,LIU D J. Quintessence with O(N) symmetry[J]. Class Quant Grav, 2002, 19:6049.

[7] HAO J G,LI X Z. Cosmological dynamics of scalar fields with O(N) symmetry[J]. Class Quant Grav,2004, 21:4771.

[8] LI X Z,HAO J G. Phantom field with O(N) symmetry in an exponential potential[J]. Phys Rev,2004, D69:107303.

[9] 孙昌波,赵一斌,孙珏岷. 具有双重指数势的 phantom 宇宙学[J]. 上海师范大学学报,2005,34:41.

Phantom cosmologies with multi- exponential potential

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Abstract: We study the dynamics of phantom model with multi – exponential potential which has N scalar fields by using phase space method. Especially, in this paper, we show that big rip attractors would exist in a given potential with the case of $N = 2$ phantom models.

Key words: phase space; phantom; big rip attractor