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Sufficient Conditions for Input-to-State Stability of Switched Systems[1)]

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Abstract Input-to-state stability of switched systems is studied. Sufficient conditions for switched systems to be input-to-state stable are given under the assumption that all subsystems are input-to-state stable. $\mathcal{KL}$ and $\mathcal{K}$ functions for switched systems are constructed by those for subsystems. For a class of switched systems a lower bound on the dwell time of switching is explicitly calculated which guarantees input-to-state stability. All results are derived constructively.

Key words Input-to-state stability, Lyapunov stability, switched systems, dwell time

1 Introduction

The notion of input-to-state stability (ISS)[1] has proved to be very useful in control systems analysis and design. In parallel with the Lyapunov stability theory, some characterizations and properties of ISS were given in[2~5]. The input-to-state control Lyapunov function (ISS-CLF) was introduced in [2] and the existence of an ISS-CLF was shown to be a necessary and sufficient condition for the existence of an ISS stabilizing state feedback law. There have been various generalizations of ISS[4,6,7].

On the other hand, there has been considerable interest in studying switched systems in recent years (for example, refer to [8] and the references therein). This is mainly because switched systems constitute a special class of hybrid dynamical systems, which have broad applications and take specific and simpler forms. The issue of stability of switched systems is considered to be of great importance and therefore has been studied extensively. The existence of a common Lyapunov function is necessary and sufficient for stability under arbitrary switching laws[8]. There have been a number of results adopting a common Lyapunov function. However, the class of switched systems that can be stabilized by arbitrary switching laws is, in general, very limited. Therefore, it has received a lot of attention to identify a restricted class of switching laws under which switched systems are stable[9]. Single Lyapunov function technique and multiple Lyapunov function technique proposed by Branicky[9] proved to be major tools in this aspect.

Compared with the vast existing results on stability of switched systems, very little attention has been paid to the study on ISS of switched systems. In [4] the ISS of integration form and the disturbance attenuation problem were studied. Robust switching controllers were constructed via ISS-Lyapunov functions [10]. Converse Lyapunov theorems were given for ISS switched systems[11].

A switched system may not be stable even though each individual subsystem is stable. Similarly, a switched system may fail to be ISS even if all of its subsystems are ISS. Therefore, looking for conditions under which a switched system is ISS is a significant problem. This paper focuses on this problem. Under the assumption that all subsystems

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are ISS we derive sufficient conditions for switched systems to be ISS. The results are given constructively in the sense that the $\mathcal{KL}$ and $\mathcal{K}$ functions, which are required for ISS of switched systems, are constructed systematically according to the corresponding functions of each active subsystem. Moreover, for a class of switched systems a lower bound on the dwell time is explicitly calculated which guarantees ISS under arbitrary switching with dwell time no less than the lower bound.

2 Preliminaries and problem statement

Denote the usual Euclidean norm by $|\cdot|$, and the supremum norm of a locally essentially measurable bounded function $\boldsymbol{u}: R_{\geqslant 0} \rightarrow R^m$ by $\|u\| = \text{ess}\sup_{t\geqslant 0}\{|\boldsymbol{u}(t)|\}$. All such functions with the finite supremum norm make a linear normed space, denoted by $\mathcal{L}_\infty^m$.

Consider the system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u}) \tag{1}$$

where $\boldsymbol{x}\in R^n$ is the state, $\boldsymbol{u}\in \mathcal{L}_\infty^m$ is the input, and f satisfies $\boldsymbol{f}(\mathbf{0},\mathbf{0})=\mathbf{0}$. By $\boldsymbol{x}(t,\boldsymbol{x}_0,\boldsymbol{u})$ we denote the trajectory of system (1) starting from the initial state $\boldsymbol{x}(0)=\boldsymbol{x}_0$ and under the control $\boldsymbol{u}$.

Definition 1[1]. System (1) is input-to-state stable (ISS) if there exist a $\mathcal{KL}$ function β and a $\mathcal{K}$ function γ, such that, for each input $\boldsymbol{u}\in\mathcal{L}_\infty^m$, $\boldsymbol{x}_0\in R^n$ and $t\geqslant 0$, the following inequality holds.

$$|\boldsymbol{x}(t,\boldsymbol{x}_0,\boldsymbol{u})| \leqslant \beta(|\boldsymbol{x}_0|,t)+\gamma(\|\boldsymbol{u}\|) \tag{2}$$

Lemma 1[2]. System (1) is ISS if and only if there exist a $\mathcal{KL}$ function β and a $\mathcal{K}$ function γ, such that for each input $\boldsymbol{u}\in\mathcal{L}_\infty^m$, $\boldsymbol{x}_0\in R^n$ and $t\geqslant 0$, the following inequality holds.

$$|\boldsymbol{x}(t,\boldsymbol{x}_0,\boldsymbol{u})| \leqslant \max\{\beta(|\boldsymbol{x}_0|,t)+\gamma(\|\boldsymbol{u}\|)\} \tag{3}$$

Throughout this paper we study switched systems of the following form

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_i(\boldsymbol{x},\boldsymbol{u}), \quad i=1,\cdots,k \tag{4}$$

where $\boldsymbol{x}\in R^n$ is the state and $\boldsymbol{u}\in\mathcal{L}_\infty^m$ is the input. By the jth subsystem we mean the system $\dot{\boldsymbol{x}}=\boldsymbol{f}_j(\boldsymbol{x},\boldsymbol{u})$. Let

$$0 = t_0 < t_1 < \cdots < t_p < \cdots \tag{5}$$

be a switching sequence, and suppose the i_jth subsystem is active on the time interval $[t_j\, t_{j+1})$. For the sake of simplicity of notations, $\boldsymbol{x}(t,\boldsymbol{x}_0,\boldsymbol{u})$ denotes the trajectory of switched system (4) starting from the initial state $\boldsymbol{x}(0)=\boldsymbol{x}_0$ under the control $\boldsymbol{u}$ and the switching sequence (5), and $\boldsymbol{x}^j(t,\boldsymbol{x}_0,\boldsymbol{u})$ stands for the trajectory of the jth subsystem of switched system (4) starting from the initial state $\boldsymbol{x}(0)=\boldsymbol{x}_0$ under the control $\boldsymbol{u}$.

Switched system (4) is said to be ISS if its trajectory satisfies (2). It is easy to see that Lemma 1 remains true for switched system (4).

Our goal is to find conditions under which switched system (4) is ISS under switching sequence (5).

Before developing the main results, several mild assumptions are required.

Assumption 1. All subsystems of switched system (4) are ISS, that is, there exist $\mathcal{KL}$ functions β_j and $\mathcal{K}$ functions γ_j, $j=1,2,\cdots,k$, such that for each $\boldsymbol{u}\in\mathcal{L}_\infty^m$, $\boldsymbol{x}_0\in R^n$ and $t\geqslant 0$ the following inequality holds.

$$|\boldsymbol{x}^j(t,\boldsymbol{x}_0,\boldsymbol{u})| \leqslant \max\{\beta_j(|\boldsymbol{x}_0|,t),\gamma_j(\|\boldsymbol{u}\|)\} \tag{6}$$

Assumption 2. There exists a constant $\delta>0$, such that for any $T>0$ there is an integer s satisfying $t_s>T$, and $t_{s+1}-t_s\geqslant\delta$.

Remark 1. Assumption 2 simply means that there exists a positive lower bound on the dwell time of some subsequence of switching. This assumption is necessary because otherwise we would have $\lim_{j\to\infty} t_{j+1}-t_j=0$, which is unacceptable in practice.

Assumption 3. For any fixed y, $\beta_j(y,0)-\beta_j(y,\delta)>0$, $j=1,2,\cdots,k$, where δ is as in

Assumption 2.

Remark 2. Since $\beta_j(y, t)\to 0$ as $t\to\infty$, this assumption is very general and mild. In fact, if $\beta_j(y,t)=\phi_j(y)e^{-a_j t}$, as frequently used in the literature, this assumption is automatically satisfied.

3 ISS conditions

If Assumptions 1~3 hold, one can easily see that $x(t,x_0,u)=x^{i_m}(t-t_m,x_m,u)$, $t_m\leqslant t<t_{m+1}$, $m=0,1,2,\cdots$, where $x_m=x^{i_{m-1}}(t_m-t_{m-1},x_{m-1},u)$. Now, we make an estimate on $x(t,x_0,u)$.

When $t_0\leqslant t<t_1$, (6) gives

$$|x(t,x_0,u)|=|x^{i_0}(t,x_0,u)|\leqslant\max\{\beta_{i_0}(|x_0|,t),\ \gamma_{i_0}(\|u\|)\}=\max\{\beta^0(|x_0|,t),\ \gamma^0(\|u\|,t)\}\tag{7}$$

where $\beta^0(y,t)=\beta_{i_0}(y,t)$, $\gamma^0(y,t)=\gamma_{i_0}(y)$. Thus,

$$|x_1|=|x^{i_0}(t_1,x_0,u)|\leqslant\max\{\beta^0(|x_0|,t_1),\gamma^0(\|u\|,t_1)\}\tag{8}$$

Since β_{i_1} is strictly increasing with respect to its first variable, when $t_1\leqslant t<t_2$, we have

$$\begin{aligned}|x(t,x_0,u)|&=|x^{i_1}(t-t_1,x_1,u)|\leqslant\max\{\beta_{i_1}(|x_1|,t-t_1),\gamma_{i_1}(\|u\|)\}\leqslant\\&\max\{\beta_{i_1}(\max\{\beta^0(|x_0|,t_1),\gamma^0(\|u\|,t_1)\},t-t_1),\gamma_{i_1}(\|u\|)\}=\\&\max\{\beta_{i_1}(\beta^0(|x_0|,t_1),t-t_1),\max\{\beta_{i_1}(\gamma^0(\|u\|,t_1),t-t_1),\gamma_{i_1}(\|u\|)\}\}=\\&\max\{\beta^1(|x_0|,t),\gamma^1(\|u\|,t)\}\end{aligned}\tag{9}$$

where $\beta^1(y,t)=\beta_{i_1}(\beta^0(y,t_1),t-t_1)$, $\gamma^1(y,t)=\max\{\beta_{i_1}(\gamma^0(y,t_1),t-t_1),\gamma_{i_1}(y)\}$. Continuing the same procedure we obtain

$$\beta^0(y,t)=\beta_{i_0}(y,t),\ \beta^s(y,t)=\beta_{i_s}(\beta^{s-1}(y,t_s),t-t_s),\ s=1,2\cdots\tag{10}$$

$$\gamma^0(y,t)=\gamma_{i_0}(y),\ \gamma^s(y,t)=\max\{\beta_{i_s}(\gamma^{s-1}(y,t_s),\ t-t_s),\ \gamma_{i_s}(y)\},\ s=1,2\cdots\tag{11}$$

$$|x(t,x_0,u)|\leqslant\max\{\beta^p(|x_0|,t),\gamma^p(\|u\|,t)\},\ t_p\leqslant t<t_{p+1},p=0,1,\cdots\tag{12}$$

We are going to construct β and γ for switched system (4) via β^j and γ^j. To this end, we introduce $\delta_j(y)$ which satisfies

$$\beta_j(y,0)=y+\delta_j(y),\ j=1,2,\cdots\tag{13}$$

Evaluating (6) at $t=0$ and $u=0$ gives rise to $|x_0|\leqslant\beta_j(|x_0|,0)$, $\forall x_0$, which means that $\delta_j(y)$ is nonnegative.

Theorem 1. Suppose the following three conditions are satisfied.

Condition (a). For any fixed y the series $\sum_{j=1}^{\infty}\delta_{i_j}(\beta^{j-1}(y,t_j))$ is convergent.

Condition (b). As a function of y the series $\sum_{j=p}^{\infty}\delta_{i_j}(\beta^{j-1}(y,t_j))$ is increasing for each $p=1,2,\cdots$.

Condition (c). For any fixed y the sequence

$$\beta_{i_{m+q}}(\beta_{i_{m+q-1}}(\cdots(\beta_{i_{m+1}}(\beta_{i_m}(y,t_{m+1}-t_m),t_{m+2}-t_{m+1}),\cdots),t_{m+q}-t_{m+q-1}),0)\tag{14}$$

is bounded with respect to $q=0,1,2,\cdots$, $m=0,1,2,\cdots$, where (14) is considered as $\beta_{i_m}(y,0)$ when $q=0$. Then, switched system (4) is ISS under the switching sequence (5) with β and γ functions defined by the following

$$\beta(y,t)=\beta^s(y,t)+\sum_{j=s+1}^{\infty}\delta_{i_j}(\beta^{j-1}(y,t_j)),\ t_s\leqslant t<t_{s+1},\ s=0,1,\cdots\tag{15}$$

$$\gamma(y)=\sup\{\gamma^j(y,t_j),\ j=1,2,\cdots\}\tag{16}$$

In order to prove this theorem we need some lemmas. The proof is simple and thus omitted.

Lemma 2. If for any fixed x, $h(x, y)$ is a continuous function of y, and for any fixed y, $h(x,y)$ is a monotone continuous function of x, then, $h(x,y)$ is continuous as a function of the whole variables (x, y).

Lemma 3. Suppose $h(x,y)$ is a continuous function defined on $a\leqslant x\leqslant b$, $0\leqslant y<\infty$, and for any fixed x, $h(x,y)$ is decreasing with respect to y. Moreover, $h(x,0)-h(x,\Delta)>0$ for some $\Delta>0$ and $x\in[a\ b]$. Then, there exists a constant $\mu>0$ such that for each $x\in[a\ b]$ and $y\geqslant\Delta$, $h(x,0)-h(x,y)\geqslant\mu$ holds.

Lemma 4. Suppose the series $\sum_{n=1}^{\infty}a_n(x)$ is convergent for all $x\in[0\ \ \infty)$ and $a_n(x)\geqslant 0$ is a continuous function of x. If $s_m(x)\triangleq\sum_{n=m}^{\infty}a_n(x)$ is increasing with respect to x for $m=1,2,\cdots$, then $s_m(x)$ is a continuous function of x.

Now, we turn to the proof of theorem 1. What we need is to show that β and γ defined by (15) and (16) are right functions for switched system (4) to be ISS under the switching sequence (5). To do this we only have to verify that β is a $\mathscr{KL}$ function and γ is a $\mathscr{K}$ function and for each t (3) holds.

First of all, we prove (3). It is evident that when $t_s\leqslant t<t_{s+1}$, it holds that

$$\gamma^s(y,t)=\max\{\beta_{i_s}(\gamma^{s-1}(y,t_s),\ t-t_s),\ \gamma_{i_s}(y)\}\leqslant$$
$$\max\{\beta_{i_s}(\gamma^{s-1}(y,t_s),0),\gamma_{i_s}(y)\}=\gamma^s(y,t_s)\leqslant\gamma(y) \tag{17}$$

By (15) we know $\beta(y,t)\geqslant\beta^s(y,t)$. Therefore (3) follows immediately from (12).

Secondly, we prove that $\beta(y,t)$ is a $\mathscr{KL}$ function. It is not difficult to know from Condition (b) of Theorem 1, Lemma 2, Lemma 4 and a direct calculation that $\beta(y,t)$ is continuous with respect to (y,t), strictly increasing with respect to y and decreasing with respect to t. Moreover, $\beta(0,t)=0$ holds for all t. Now, we claim that $\lim_{t\to\infty}\beta(y,t)=0$ holds for any y, and thus $\beta(y,t)$ is a $\mathscr{KL}$ function. If this is not true, there must exist some y satisfying $\lim_{t\to\infty}\beta(y,t)=c>0$. Choose $T>0$ such that for $t\geqslant T$, $c\leqslant\beta(y,t)<1.5c$. Choose an integer K such that for all $s\geqslant K$,

$$0\leqslant\sum_{j=s}^{\infty}\delta_{i_j}(\beta^{j-1}(y,t_j))<0.5c \tag{18}$$

For T chosen above, there exits s_0 satisfying $t_{s-1}\geqslant T$ for all $s\geqslant s_0$. Therefore, when $s\geqslant s_0$ it holds that

$$c\leqslant\beta(y,t)=\beta^s(y,t)+\sum_{j=s+1}^{\infty}\delta_{i_j}(\beta^{j-1}(y,t_j))<1.5c,\ t_s\leqslant t<t_{s+1} \tag{19}$$

Let $s*=\max\{K,\ s_0\}$. If $s\geqslant s^*$, or equivalently, $t\geqslant t_{s^*}$, (18) and (19) must hold. Therefore,

$$0.5c\leqslant c-\sum_{j=s+1}^{\infty}\delta_{i_j}(\beta^{j-1}(y,t_j))\leqslant\beta^s(y,t)\leqslant 1.5c \tag{20}$$

Assumption 2 guarantees that a sequence $s_r>s^*$ and an integer p exist with the property $i_{s_r}=p$, $r=1,2,\cdots$, and $t_{s_{r+1}}-t_{s_r}\geqslant\delta>0$. It follows from Assumption 3 and Lemma 3 that a constant $\mu>0$ exists such that for $\forall z\in[0.5c\ \ 1.5c]$

$$\beta_p(z,0)-\beta_p(z,t_{s_{r+1}}-t_{s_r})\geqslant\beta_p(z,0)-\beta_p(z,\delta)\geqslant\mu \tag{21}$$

(20) indicates $\beta^{s_r-1}(y,t_{s_r})\in[0.5c\ \ 1.5c]$. Taking (15), (10) and (13) into account, we have

$$\beta(y,t_{s_r})-\beta(y,t_{s_{r+m+1}})=\sum_{q=r}^{r+m}(\beta(y,t_{s_q})-\beta(y,t_{s_q+1}))+\sum_{q=r}^{r+m}(\beta(y,t_{s_q+1})-\beta(y,t_{s_{q+1}}))\geqslant$$
$$\sum_{q=r}^{r+m}(\beta(y,t_{s_q})-\beta(y,t_{s_{q+1}}))=$$
$$\sum_{q=r}^{r+m}(\beta^{s_q}(y,t_{s_q})-\beta^{s_{q+1}}(y,t_{s_{q+1}})+\delta_{i_{s_{q+1}}}(\beta^{s_q}(y,t_{s_{q+1}})))=$$
$$\sum_{q=r}^{r+m}(\beta_{i_{s_q}}(\beta^{s_q-1}(y,t_{s_q}),0)-\beta_{i_{s_{q+1}}}(\beta^{s_q}(y,t_{s_{q+1}}),0)+$$

$$\beta_{i_{s_{q+1}}}(\beta^{s_q}(y,t_{s_{q+1}}),0)-\beta^{s_q}(y,t_{s_{q+1}}))=$$
$$\sum_{q=r}^{r+m}(\beta_{i_{s_q}}(\beta^{s_{q-1}}(y,t_{s_q}),0)-\beta_{i_{s_q}}(\beta^{s_{q-1}}(y,t_{s_q}),t_{s_{q+1}}-t_{s_q}))\geqslant$$
$$(m+1)\mu \tag{22}$$

By letting $m\to\infty$ we have $\beta(y,t_{s_r})-c=\infty$. This is a contradiction.

Finally, we show that $\gamma(y)$ is a $\mathcal{K}$ function. While this can be easily seen from the following expression and Condition (c) of Theorem 1.

$$\gamma^s(y,t)=\max\{\gamma_{i_s}(y),\ \beta_{i_s}(\beta_{i_{s-1}}(\cdots(\beta_{i_{s-p+1}}(\beta_{i_{s-p}}(\gamma_{i_{s-p-1}}(y),t_{s-p+1}-t_{s-p}),$$
$$t_{s-p+2}-t_{s-p+1}),\cdots,),t_s-t_{s-1}),t-t_s),\ p=0,1,\cdots,s-1\} \tag{23}$$

Corollary 1. If $\beta_i(y,t)$ has the form $\beta_i(y,t)=M_i y\phi_i(t)$, where $M_i\geqslant 1$, $\phi_i(t)$ is decreasing to zero as $t\to\infty$ and $\phi_i(0)=1$, then there exists $\delta>0$, such that switched system (4) is ISS under any switching sequence (5) satisfying $t_{j+1}-t_j\geqslant\delta$, $j=0,1,\cdots$.

4 A lower bound on dwell time

In this section, we derive a lower bound on dwell time of switching sequence for β of the mostly used form: $\beta(y,t)=Mye^{-\alpha t}$, so that switched system (4) is ISS for arbitrary switching with dwell time no less than the lower bound. In this section we do not need Assumptions 2 and 3.

Theorem 2. Suppose each subsystem of (4) is ISS and $\beta_j(y,t)=M_j ye^{-\alpha_j t}$, where $M_j>0$, $\alpha_j>0$. Denote $\delta_*=\max_j\{\alpha_{i_j}^{-1}\lg M_{i_{j+1}}\}$. Let δ be any constant satisfying $\delta>\delta_*$. Then switched system (4) is ISS under arbitrary switching sequence (5) satisfying $t_{j+1}-t_j\geqslant\delta$, $j=1,2,\cdots$.

Proof. From (6) $M_j\geqslant 1$ must hold. Since $\delta_j(y)=\beta_j(y,0)-y=(M_j-1)y$, a straightforward calculation gives

$$\beta^s(y,t_{s+1})=M_{i_0}M_{i_1}\cdots M_{i_s}ye^{-(\alpha_{i_0}t_1+\alpha_{i_1}(t_2-t_1)+\cdots+\alpha_{i_s}(t_{s+1}-t_s))} \tag{24}$$

Therefore,

$$\delta_{i_j}(\beta^{j-1}(y,t_j))\leqslant M_{i_0}M_{i_1}\cdots M_{i_j}ye^{-(\alpha_{i_0}t_1+\alpha_{i_1}(t_2-t_1)+\cdots+\alpha_{i_{j-1}}(t_j-t_{j-1}))}\triangleq a_j(y) \tag{25}$$

Noticing that the definition of δ_* implies $M_{i_{j+1}}\leqslant e^{\alpha_{i_j}\delta_*}$, we have

$$\frac{a_{j+1}(y)}{a_j(y)}=M_{i_{j+1}}e^{-\alpha_{i_j}(t_{j+1}-t_j)}\leqslant M_{i_{j+1}}e^{-\alpha_{i_j}\delta}\leqslant e^{-\alpha_{i_j}(\delta-\delta_*)}\leqslant e^{-\alpha(\delta-\delta_*)}<1 \tag{26}$$

where $\alpha=\min\{\alpha_1,\cdots,\alpha_k\}$. (26) indicates that the series $\sum_{i=1}^{\infty}a_i(y)$ is convergent for $\forall y$, which in turn implies that Condition (a) of Theorem 1 must hold. Since $\delta_j(y)=(M_j-1)y$, Condition (b) of Theorem 1 follows. It is easy to deduce the following explicit expression for (14).

$$a_{m,q}(y)=M_{i_{m+q}}M_{i_{m+q-1}}\cdots M_{i_m}ye^{-(\alpha_{i_m}(t_{m+1}-t_m)+\alpha_{i_{m+1}}(t_{m+2}-t_{m+1})+\cdots+\alpha_{i_{m+q-1}}(t_{m+q}-t_{m+q-1}))} \tag{27}$$

where $a_{m,0}(y)=\beta_{i_m}(y,0)=M_{i_m}y$. Therefore,

$$a_{m,q}(y)\leqslant a_{m,1}(y)=M_{i_m}M_{i_{m+1}}ye^{-\alpha_{i_m}(t_{m+1}-t_m)}\leqslant M_{i_m}M_{i_{m+1}}y,\ q=1,2,\cdots \tag{28}$$

Thus, $a_{m,q}\leqslant y(\max\{M_1,\cdots,M_K\})^2$, which guarantees that Condition (c) of Theorem 1 is satisfied. □

5 Conclusions

We have addressed the problem of achieving ISS of switched systems under the assumption that all subsystems are ISS. Sufficient conditions have been derived by use of ISS properties of each active subsystem. We avoided employing ISS Lyapunov functions of subsystems. This was based on the following consideration. First of all, it is often very difficult to find an ISS Lyapunov function for a given system even if we are sure that the system is indeed ISS. Secondly, we were interested in deriving constructive results. If ISS

Lyapunov functions of subsystems were involved in the construction of ISS conditions for switched systems, the results would fail to be constructive if an ISS Lyapunov function of some active subsystem is unavailable.

A lower bound on the dwell time was given to guarantee ISS property, which provides a big freedom for the design of switching signal.

References

1 Sontag E D. Smooth stabilization implies coprime factorization. *IEEE Transactions on Automatic Control*, 1989, **34**(4):435~443

2 Sontag E D, Wang Y. On characterizations of the input-to-state stability property. *Systems & Control Letters*, 1995, **24**(3): 351~359

3 Freeman R A, Kokotovic P V. Inverse optimality in robust stabilization. *SIAM Journal of Control and Optimization*, 1996, **34**(4): 1365~1391

4 Liberson D. ISS and integral-ISS disturbance attenuation with bounded controls. In: Proceedings of the 38th Conference on Decision & Control, Phoenix, Arizona: Omnipress, 1999. 2501~2506

5 Wang Y A. Converse Lyapunov theorem with applications to ISS-disturbance attenuation. In: Proceedings of the 13th IFAC World Congress, Los Angeles: IFAC, 1996. 79~84

6 Liberzon D, Sontag E D, Wang Y. On integral-input-to-state stabilization. In: Proceedings of American Control Conference, San Diego, California: Omnipress, 1999. 1598~1602

7 Sontag E D, Wang Y. Output-to-state stability and detectability of nonlinear systems. *Systems & Control Letters*, 1997, **29**(3): 279~290

8 Liberzon D, Morse A S. Basic problems in stability and design of switched systems. *IEEE Control Systems Magazine*, 1999, **37**(3): 117~122

9 Branicky M S. Multiple Lyapunov functions and other analysis tools for switched and hybrid systems. *IEEE Transactions on Automatic Control*, 1998, **43**(4): 475~482

10 Angeli D, Mosca E. Lyapunov-based switching supervisory control of nonlinear uncertain systems. In: Proceedings of American Control Conference, Arlington, Virginia: Omnipress, 2001. 967~972

11 Mancilla-Aguilar J L, Garcia R A. On converse Lyapunov theorems for ISS and iISS switched nonlinear systems. *Systems & Control Letters*, 2001, **42**(1): 47~53

12 Zhao J, Spong M W. Hybrid control for global stabilization of the cart-pendulum system. *Automatica*, 2001, **37**(12): 1941~1951

13 Xie G, Zheng D. System stabilizationable condition and stabilization algorithm of linear switched systems based on vector norm theory. *Acta Automatica Sinica*, 2001, **27**(1): 115~119(in Chinese)

14 Fei S, Wang Z, Feng C. On the optimal design via the switching control strategy for a class of switching systems. *Acta Automatica Sinica*, 2001, **27**(2):247~251(in Chinese)

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切换系统输入对状态稳定性的充分条件

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摘 要 研究了切换系统输入对状态的稳定性. 在所有子系统都是输入对状态稳定的条件下，利用各子系统的$\mathcal{KL}$函数和$\mathcal{K}$函数构造出切换系统所需的$\mathcal{KL}$函数和$\mathcal{K}$函数，从而给出了切换系统输入对状态稳定的充分条件. 对于一类常见的切换系统，计算出保证输入对状态稳定性的切换停留时间的下界. 所有结果都是构造性的.

关键词 输入对状态的稳定性，Lyapunov 稳定性，切换系统，停留时间

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