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Harmonic Control Between Two Systems[1)]

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Abstract The concept of harmonic control between two systems is presented. Necessary and sufficient conditions are given for harmonical stabilization of two systems. These results can be generalized to harmonic stability problems for more than two systems. Examples are given to illustrate the results. The results in this paper show that two systems can be easily harmonically stabilized under self-feedback and interconnected feedback. It is possible for the control framework given here to play a useful role in the stabilization of composite systems.

Key words Fixed modes, harmonic stable, interconnected systems

1 Introduction

When control theory is applied to solving problems of electric power systems, socioeconomic systems, etc., large scale systems with many state variables often appear. A basic method for large scale systems is the decentralized control theory. Due to its advantages in computation and implementation of control laws, the decentralized control theory has attracted a great amount of interest[1~11] since 1960s. But it is very hard to obtain necessary and sufficient conditions for decentralized stabilization of large scale systems. Therefore, necessary conditions and sufficient conditions have to be studied separately. For several decades, the study of necessary conditions have always centered on the concept of fixed mode[1~3] which is a natural generalization of the well-known concept of uncontrollable modes and unobservable modes occuring in the usual centralized control problems; the study of sufficient conditions have always centered on special large scale systems[4~11]. First, it was shown by [4] that a system composed of essentially companion-form subsystems which are interconnected through the inputs of subsystems can always be stabilized by local feedback. This result was extended by [5~9]. In addition, a class of large scale systems with symmetrically interconnected subsystems were also studied extensively[10~12]. In order to decentrally stabilize large scale systems, these methods always ignore or reduce the influence of interconnections under the condition that the subsystems are controllable, therefore, the closed-loop systems are always connectively stable[5~7]. Although the idea of harmonic control appeared very early[13,14], there has been little development in this line of research to the authors' knowledge. How to utilize the effects of interconnections to improve system stability is still an open problem.

For sake of studying the effects of interconnections and making use of the idea of harmonic control acting on large scale systems, we study harmonic stability problems among independent subsystems and present necessary and sufficient conditions for it. Harmonic control in this paper means that the closed-loop system stability is attained by constructing harmonic interconnections using the states of subsystems.

Undoubtedly, there are many collaborative phenomena in the real world. For example, two handicapped people, one is a blind and the other is a lame, can cooperate with each other very well. In this paper, we study collaborative problems for the concrete con-

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trol systems. According to the popular cooperative ideas in the real world, we have reasons to believe that two unstable subsystems can make up a stable system after interactions under some necessary conditions.

Throughout this paper, the superscript T means transpose for real matrices. Capital letters denote matrices, small letters denote vectors, and Greek alphabets denote scalars.

2 Main results

Given two independent subsystems $\sum_1: \dot{x}_1 = A_1 x_1 + B_{11} u_{11}$, and $\sum_2: \dot{x}_2 = A_2 x_2 + B_{22} u_{22}$. An interconnected system can be formed through interactions $B_{12} u_{12}$, and $B_{21} u_{21}$:

$$\begin{cases} \dot{x}_1 = A_1 x_1 + B_{11} u_{11} + B_{12} u_{12} \\ \dot{x}_2 = A_2 x_2 + B_{21} u_{21} + B_{22} u_{22} \end{cases} \tag{1}$$

where $u_{11} = K_{11} x_1$, $u_{12} = K_{12} x_2$, $u_{21} = K_{21} x_1$, and $u_{22} = K_{22} x_2$; A_1, A_2, B_{ij}, K_{ij}, $i=1,2$ are real matrices with compatible dimensions. In Fig. 1, there are not only information feedback in subsystems themselves, but also information interchange between two subsystems. It means that two systems are cooperating.

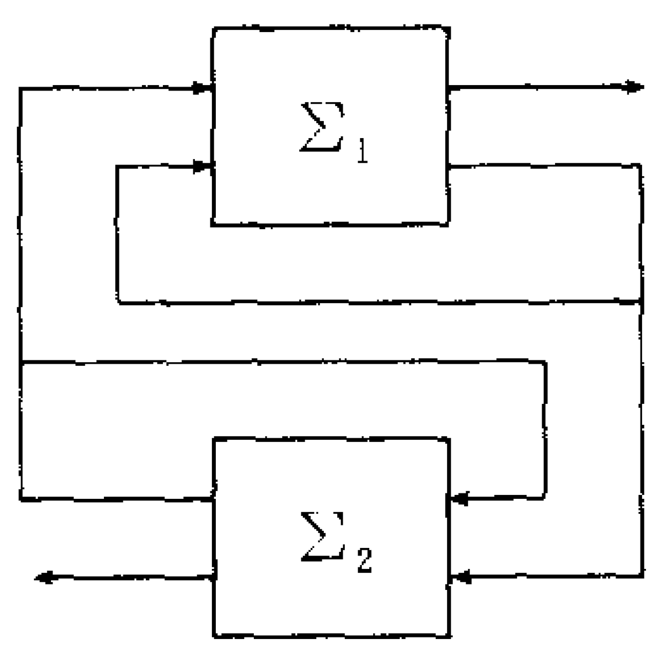

Fig. 1 Interconnected system

Set $\widetilde{A} = \mathrm{diag}(A_1, A_2)$, $\widetilde{B} = \mathrm{diag}(B_{11}, B_{22})$, $B^c = \mathrm{diag}(B_{12}, B_{21})$, $\widetilde{K} = \mathrm{diag}(K_{11}, K_{22})$, $K^c = \begin{pmatrix} 0 & K_{12} \\ K_{21} & 0 \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. System (1) can be written more compactly as follows:

$$\dot{x} = Ax, \; A = \widetilde{A} + \widetilde{B}\widetilde{K} + B^c K^c = \begin{pmatrix} A_1 + B_{11} K_{11} & B_{12} K_{12} \\ B_{21} K_{21} & A_2 + B_{22} K_{22} \end{pmatrix} \tag{2}$$

Remark 1. System (2), from the information flow point of view, can not be reduced to the usual centralized control problem:

$$\dot{x} = \widetilde{A}x + \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} u$$

and it is aslo different from the usual decentralized control problem:

$$\dot{x} = \widetilde{A}x + \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

In this paper, we study harmonic stability problems via interactions $B_{12} K_{12}$ and $B_{21} K_{21}$. The main problem is to find K_{11}, K_{22}, K_{12}, and K_{21} such that System (2) is stable, i. e., A is stable(all eigenvalues of A are in the open left-half complex plane C^-). The following definitions are needed to state the main theorems of this paper.

Definition 1. If there are $\widetilde{K}$ and K^c such that A is stable, we say that subsystems $\sum_1$ and $\sum_2$ can be harmonically stabilized.

In order to discuss whether System (2) can be stabilized or not by this kind of harmonic feedback, we need to introduce the concept of fixed mode.

Definition 2. Complex number set

$$\Lambda(\widetilde{A}, [\widetilde{B}, B^c]) = \bigcap_{\widetilde{K}, K^c} \sigma(\widetilde{A} + \widetilde{B}\widetilde{K} + B^c K^c)$$

is called the set of fixed modes of $(\widetilde{A}, [\widetilde{B}, B^c])$, where $\sigma(.)$ denotes the set of eigenvalues of the corresponding matrix.

Clearly, $\Lambda(\widetilde{A}, [\widetilde{B}, B^c]) \subset \sigma(\widetilde{A})$. If $\Lambda(\widetilde{A}, [\widetilde{B}, B^c]) \neq \varnothing$, it means that this set of eigenvalues can not be changed by the harmonic feedback introduced above.

First, we give a result for fixed modes test.

Theorem 1. $(\widetilde{A}, [\widetilde{B}, B^c])$ has no fixed modes, i. e., $\Lambda(\widetilde{A}, [\widetilde{B}, B^c]) = \bigcap_{\widetilde{K}, K^c} \sigma(\widetilde{A} + \widetilde{B}\widetilde{K} + B^c K^c) = \varnothing$ if and only if the following two conditions hold simultaneously:

i) $(A_1, [B_{11}, B_{12}])$ is controllable, and $\forall \lambda \in \sigma(A_1), \text{rank}[\lambda I_1 - A_1, B_{11}] + \text{rank}(B_{21}) \geqslant n_1$, and

ii) $(A_2, [B_{21}, B_{22}])$ is controllable, and $\forall \lambda \in \sigma(A_2), \text{rank}[\lambda I_2 - A_2, B_{22}] + \text{rank}(B_{12}) \geqslant n_2$, where n_1 and n_2 are the orders of A_1 and A_2, respectively, I_1 and I_2 are unit matrices with the same orders as A_1 and A_2, respectively.

Proof. See Appendix A.

Necessary and sufficient conditions for harmonical stabilizability of two independent subsystems can be given by using the definition of fixed modes. □

Theorem 2. Subsystems Σ_1 and Σ_2 can be harmonically stabilized via interconnected state feedback(B_{11} and B_{22} are not equal to zero matrices simultaneously), i. e., there are $\widetilde{K}$ and K^c such that $A = \widetilde{A} + \widetilde{B}\widetilde{K} + B^c K^c$ is stable if and only if $\Lambda(\widetilde{A}, [\widetilde{B}, B^c]) \subset C^-$.

Proof. See Appendix A. □

Remark 2. In Theorems 1 and 2, we do not require that (A_1, B_{11}) and (A_2, B_{22}) be controllable or stabilizable. This shows that the subsystems may be unstable or unstabilizable, and that for sake of the stability of the interconnected systems, two subsystems must be cooperative.

In the following, we give an algorithm step by step for constructing $\widetilde{K}$ and K^c such that $A = \widetilde{A} + \widetilde{B}\widetilde{K} + B^c K^c$ is stable. First, we study the case in which there is no self-feedback in subsystems.

2.1 The case without self-feedback

In the case that there is self-feedback in subsystems, suppose we have designed their self-feedback such that the closed-loop of the subsystems possess the characters that they should have. In the following, suppose $B_{11} = 0$ and $B_{22} = 0$, that is, $\widetilde{B}$ is a zero matrix. Then we study the problems: Under what conditions is there K^c such that $A = \widetilde{A} + B^c K^c$ is stable? How to construct K^c?

2.1.1 The case of single input

Suppose $\boldsymbol{b}_{12} \in R^{n_1}$ and $\boldsymbol{b}_{21} \in R^{n_2}$, n_1, n_2 are the orders of A_1, A_2, respectively.

Theorem 3. $(\widetilde{A}, B^c)$ has no fixed modes ($B^c = \text{diag}(\boldsymbol{b}_{12}, \boldsymbol{b}_{21})$), i. e., $\Lambda(\widetilde{A}, B^c) = \bigcap_{K^c} \sigma(\widetilde{A} + B^c K^c) = \varnothing$, if and only if $(A_1, \boldsymbol{b}_{12})$ and $(A_2, \boldsymbol{b}_{21})$ are controllable.

Proof. By the knowledge of controlability, for the column vector $\boldsymbol{b}_{12}$, the controllability of $(A_1, \boldsymbol{b}_{12})$ implies that every eigenvalue of A_1 corresponds to only one Jordan block. It is similar for A_2 and $\boldsymbol{b}_{21}$. Then the proposition follows by using Theorem 1. □

The following lemma is needed in order to prove the main theorems of this paper.

Lemma 1. Given n numbers $\alpha_1, \cdots, \alpha_n$, where $\alpha_1, \cdots, \alpha_{n-1}$ are different from each other, α_n is a real number, and $\boldsymbol{a} = \{\alpha_1, \cdots, \alpha_{n-1}\}$ is a complex conjugate set, i. e. $\boldsymbol{a}$ is composed of real numbers and complex conjugate numbers. Given another complex conjugate set $\Lambda = \{\lambda_1, \cdots, \lambda_n\}$ with $\sum_{i=1}^{n} \lambda_i = \sum_{i=1}^{n} \alpha_i$, then one can choose $\boldsymbol{b}_1 = (\beta_1, \cdots, \beta_{n-1})^{\mathrm{T}}$ and $\boldsymbol{c}_1 = (\theta_1, \cdots, \theta_{n-1})$ such that the set of eigenvalues of $A = \begin{pmatrix} A_1 & \boldsymbol{b}_1 \\ \boldsymbol{c}_1 & \alpha_n \end{pmatrix}$ is Λ, where $A_1 = \text{diag}(\alpha_1, \cdots, \alpha_{n-1})$.

Proof. Set $f(s)=\prod_{i=1}^{n}(s-\lambda_i)$, $g(s)=\prod_{i=1}^{n-1}(s-\alpha_i)$, $g'(\alpha_k)=\prod_{i=1,\, i\neq k}^{n-1}(\alpha_k-\alpha_i)$. Clearly,

$$\det(sI_n - A) = \prod_{i=1}^{n-1}(s-\alpha_i)\left(s-\alpha_n-\sum_{i=1}^{n-1}\frac{\beta_i\theta_i}{s-\alpha_i}\right) \tag{3}$$

Furthermore, the polynomial equation with degree $n-2$

$$f(s)-g(s)(s-\alpha_n)=\sum_{i=1}^{n-1}\frac{f(\alpha_i)g(s)}{g'(\alpha_i)(s-\alpha_i)}$$

holds at $n-1$ distinct points $\alpha_1,\cdots,\alpha_{n-1}$, so the equation follows identically. Therefore,

$$f(s)=g(s)(s-\alpha_n)+\sum_{i=1}^{n-1}\frac{f(\alpha_i)g(s)}{g'(\alpha_i)(s-\alpha_i)} \tag{4}$$

By comparing (3) with (4), we know that the proposition follows if we choose β_i and θ_i satisfying

$$\beta_i\theta_i=-\frac{f(\alpha_i)}{g'(\alpha_i)},\ i=1,\cdots,n-1 \tag{5}$$

□

Remark 3. According to the eigenvalue assignment theory[15], the conclusion above holds clearly. In this lemma, only the method for taking β_i, θ_i is given. One can also refer to [16] for some details. In addition, when α_i is a real number, β_i, θ_i can be chosen as real numbers; when α_i, α_{i+1} are complex conjugate numbers $\alpha+\beta i$, $\alpha-\beta i$, β_i, β_{i+1}, θ_i, θ_{i+1} can be chosen properly such that β_i, β_{i+1}, θ_i, θ_{i+1} are changed into real numbers when $\begin{pmatrix}\alpha+\beta i & 0\\ 0 & \alpha-\beta i\end{pmatrix}$ is changed into the real standard Jordan form $\begin{pmatrix}\alpha & \beta\\ -\beta & \alpha\end{pmatrix}$ under similarity transformation.

Theorem 4. Suppose $\boldsymbol{b}_{12}\in R^{n_1}$, $\boldsymbol{b}_{21}\in R^{n_2}$, $(\widetilde{A}, B^c)$ has no fixed modes, and $\mathrm{tr}(A_1)+\mathrm{tr}(A_2)<0$, where tr(.) denotes the trace of the corresponding matrix. Then there is K^c such that $A=\widetilde{A}+B^cK^c=\begin{pmatrix}A_1 & \boldsymbol{b}_{12}\boldsymbol{k}_{12}\\ \boldsymbol{b}_{21}\boldsymbol{k}_{21} & A_2\end{pmatrix}$ is stable.

Proof. See Appendix A.

Remark 4. From the proof of the theorem, we know that the eigenvalues of A can be assigned properly with the only constraint that $\boldsymbol{a}=\{\alpha_{n_1+1},\cdots,\alpha_{n_1+n_2-1}\}$ is a complex conjugate set, and that the elements of $\boldsymbol{a}$ are distinct(generally, this holds naturally). When either n_1 or n_2 is an odd number, one can get a complex conjugate set $\boldsymbol{a}$ by Equation (A1) naturally; when both n_1 and n_2 are even numbers, the eigenvalues $\lambda_1,\cdots,\lambda_{n_1+n_2}$ of A must be chosen properly in the open left-half complex plane such that Equation (A1) has a real root. This can be completed easily.

2.1.2 Multi-input case

Before presenting the main results of this section, we have some discussion on the controllable canonical form of linear systems[17].

Lemma 2[18]. Given (A,B) controllable, suppose B is of full column rank r. Based on the basic column transformation of B, one can get the second Wonham canonical controllable form as follows:

$$A=\begin{bmatrix}A_{11} & & & \\ A_{21} & A_{22} & & \\ \vdots & \vdots & \ddots & \\ A_{r1} & A_{r2} & \cdots & A_{rr}\end{bmatrix},\ A_{ii}=\begin{bmatrix}0 & 1 & & \\ \vdots & & \ddots & \\ 0 & & & 1\\ * & * & \cdots & *\end{bmatrix},\ i=1,2,\cdots,r$$

$$A_{ij}=\begin{bmatrix}0 & \cdots & 0\\ \vdots & & \vdots\\ 0 & \cdots & 0\\ * & \cdots & *\end{bmatrix},\ i>j,\ B=\begin{bmatrix}B_1\\ B_2\\ \vdots\\ B_r\end{bmatrix},\ B_i=\begin{bmatrix}0 & \cdots & 0 & 0 & 0 & \cdots & 0\\ \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots\\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0\\ 0 & \cdots & 0 & 1 & 0 & \cdots & 0\end{bmatrix}$$

When the cyclic index of A is also r, one can choose the controllable form as above such that order$(A_{11}) \leqslant$ order(A_{ii}), $i=2,\cdots,r$, where order(.) denotes the order of the corresponding matrix.

Lemma 3[18]. For any nonsingular block matrix $P=\begin{pmatrix} P_{st} & P_{ss} \\ P_{tt} & P_{ts} \end{pmatrix}$, where P_{ss} and P_{tt} are square matrices with orders s and t, respectively, suppose $P^{-1}=\begin{pmatrix} Q_{ts} & Q_{tt} \\ Q_{ss} & Q_{st} \end{pmatrix}$. Then P_{ss} is nonsingular if and only if Q_{tt} is nonsingular.

In the following, we will use Lemma 2 and Lemma 3 to transform the multi-input case into the single-input case by assigning some eigenvalues and eigenvectors. Suppose $B_{12} \in R^{n_1 \times m}$, $B_{21} \in R^{n_2 \times p}$, and $\max(\mathrm{rank}(B_{12}), \mathrm{rank}(B_{21})) \geqslant 2$.

Theorem 5. If the following conditions hold,

i) (A_1, B_{12}) is controllable, $\forall \lambda \in \sigma(A_1)$, $\mathrm{rank}(\lambda I_1 - A_1) + \mathrm{rank}(B_{21}) \geqslant n_1$,

ii) (A_2, B_{21}) is controllable, $\forall \lambda \in \sigma(A_2)$, $\mathrm{rank}(\lambda I_2 - A_2) + \mathrm{rank}(B_{12}) \geqslant n_2$,

iii) $\mathrm{tr}(A_1) + \mathrm{tr}(A_2) < 0$,

then there is K^c such that $A = \tilde{A} + B^c K^c$ is stable.

Proof. See appendix A.

2.2 The case with self-feedback

From the above discussion, we know that $\mathrm{tr}(A_1) + \mathrm{tr}(A_2) < 0$ is a critical condition, but in the case that there is self-feedback in subsystems, it is easy to find K_{11} and K_{22} such that $\mathrm{tr}(A_1 + B_{11} K_{11}) + \mathrm{tr}(A_2 + B_{22} K_{22}) < 0$ when the controllable subspaces $\langle A_1 \mid B_{11} \rangle$ and $\langle A_2 \mid B_{22} \rangle$ are not equal to zero spaces simultaneously. Therefore, the trace condition is easy to be satisfied, see Theorem 2 for the main result.

3 Examples

Example 1.

$$A_1 = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 3 & -1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -10 \end{bmatrix}, \quad \boldsymbol{b}_{12} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad \boldsymbol{b}_{21} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Clearly, $(A_1, \boldsymbol{b}_{12})$, and $(A_2, \boldsymbol{b}_{21})$ are controllable, and $\mathrm{tr}(A_1) + \mathrm{tr}(A_2) = -6 < 0$. Take

$$\lambda_1 = -0.75, \lambda_2 = -0.5, \lambda_3 = -0.6, \lambda_4 = -0.4$$
$$\lambda_5 = -1, \lambda_6 = -0.85, \lambda_7 = -0.7, \lambda_1 = -1.2$$

By using Theorem 4, one can get

$\boldsymbol{k}_{12} = [-0.267658664321\ 0.16714623386576\ -0.05145346051291\ 0.88430722664716]$

$\boldsymbol{k}_{21} = [-45.222097600732\ -105.357380908421\ -79.631199687734\ 0.123699687733]$

such that $\mathrm{A} = \begin{pmatrix} \mathrm{A}_1 & \boldsymbol{b}_{12} \boldsymbol{k}_{12} \\ \boldsymbol{b}_{21} \boldsymbol{k}_{21} & A_2 \end{pmatrix}$ is stable, and its eigenvalues are $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8$.

Example 2.

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -12 & 8 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 8 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -9 & -6 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -0.75 & 2 \end{bmatrix}$$

$$B_{12} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}^{\mathrm{T}}, \quad B_{21} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{\mathrm{T}}$$

Clearly, (A_1, B_{12}) and (A_2, B_{21}) are controllable and $\mathrm{tr}(A_1) + \mathrm{tr}(A_2) = -5 < 0$. By using Theorem 5, one can get

$$K_{12}=\begin{pmatrix}1 & 1 & 2.87727709866887 & 1\\ 7.8 & 4.1 & -2.87727709866887 & -1\end{pmatrix},$$

$$K_{21}=\begin{pmatrix}9.2 & -11.8 & -5.9 & -8.2 & 1.1\\ 332.14531117781 & 94.585565285673 & 24.198567777589 & 6.994750006238 & 49.568566203367\end{pmatrix}$$

such that $A=\begin{pmatrix}A_1 & B_{12}K_{12}\\ B_{21}K_{21} & A_2\end{pmatrix}$ is stable.

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Appendix A

Proof of Theorem 1. (Necessity) $(\tilde{A}, [\tilde{B}, B^c])$ has no fixed modes, i.e., $\forall\lambda\in\sigma(\tilde{A})$, $\exists\tilde{K}, K^c$, *s.t.* $\lambda\notin\sigma(A)$. So $\mathrm{rank}([\lambda I_1-(A_1+B_{11}K_{11}), B_{12}K_{12}])=n_1$, where n_1 is the order of A_1. Then $\mathrm{rank}([\lambda I_1-(A_1+B_{11}K_{11}), B_{12}])=n_1$. Therefore, $\langle A_1+B_{11}K_{11}|B_{12}\rangle=R^{n_1}$. By using $\langle A_1+B_{11}K_{11}|B_{12}\rangle\subset\langle A_1|B_{11}\rangle+\langle A_1|B_{12}\rangle$, one can get that $(A_1,[B_{11},B_{12}])$ is controllable, where $\langle A|B\rangle$ is the controllable subspace of (A,B). On the other hand, one can get $\mathrm{rank}[\lambda I_1-A_1, B_{11}]+\mathrm{rank}(B_{21})\geqslant n_1$ from $\mathrm{rank}\begin{pmatrix}\lambda I_1-A_1-B_{11}K_{11}\\ B_{21}K_{21}\end{pmatrix}=n_1$. ii) is similar to i).

(Sufficiency) Take standard controllable decomposition for (A_1, B_{11}) and (A_2, B_{22}). Then it is clear that one just needs to test the proposition for uncontrollable modes of (A_1, B_{11}) and (A_2, B_{22}). Therefore, suppose $B_{11}=0$ and $B_{22}=0$ without losing generality. For any $\lambda\in\sigma(\tilde{A})$, suppose that A_1 and A_2 are the matrices with the standard Jordan form, and $\mathrm{rank}(\lambda I_1-A_1)=n_1-p$, $\mathrm{rank}(\lambda I_2-A_2)=n_2-q$, $p\geqslant q$. By i) and ii), we know that (A_1, B_{12}) and (A_2, B_{21}) are controllable respectively, and $\mathrm{rank}(\lambda I_1 - A_1) + \mathrm{rank}(B_{21}) \geqslant n_1$, $\mathrm{rank}(\lambda I_2 - A_2) + \mathrm{rank}(B_{12}) \geqslant n_2$. Noticing that K^c depends on λ, for this λ, $\mathrm{rank}(B_{12}) \geqslant p$ and $\mathrm{rank}(B_{21}) \geqslant p$ hold according to the rank conditions. So it is easy to construct K^c such that $\lambda \notin \sigma\left(\begin{bmatrix} A_1 & B_{12}K_{12} \\ B_{21}K_{21} & A_2 \end{bmatrix}\right)$. Therefore, with the conditions i) and ii), for any $\lambda \in \sigma(\widetilde{A})$, one can construct $\widetilde{K}$ and K^c such that $\lambda \notin \sigma(A)$. □

Proof of Theorem 4. Suppose that A_1 and A_2 are the matrices with the standard Jordan form.

i) Suppose A_1 is a real diagonal matrix, $A_1 = \mathrm{diag}(\alpha_1, \cdots, \alpha_{n_1})$, and $\boldsymbol{b}_{12} = (\beta_1, \cdots, \beta_{n_1})^{\mathrm{T}}$. In the following, we prove that one can choose $\boldsymbol{k}_{21} = (\zeta_1, \cdots, \zeta_{n_1})$, $\boldsymbol{t}_1 = (\theta_1, \cdots, \theta_{n_2-1})$, $\boldsymbol{t}_2 = (\delta_1, \cdots, \delta_{n_2-1})$, $\alpha_{n_1+1}, \cdots, \alpha_{n_1+n_2}$, where $\alpha_{n_1+n_2}$ is a real number, and $\boldsymbol{a} = \{\alpha_{n_1+1}, \cdots, \alpha_{n_1+n_2-1}\}$ is a complex conjugate set and α_{n_1+i}, $i = 1, \cdots, n_2 - 1$ are distinct from each other, such that $\overline{A} = \begin{pmatrix} \mathrm{diag}(A_1, C) & (\boldsymbol{b}_{12}^{\mathrm{T}}, \boldsymbol{t}_1)^{\mathrm{T}} \\ (\boldsymbol{k}_{21}\,\boldsymbol{t}_2) & \alpha_{n_1+n_2} \end{pmatrix}$ is stable, and $A_2' = \begin{pmatrix} C & \boldsymbol{t}_1^{\mathrm{T}} \\ \boldsymbol{t}_2 & \alpha_{n_1+n_2} \end{pmatrix}$ is similar to A_2, where $C = \mathrm{diag}(\alpha_{n_1+1}, \cdots, \alpha_{n_1+n_2-1})$.

Suppose the eigenvalues of $\overline{A}$ are $\lambda_1, \cdots, \lambda_{n_1+n_2}$, the eigenvalues of A_2 are $\gamma_1, \cdots, \gamma_{n_2}$, and the eigenvalues of A_1 and A_2 do not appear among $\lambda_1, \cdots, \lambda_{n_1+n_2}$. Clearly, $\sum_{i=1}^{n_1+n_2} \lambda_i = \mathrm{tr}(A_1) + \mathrm{tr}(A_2)$, $\sum_{i=1}^{n_2} \alpha_{n_1+i} = \sum_{i=1}^{n_2} \gamma_i = \mathrm{tr}(A_2)$. Set

$$f(s) = \prod_{i=1}^{n_1+n_2} (s - \lambda_i), \quad g(s) = \prod_{i=1}^{n_1+n_2-1} (s - \alpha_i)$$

$$f_1(s) = \prod_{i=1}^{n_2} (s - \gamma_i), \quad g_1(s) = \prod_{i=1}^{n_2-1} (s - \alpha_{n_1+i})$$

Then (See Lemma 1)

$$\theta_i \delta_i = -\frac{f(\alpha_{n_1+i})}{g'(\alpha_{n_1+i})} = -\frac{f_1(\alpha_{n_1+i})}{g_1'(\alpha_{n_1+i})}, \quad i = 1, \cdots, n_2 - 1$$

where $g'(.)$ and $g_1'(.)$ are defined as $g'(.)$ given in Lemma 1. Therefore, α_{n_1+i}, $i = 1, \cdots, n_2 - 1$ should satisfy

$$\prod_{k=1}^{n_1+n_2} (\alpha_{n_1+i} - \lambda_k) = \prod_{h=1}^{n_2} (\alpha_{n_1+i} - \gamma_h) \prod_{j=1}^{n_1} (\alpha_{n_1+i} - \alpha_j)$$

So α_{n_1+i}, $i = 1, \cdots, n_2 - 1$ should be chosen as the roots of polynomial equation

$$\prod_{k=1}^{n_1+n_2} (s - \lambda_k) = \prod_{h=1}^{n_2} (s - \gamma_h) \prod_{j=1}^{n_1} (s - \alpha_j) \tag{A1}$$

In addition, $\lambda_1, \cdots, \lambda_{n_1+n_2}$ can be adjusted in the open left-half complex plane such that $\boldsymbol{a} = \{\alpha_{n_1+1}, \cdots, \alpha_{n_1+n_2-1}\}$ is a complex conjugate set and the elements of $\boldsymbol{a}$ are distinct from each other. When $\boldsymbol{a}$ is chosen properly, $\alpha_{n_1+n_2}$ can be determined by $\sum_{i=1}^{n_2} \alpha_{n_1+i} = \mathrm{tr}(A_2)$. Using the method in Lemma 1, $\boldsymbol{k}_{21}$, θ_i, δ_i, $i = 1, \cdots, n_2 - 1$ can be chosen such that the eigenvalues of $\overline{A}$ are $\lambda_1, \cdots, \lambda_{n_1+n_2}$ and the eigenvalues of A_2' are $\gamma_1, \cdots, \gamma_{n_2}$, where θ_i and δ_i satisfy the requirements of Remark 3. By the distinction of $\alpha_{n_1+1}, \cdots, \alpha_{n_1+n_2-1}$, every eigenvalue of A_2' corresponds to only one Jordan block, so A_2' is similar to A_2. Using similarity transformation, transform A_2' into A_2 (A_2 is with the real standard Jordan form). Then one can get that

$$A' = \begin{pmatrix} A_1 & \boldsymbol{b}_{12}\boldsymbol{k}_{12}' \\ \boldsymbol{b}_{21}'\boldsymbol{k}_{21} & A_2 \end{pmatrix}$$

is stable. When $\boldsymbol{b}_{21}' = \boldsymbol{b}_{21}$, the proof is completed. When $\boldsymbol{b}_{21}' \neq \boldsymbol{b}_{21}$, because the eigenvalues of A_1 and A_2 do not appear among $\lambda_1, \cdots, \lambda_{n_1+n_2}$, $(A_2, \boldsymbol{b}_{21}')$ is controllable and $(A_2, \boldsymbol{k}_{12}')$ is observable. Then taking $\boldsymbol{k}_{12}$ such that $\boldsymbol{k}_{12}(sI_2 - A_2)^{-1}\boldsymbol{b}_{21} = \boldsymbol{k}_{12}'(sI_2 - A_2)^{-1}\boldsymbol{b}_{21}'$, one can get that $\begin{pmatrix} A_1 & \boldsymbol{b}_{12}\boldsymbol{k}_{12} \\ \boldsymbol{b}_{21}\boldsymbol{k}_{21} & A_2 \end{pmatrix}$ is stable, and its eigenvalues are the same as the ones of A'.

ii) When A_1 is not a diagonal matrix, suppose A_1 has only one Jordan block and the other eigenvalues of A_1 are distinct from each other, i. e. ,

$$A_1 = \mathrm{diag}(J_1, \alpha_{r+1}, \cdots, \alpha_{n_1}), \quad \boldsymbol{J}_1 = \begin{bmatrix} \alpha_1 & 1 & & \\ & \alpha_2 & \ddots & \\ & & \ddots & 1 \\ & & & \alpha_r \end{bmatrix}, \quad \alpha_1 = \cdots = \alpha_r$$

and suppose $b_{12}=(0, \cdots, 0, \beta_r, \beta_{r+1}, \cdots, \beta_{n_1})^T$ without losing generality. Construct $A'=\begin{pmatrix} A_0 & c_1 \\ d_1 & \alpha_{n_1+n_2} \end{pmatrix}$ by the method in i), where $A_0=\mathrm{diag}(A_1, \alpha_{n_1+1}, \cdots, \alpha_{n_1+n_2-1})$, $c_1=(b_{12}^T, \theta_1, \cdots, \theta_{n_2-1})^T$, $d_1=(\zeta_1, \cdots, \zeta_r, \zeta_{r+1}, \cdots, \zeta_{n_1}, \delta_1, \cdots, \delta_{n_2-1})$. $\zeta_{r+1}, \cdots, \zeta_{n_1}, \theta_1, \cdots, \theta_{n_2-1}, \delta_1, \cdots, \delta_{n_2-1}, \alpha_{n_1+1}, \cdots, \alpha_{n_1+n_2-1}$ can be chosen by using the method in i), $\zeta_1, \cdots, \zeta_r$ can be chosen according to the polynomial equation $f(s)=\det(sI-A')$. Then taking similarity transformation as in i), one can get that $A=\begin{pmatrix} A_1 & b_{12}k_{12} \\ b_{21}k_{21} & A_2 \end{pmatrix}$ is stable.

When b_{12} is not of the form $(0, \cdots, 0, \beta_r, \beta_{r+1}, \cdots, \beta_{n_1})^T$, by the controllability of (A_1, b_{12}) and the observability of (A_1, k_{21}), one can change b_{12} by adjusting k_{21}.

When A_1 has complex conjugate eigenvalues, one can use the real Jordan form and Equation $f(s)=\det(sI-A')$ to decide k_{12} and k_{21} such that A is stable. □

Proof of Theorem 5. In fact, i) and ii) are necessary and sufficient conditions for(see Theorem 1) $\Lambda(\tilde{A}, B^c)=\varnothing$. In the following, we mainly present an algorithm for constructing K^c. The cyclic index of A_1 and A_2 will play an important role in the algorithm, therefore, suppose cyclic$(A_1)\leqslant$cyclic$(A_2)=r$, the column number and the column rank of B_{12} and B_{21} are also r. Next we mainly discuss the case $r=2$. By using Lemma 2, (A_1, B_{12}) and (A_2, B_{21}) can be changed into the second Wonham canonical form as follows:

$$A_1=\begin{pmatrix} A_{11}^1 & 0 \\ A_{21}^1 & A_{22}^1 \end{pmatrix}, \quad A_2=\begin{pmatrix} A_{11}^2 & 0 \\ A_{21}^2 & A_{22}^2 \end{pmatrix}$$

$$A_{ii}^j=\begin{bmatrix} 0 & 1 & & \\ \vdots & & \ddots & \\ 0 & & & 1 \\ * & * & \cdots & * \end{bmatrix}, \quad A_{21}^j=\begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \\ * & * & \cdots & * \end{bmatrix}, \quad i,j=1,2$$

B_{12} and B_{21} are the matrices of the form $\begin{pmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix}^T$, where $*$ denotes the element needn't to be written out.

Let the orders of $A_{11}^1, A_{22}^1, A_{11}^2, A_{22}^2$ be v_1, v_2, u_1, u_2, respectively, and $u_1\leqslant u_2$. When $A_{11}^1=A_{22}^1$, $A_{11}^2=A_{22}^2$ and $A_{21}^1=0$, $A_{21}^2=0$, K^c can be constructed by using the method for the case with single input. In the following, we construct K_{12} and K_{21} for the case $u_1<\min\{v_1, v_2\}$. Set

$$K_{12}=\begin{pmatrix} \eta_{11} & \cdots & \eta_{1u_1} & \cdots & \eta_{1u_1+u_2} \\ \eta_{21} & \cdots & \eta_{2u_1} & \cdots & \eta_{2u_1+u_2} \end{pmatrix}, \quad K_{21}=\begin{pmatrix} \zeta_{11} & \cdots & \zeta_{1v_1} & \cdots & \zeta_{1v_1+v_2} \\ \zeta_{21} & \cdots & \zeta_{2v_1} & \cdots & \zeta_{2v_1+v_2} \end{pmatrix}$$

The procedure of designing K_{12} and K_{21} is changed into two parts.

1) Set $B_{21}=\begin{pmatrix} b^{21} & 0 \\ 0 & b_{21}^2 \end{pmatrix}$. First, choose a complex conjugate set $\Lambda_1=\{\lambda_1, \cdots, \lambda_{u_1}\}$ in the open left-half complex plane such that $tr(A_1)+tr(A_2)-(\lambda_1+\cdots+\lambda_{u_1})<0$, the elements of Λ_1 are distinct from each other, and there is no intersection between Λ_1 and the set of eigenvalues of A_1 and A_2. Then take $k_1=(\zeta_{11}, \cdots, \zeta_{1v_1+v_2})$, $H_1=\begin{pmatrix} \eta_{11} & \cdots & \eta_{1u_1} \\ \eta_{21} & \cdots & \eta_{2u_1} \end{pmatrix}$ such that $B=\begin{pmatrix} A_1 & B_{12}H_1 \\ b_{21}^1 k_1 & A_{11}^2 \end{pmatrix}$ satisfies

a) The set of eigenvalues of B includes Λ_1. And the matrix composed of the row eigenvectors corresponding to $\lambda_1, \cdots, \lambda_{u_1}$ is $P=(P_1\ P_2\ P_3)$, where P_1, P_2, P_3 are $u_1\times v_1$, $u_1\times v_2$, $u_1\times u_1$ matrices, respectively, and all elements of the last column of P_1, P_2, P_3 are equal to 1.

b) The v_1-th, (v_1+v_2)-th elements of the row eigenvectors corresponding to the other eigenvalues of B are not equal.

c) Every eigenvalue of B corresponds to only one Jordan block.

In fact, the first step is to choose H_1 and k_1 such that $PB=\mathrm{diag}(\lambda_1, \cdots, \lambda_{u_1})P$. By the assumptions above, this step can be completed easily by solving linear equations.

2) When 1) is completed, suppose $S^{-1}BS=\mathrm{diag}(J_0, J)$, where J is a matrix of standard Jordan form, and $J_0=\mathrm{diag}(\lambda_1, \cdots, \lambda_{u_1})$. And P is the matrix composed of the first u_1 rows of S^{-1}. Set S_1 be the matrix composed of the first v_1+v_2 rows of S, F the matrix composed of the v_1-th, v_1+v_2-th columns of S^{-1}. Then take

$$H_2=\begin{pmatrix} \eta_{1u_1+1} & \cdots & \eta_{1u_1+u_2} \\ \eta_{2u_1+1} & \cdots & \eta_{2u_1+u_2} \end{pmatrix}=\begin{pmatrix} 1 \\ -1 \end{pmatrix}(\xi_1, \cdots, \xi_{u_2}), \quad k_2=(\zeta_{21}, \cdots, \zeta_{2v_1+v_2})$$

The original problem is changed into choosing H_2 and k_2 such that

$$A'=\begin{pmatrix} \mathrm{diag}(J_0, J) & FH_2 \\ b_{21}^2 k_2 S_1+b_{21}^2 c & A_{22}^2 \end{pmatrix}$$

is stable, where $\boldsymbol{c}$ is a row vector which is related to A_{21}^2 and S. The elements of $\boldsymbol{c}$ need not to be written out. According to the method of taking H_2 and the characters of P, we know that $\lambda_1, \cdots, \lambda_{u_1}$ belong to $\sigma(A')$. Therefore, the stability of A' is equivalent to the stability of $A''=\begin{pmatrix} J & F_1 H_2 \\ \boldsymbol{b}_{21}^2 \boldsymbol{k}_2 S_1' + \boldsymbol{b}_{21}^2 \boldsymbol{c}' & A_{22}^2 \end{pmatrix}$, where $(\boldsymbol{f}_1\ \boldsymbol{f}_2)=F_1$ is composed of the last v_1+v_2 rows of F, S_1' is the matrix composed of the last v_1+v_2 columns of S_1, $\boldsymbol{c}'$ is the row vector composed of the last v_1+v_2 elements of $\boldsymbol{c}$. By b) of 1), $(J, \boldsymbol{f}_1-\boldsymbol{f}_2)$ is controllble. In addition, by using Lemma 3, S_1' is nonsingular, because P_3 is clearly nonsingular. Therefore, $\boldsymbol{k}_2 S_1'+\boldsymbol{c}'$ can be chosen deliberately. Choosing H_2, $\boldsymbol{k}_2$ such that A'' is stable is a problem discussed above with single input.

Please refer to [18] for the detailed discussions for the case $u_1 \not< \min\{v_1, v_2\}$ and the case $r \geqslant 3$. □

Proof of Theorem 2. (Necessity) Clearly.

(Sufficiency) The fixed modes can be separated easily from $\widetilde{A}$ by taking similarity transformation, therefore, suppose $\Lambda(\widetilde{A}, [\widetilde{B}, B^c]) = \varnothing$ without losing generality. Then one can choose K_{11} and K_{22} such that

i) $(A_1+B_{11}K_{11}, B_{12})$ is controllable, and $\forall \lambda \in \sigma(A_1+B_{11}K_{11})$, $\mathrm{rank}[\lambda I_1 - A_1 - B_{11}K_{11}] + \mathrm{rank}(B_{21}) \geqslant n_1$.

ii) $(A_2+B_{22}K_{22}, B_{21})$ is controllable, and $\forall \lambda \in \sigma(A_2+B_{22}K_{22})$, $\mathrm{rank}[\lambda I_2 - A_2 - B_{22}K_{22}] + \mathrm{rank}(B_{12}) \geqslant n_2$.

iii) $\mathrm{tr}(A_1+B_{11}K_{11}) + \mathrm{tr}(A_2+B_{22}K_{22}) < 0$.

Then one can get the conclusion by using Theorem 5. □

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两个系统间的协调控制问题

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摘　要　提出了两个系统协调控制的概念,主要研究了不同系统间的互助合作问题,给出两个系统间可协调稳定的充要条件及构造协调控制律的算法,并举例说明了所给结果的有效性,这些结果都可以向多系统协调稳定问题推广. 本文结果表明,在自反馈与关联反馈下两个系统之间很容易实现协调稳定. 本文所给出的控制机制有可能在由子系统集成组合系统的过程中起到作用.

关键词　固定模,协调稳定,关联反馈

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