Contributions to the Credibility Theory

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Abstract. The paper presents the mathematical theory of some credibility models, involving complicated properties of conditional expectations and of conditional covariances. The fact that it is based on complicated mathematics will give more insight and understanding of the theoretical aspects and will point the way to the practical possibilities of the credibility models.

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1 The Bühlmann's model

In this section we first give Bühlmann's original model, which involves only one isolated contract. We derive the optimal linearized credibility estimate for the risk premium for this case. It turns out that this procedure does not provide us with a statistic computable from the observations, since the result involves unknown parameters of the structure function. To obtain estimates for these structure parameters, for Bühlmann's classical model we embed the contract in a collective of contracts, all providing independent information on the structure distribution (see Section 2). In the original credibility model of Bühlmann, we consider one contract with unknown and fixed risk parameter θ, during a period of t years. The yearly claim amounts are denoted by $X_1, \ldots, X_t$. The risk parameter θ is supposed to be drawn from some structure distribution $U(\cdot)$. It is assumed that, for given $\theta = \theta$, the claims are conditionally independent and identically distributed with known common distribution function $F_{X|\theta}(x, \theta)$. For this model we want to estimate the net premium $\mu(\theta) = E[X_r|\theta = \theta]$, $r = \overline{1,t}$ as well as X_{t+1} for a contract with risk parameter θ.

Theorem 1.1 (Bühlmann's optimal credibility estimator). *Suppose $X_1, \ldots, X_t$ are random variables with finite variance, which are, for given $\theta = \theta$, conditionally independent and identically distributed with known common distribution function $F_{X|\theta}(x, \theta)$. The structure distribution function is $U(\theta) = P[\theta \leq \theta]$. Let D denote the set of non-homogeneous linear combinations $g(\cdot)$ of the observable random variables $X_1, X_2, \ldots, X_t$:*

$$g(\underline{X}') = c_0 + c_1 X_1 + c_2 X_2 + \ldots + c_t X_t \tag{1.1}$$

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Then the solution of the problem:

$$\underset{g\in D}{Min}\, E\{[\mu(\theta) - g(X_1, \ldots, X_t)]^2\} \tag{1.2}$$

is:

$$g(X_1, \ldots, X_t) = z\overline{X} + (1-z)m \tag{1.3}$$

where $\overline{X}' = (X_1, \ldots, X_t)$ *is the vector of observations,* $z = at/(s^2+at)$, *is the resulting credibility factor,* $\overline{X} = \frac{1}{t}\sum_{i=1}^{t} X_i$ *is the individual estimator, and* a, s^2 *and* m *are the structural parameters as defined in (1.4)*:

$$\left.\begin{aligned} m &= E[X_r] = E[\mu(\theta)], r = \overline{1,t} \\ a &= \mathrm{Var}\{E[X_r|\theta]\} = \mathrm{Var}[\mu(\theta)], r = \overline{1,t}, \\ \sigma^2(\theta) &= \mathrm{Var}[X_r|\theta = \theta], r = \overline{1,t}, \\ s^2 &= E\{\mathrm{Var}[X_r|\theta]\} = E[\sigma^2(\theta)], r = \overline{1,t}. \end{aligned}\right\} \tag{1.4}$$

Remark 1.1. If $\mu(\theta)$ is replaced by X_{t+1} in (1.2), exactly the same solution (1.3) is obtained, since the covariances with $\underline{X}$ are the same.

Applications of Theorem 1.1. 1) Suppose the claims are integer-valued and *Poisson* (θ) distributed, so

$$dF_{X|\theta}(x, \theta) = \theta^x e^{-\theta}/x!, \quad x = 0, 1, \ldots \tag{1.5}$$

and suppose the structure distribution of θ to be a *Gamma* distribution

$$u(\theta) = \theta^{\beta-1} e^{-\alpha\theta} \alpha^\beta / \Gamma(\beta), \quad \theta > 0. \tag{1.6}$$

In this case the best linearized credibility estimator for $\mu(\theta)$ can be written as follows

$$z\bar{x} + (1-z)m = (v+\beta)/(t+\alpha). \tag{1.7}$$

Since in this case $m = E[X] = E\{E[X|\theta]\} = E[\theta] = \beta/\alpha$, and for the ratio of the structure parameters a and s^2 we have:

$$s^2/a = E\{Var[X|\theta]\}/Var\{E[X|\theta]\} = E[\theta]/Var[\theta] = (\beta/\alpha)/(\beta/\alpha^2) = \alpha,$$

we find $z = at/(s^2+at) = t/(t+\alpha)$, so the best linearized credibility estimator for $\mu(\theta)$ can be written in the form (1.7), where $v = \sum_{i=1}^{t} x_i$.

2) Suppose the claims are a *Negative Binomial* (θ) distribution, so:

$$dF_{X|\theta}(x, \theta) = \theta^x (1-\theta)^{1-x}, \quad x \in \{0, 1\} \tag{1.8}$$

and suppose the structure distribution of θ to be a *Beta* distribution:

$$u(\theta) = \theta^{\alpha-1}(1-\theta)^{\beta-1}/\beta(\alpha,\beta), \quad \theta \in (0,1). \tag{1.9}$$

In this case the best linearized credibility estimator for $\mu(\theta)$ can be written as follows:

$$z\overline{x} + (1-z)m = [t/(t+\alpha+\beta)\overline{x}] + [\alpha/(t+\alpha+\beta)]. \tag{1.10}$$

Since in this case $m = E[X] = E\{E[X|\theta]\} = E[\theta] = \alpha/(\alpha+\beta)$, and for the ratio of the structure parameters a and s^2 we have:

$$s^2/a = E\{Var[X|\theta]\}/Var\{E[X|\theta]\} = E[\theta(1-\theta]/Var[\theta] = [E(\theta) - E(\theta^2)]/Var(\theta) =$$
$$= \{[\alpha/(\alpha+\beta)] - [\alpha(\alpha+1)/(\alpha+\beta+1)]\}/\{[(\alpha\beta)/(\alpha+\beta)^2(\alpha+\beta+1)]\} =$$
$$= [\alpha\beta/(\alpha+\beta+1)]/[\alpha\beta/(\alpha+\beta)^2(\alpha+\beta+1)] = \alpha+\beta,$$

we find $z = at/(s^2+at) = at/\{a[(s^2/a)+t]\} = t/(t+\alpha+\beta)$, so the best linearized credibility estimator for $\mu(\theta)$ can be written in the form (1.10).

3) Suppose the claims are a *Exponential* (θ) distribution, so:

$$dF_{X|\theta}(x,\theta) = \theta e^{-\theta x}, \quad x > 0 \tag{1.11}$$

and suppose the structure distribution of θ to be a Gamma distribution:

$$u(\theta) = \theta^{\beta-1}e^{-\alpha\beta}\alpha^{\beta}/\Gamma(\beta), \quad \theta > 0. \tag{1.12}$$

In this case the best linearized credibility estimator for $\mu(\theta)$ can be written as follows:

$$z\overline{x} + (1-z)m = (v+\alpha)/(t+\beta-1), \quad \text{if} \quad \beta > 2. \tag{1.13}$$

Since in this case $m = E[X] = E\{E[X|\theta]\} = E[1/\theta] = \alpha/(\beta-1)$, if $\beta > 1$, and for the ratio of the structure parameters a and s^2 we have:

$$s^2/a = E\{Var[X|\theta]\}/Var\{E[X|\theta]\} = E[1/\theta^2]/Var(1/\theta) = \beta - 1,$$

if $\beta > 2$, we find

$$z = at/(s^2+at) = at/\{a[(s^2/a)+t]\} = [t/(t+\beta-1)][v/t] = v/[t+\beta-1],$$

if $\beta > 2$, so the best linearized credibility estimator for $\mu(\theta)$ can be written in the form (1.13), where $v = \sum_{i=1}^{t} x_i$.

4) Suppose the claims are a *Normal* (θ, σ^2) distribution, so:

$$dF_{X|\theta}(x,\theta) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\theta}{\sigma}\right)^2}, \quad x \in \mathbf{R} \tag{1.14}$$

and suppose the structure distribution of θ to be a *Normal* (μ_0, σ_0^2) distribution:

$$u(\theta) = \frac{1}{\sigma_0\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu_0}{\sigma_0}\right)^2}, \quad \theta \in \mathbf{R}. \tag{1.15}$$

In this case the best linearized credibility estimator for $\mu(\theta)$ can be written as follows:

$$z\overline{x}+(1-z)m=\left[\frac{\nu}{\sigma^2}+\frac{\mu_0}{\sigma_0^2}\right]/\left[\frac{t}{\sigma^2}+\frac{1}{\sigma_0^2}\right]. \tag{1.16}$$

Since in this case $m=E[X]=E\{E[X|\theta]\}=E(\theta)=\mu_0$ and for the ratio of the structure parameters a and s^2 we have:

$$s^2/a=E\{Var[X|\theta]\}/Var\{E[X|\theta]\}=E(\sigma^2)/Var(\theta)=\sigma^2/\sigma_0^2,$$

we find $z=at/(s^2+at)=at/\{a[(s^2/a)+t]\}=t/[(\sigma^2/\sigma_0^2)+t]$, so the best linearized credibility estimator for $\mu(\theta)$ can be written in the form (1.16), where $\nu=\sum_{i=1}^{t}x_i$.

5)

Theorem 1.2 (Credibility estimator minimizes mean squared error for exponential family with natural parametrization and prior): - consider the exponential family of distributions with natural parametrization:

$$f_{X|\theta}(x,\theta)=p(x)e^{-\theta x}/q(\theta),\ x>0,\ \theta>0 \tag{1.17}$$

together with the natural conjugate priors with density

$$u(\theta)=q(\theta)^{-t_0}e^{-\theta x_0}/c(t_0,x_0),\quad \theta>0 \tag{1.18}$$

where $p(x)$ is an arbitrary non - negative function, t_0 and x_0 are positive constants, and $c(t_0,x_0)$ is a normalization constant. For this case, the linearized credibility estimator is

$$z\overline{x}+(1-z)m=\left(x_0+\sum_{i=1}^{t}x_i\right)/(t_0+t), \tag{1.19}$$

where $m=E[\mu(\theta)]=x_0/t_0$, $s^2/a=t_0$, $z=t/(t+t_0)$. Indeed: - by Theorem 1.1 we only have to prove that the optimal estimator

$$E[\mu(\theta)|\underline{X}]=\left[\int\mu(\theta)\prod_{i=1}^{t}f_{X|\theta}(x_i,\theta)dU(\theta)\right]/\left[\int\prod_{i=1}^{t}f_{X|\theta}(x_i,\theta)dU(\theta)\right]. \tag{1.20}$$

First we express $E[\mu(\theta)]$ in the prior parameters x_0 and t_0, then the theorem follows because of the special form of the posterior distribution. Because $q(\theta)$ is the normalizing constant of the distribution (1.17) one has

$$q(\theta)=\int_0^{+\infty}p(x)e^{-\theta x}dx. \tag{1.21}$$

So

$$q'(\theta)=-\int_0^{+\infty}xp(x)e^{-\theta x}dx=-q(\theta)E[X|\theta=\theta], \tag{1.22}$$

since

$$E[X|\theta=\theta] = \int_0^{+\infty} x f_{X|\theta}(x,\theta)dx = \left[\int_0^{+\infty} xp(x)e^{-\theta x}dx\right]/q(\theta).$$

Therefore the risk premium when $\theta = \theta$ equals:

$$\mu(\theta) = E[X|\theta=\theta] = -q'(\theta)/q(\theta). \tag{1.23}$$

Taking the first derivative of (1.18) with respect to θ gives, using (1.23):

$$u'(\theta) = [-t_0 q(\theta)^{-t_0-1} q'(\theta)e^{-\theta x_0}]/c(t_0,x_0) + [q(\theta)^{-t_0}e^{-\theta x_0}(-x_0)]/c(t_0,x_0) =$$
$$= t_0[-q'(\theta)/q(\theta)][q(\theta)^{-t_0}e^{-\theta x_0}/c(t_0,x)0)] - x_0[q(\theta)^{-t_0}e^{-\theta x_0}/c(t_0,x_0)] =$$
$$= t_0 u(\theta) - x_0 u(\theta) = [t_0\mu(\theta) - x_0]u(\theta).$$

So

$$u'(\theta) = [t_0\mu(\theta) - x_0]u(\theta). \tag{1.24}$$

Integrating this derivative over θ gives zero for the left hand side, since

$$\int_0^{+\infty} u'(\theta)d\theta = u(+\infty) - u(0) = 0. \tag{1.25}$$

So the right hand side of (1.24) results in:

$$m = E[\mu(\theta)] = \int_0^{+\infty} \mu(\theta)u(\theta)d\theta = x_0/t_0, \tag{1.26}$$

since: (1.24) $\wedge$ (1.25) $\Rightarrow$

$$\int_0^{+\infty} [t_0\mu(\theta) - x_0]u(\theta)d\theta = 0 \Leftrightarrow t_0 E[\mu(\theta)] - x_0 \int_0^{+\infty} u(\theta)d\theta = 0 \Leftrightarrow$$
$$\Leftrightarrow t_0 E[\mu(\theta)] - x_0 \cdot 1 = 0 \Leftrightarrow E[\mu(\theta)] = x_0/t_0.$$

The conditional density of θ, given $\underline{X} = \underline{x}$ (posterior density) is, apart from a normalizing function of $x_1, \ldots, x_t$:

$$f_{\theta|\underline{X}}(\theta,\underline{x}) = f_{\underline{X}|\theta}(\underline{x},\theta)f_\theta(\theta)/f_{\underline{X}}(\underline{x}) ::$$
$$u(\theta)\prod_{i=1}^{t}\{p(x_i)e^{-\theta x_i}/q(\theta)\} :: q(\theta)^{-(t_0+t)}e^{-\left(x_0+\sum_i x_i\right)}. \tag{1.27}$$

Density (1.27) is of the same type as the original structure density (1.18), with x_0 replaced by $\left(x_0 + \sum_i x_i\right)$ and t_0 by (t_0+t). So by (1.26) the posterior mean (1.20), which is the mean squared error - optimal estimator for $\mu(\theta)$, is:

$$E[\mu(\theta)|X_1,\ldots,X_t] = \left(x_0 + \sum_i x_i\right)/(t_0+t). \tag{1.28}$$

This is indeed a non - homogeneous linear combination of $X_1, \ldots, X_t$. By (1.26) we have $m = x_0/t_0$, and comparing (1.28) and (1.3) we see that $t_0 = s^2/a$ and $z = t/(t + t_0)$.

Remark 1.2. The parametrization is called natural because the exponent part is a linear function of θ, and by taking a natural conjugate prior the posterior distribution is of the same type as the prior distribution. We restrict ourselves to $x > 0$ and $\theta > 0$, and suppose furthermore that at the end point of the intervals the densities are zero. These restrictions are not strictly necessary.

Remark 1.3. It should be noted that the solution (1.3) of the linearized credibility problem only yields a statistic computable from the observations, if the structure parameters m, s^2 and a are known. Generally, however, the structure function $U(\cdot)$ is not known. Then the "estimator" as it stands is not a statistic. Its interest is merely theoretical, but it will be the basis for further results on credibility. In the following section we consider different contracts, each with the same structure parameters a, m and s^2, so we can estimate these quantities using the statistics of the different contracts.

2 Further developments

For this estimation, assume that we have a portfolio of k identical and independent policies that have been observed for $t(\geq 2)$ years, and let X_{jr} denote the total claim amount of policy j in year r. Let:

$$M_j = \overline{X}_j = \frac{1}{t}\sum_{s=1}^{t} X_{js},\ j = \overline{1,k},\ M_0 = \overline{\overline{X}}_{..} = \frac{1}{k}\sum_{j=1}^{k} M_j. \tag{2.1}$$

For m we propose the unbiased estimator:

$$\hat{m} = M_0 = \overline{\overline{X}}_{..} \tag{2.2}$$

For each policy j, the empirical variance:

$$\frac{1}{t-1}\sum_{r=1}^{t}(X_{jr} - M_j)^2 \tag{2.3}$$

is an unbiased estimator of $Var(X_{jr}|\theta_j)$, and thus:

$$\hat{s}^2 = \frac{1}{k(t-1)}\sum_{j=1}^{k}\sum_{r=1}^{t}(X_{jr} - M_j)^2 \tag{2.4}$$

is an unbiased estimator of s^2. The empirical variance:

$$\frac{1}{k-1}\sum_{j=1}^{k}(M_j - M_0)^2 \tag{2.5}$$

is an unbiased estimator of $Var(M_j)$, and as:

$$Var(M_j) = \frac{\hat{s}^2}{t} + a, \tag{2.6}$$

we introduce the unbiased estimator:

$$\hat{a} = \frac{1}{k-1} \sum_{j=1}^{k} (M_j - M_0)^2 - \frac{\hat{s}^2}{t} \tag{2.7}$$

for a. This estimator has the weakness that it may take negative values whereas a is non - negative. Therefore, we replace a by the estimator:

$$a^* = \max(0, \hat{a}), \tag{2.8}$$

thus losing unbiasedness, but gaining admissibility. Note that $\hat{m}, \hat{s}^2$ and a^* are consistent as $k \to +\infty$.

3 The credibility estimator

Let us look a bit at the credibility estimator (1.3). This estimator has been criticized because it gives the claim amounts from all previous years the same weight; intuitively one should believe that new claims should have more weight than old claims. However, as the claim amounts of different years were assumed to be exchangeable, it was only reasonable that the claim amounts should have equal weights. The following model (which is called "*Recursive credibility estimation*") is *an attempt to amend for this intuitive weakness*, and thus *an application* of *the model* from *Section 1.* We assume that $X_1, X_2, \ldots$ are conditionally independent given an unknown random sequence $\theta = \{\theta_i\}_{i=1}^{+\infty}$, and that for all i X_i depends on θ only through θ_i. This means that for each year i there is a separate risk parameter θ_i containing the risk characteristics of the policy in that year. The model of Section 1 appears as a special case by assuming $\theta_i = \theta_1$ for all i. We assume that:

$$E(X_i|\theta_i) = \mu(\theta_i)$$

with the function μ independent of i. Assumption (3.1):

$$Cov[\mu(\theta_i), \mu(\theta_j)] = \rho^{|i-j|}\lambda \tag{3.1}$$

with $0 < \rho < 1$ and $\lambda > 0$ (λ greater than zero), means that the correlation between claim amounts from different years decreases when the time distance between the years increases, which is intuitively appealing. Furthermore that:

$$\mu = E[\mu(\theta_i)]$$

$$\varphi = E[Var(X_i|\theta_i)]$$

$$\lambda = Var[\mu(\theta_i)]$$

for all i.

Our motivation for introducing the present model was that *we wanted new claims to have more weight than older claims.* The following theorem shows that this wish has been satisfied.

Theorem 3.1. *Let the coefficients $\alpha_{t0}, \alpha_{t1}, \dots, \alpha_{tt}$ be defined by*

$$\hat{\mu}(\theta_{t+1}) = \alpha_{t0} + \sum_{j=1}^{t} \alpha_{tj} X_j$$

and assume that $\rho < 1$. Then

$$0 < \alpha_{t1} < \alpha_{t2} < \dots < \alpha_{tt} < 1. \tag{3.2}$$

4 Risk volumes

In the simple model of Section 1 we assumed that the risk volume was the same for all years. Often, in particular in reinsurance, one wants to allow for varying risk volumes, and for that purpose we will introduce *the credibility model incorporating risk volumes.* We consider a ceded insurance portfolio. Let S_j denote the total claim amount of year j and P_j some measure of the risk volume in year j. By the loss ratio of year j we shall mean $X_j = S_j/P_j$. We assume that $X_1, X_2, \dots$ are conditionally independent given an unknown random risk parameter θ, that:

$$E(X_j|\theta) = \mu(\theta)$$

independent of j, and that

$$Var(X_j|\theta) = \frac{s^2(\theta)}{P_j}, \quad j = \overline{1,t}. \tag{4.1}$$

We introduce the structural parameters:

$$\mu = E[\mu(\theta)], \quad \varphi = E[s^2(\theta)], \quad \lambda = Var[\mu(\theta)].$$

The assumption (4.1) is perhaps most reasonable if P_j is the number of risks in the portfolio in year j. If we assume that the claim amounts $Y_{j1}, \dots, P_{jP_j}$ of the P_j risks in year j are conditionally independent and identically distributed given θ, then:

$$Var(X_j|\theta) = Var\left(\frac{1}{P_j}\sum_{k=1}^{P_j} Y_{jk}|\theta\right) = \frac{Var(Y_{j1}|\theta)}{P_j},$$

and (4.1) boils down to the assumption that

$$Var(Y_{j1}|\theta) = s^2(\theta)$$

independent of j. We have the following result.

Theorem 4.1. *The credibility estimator $\hat{\mu}(\theta)$ of $\mu(\theta)$ based on $\underline{X}' = (X_1, X_2, \dots, X_t)$ is given by*

$$\hat{\mu}(\theta) = \frac{P}{P+K}\overline{X}_t + \frac{K}{P+K}\mu \tag{4.2}$$

with $P = \sum_{j=1}^{t} P_j$, $\overline{X}_t = \frac{1}{P}\sum_{j=1}^{t} P_j X_j$, $K = \frac{\varphi}{\lambda}$.

5 Variation of $E[Var(X_i|\theta)]$

In Section 4 we allowed $E[Var(X_i|\theta)]$ to vary. In the present section we are going to allow $E(X_j)$ to vary. Let $\underline{X} = (X_1, \ldots, X_t)'$ be an observed random $(t \times 1)$ vector and θ an unknown random risk parameter. Instead of assuming time independence in the net risk premium:

$$\mu(\theta) = E(X_j|\theta), \quad j = \overline{1,t} \tag{5.1}$$

one could assume that the conditional expectation of the claims on a contract changes in time, as follows:

$$\mu_j(\theta) = E(X_j|\theta), \quad j = \overline{1,t}. \tag{5.2}$$

Section 5 contains a *description* of *the credibility regression model allowing for effects like inflation.* Often it is unrealistic to assume that, given θ, the $\underline{X}' = (X_1, \ldots, X_t)$ are i.i.d.. To avoid this restriction, we will introduce the regression technique. The variables describing the contract are $(\theta, \underline{X}')$. Using the conventions for matrix and vector notation, we have as a direct generalization of the Bühlmann hypothesis:

$$\mu_j(\theta) = E(X_j|\theta), \quad j = \overline{1,t}$$

or

$$\underline{\mu}^{(t,1)}(\theta) = E(\underline{X}|\theta) = (\mu_1(\theta), \ldots, \mu_t(\theta))'. \tag{5.3}$$

We restrict the class of admissible functions $\mu_j(\cdot)$ to:

$$\underline{\mu}^{(t,1)}(\theta) = x^{(t,n)}\underline{\beta}^{(n,1)}(\theta) \tag{5.4}$$

where $x^{(t,n)}$ is a matrix given in advance, the so - called design matrix, having full rank $n \leq t$ and where the $\underline{\beta}^{(n,1)}(\theta)$ are the unknown regression constants. It is assumed that the matrices:

$$Cov[\underline{\beta}^{(n,1)}] = a = a^{(n,n)} \tag{5.5}$$

$$E[Cov(\underline{X}|\theta)] = \Phi = \Phi^{(t,t)} \tag{5.6}$$

are positive definite. We finally introduce:

$$\underline{b} = \underline{b}^{(n,1)} = E[\underline{\beta}^{(n,1)}(\theta)]. \tag{5.7}$$

So, let:

$$\mu_j(\theta) = \underline{x}_j'\underline{\beta}(\theta), \tag{5.8}$$

where the non - random $(1 \times q)$ vector $\underline{x}_j'$ is known, and let $\hat{\mu}_j(\theta)$ be the credibility estimator of $\mu_j(\theta)$ based on $\underline{X}'$, with $j = \overline{1,t}$.

Theorem 5.1. *The credibility estimator $\hat{\mu}_j(\theta)$ is given by:*

$$\hat{\mu}_j(\theta) = \underline{x}_j'[Z\hat{\underline{b}} + (I - Z)\underline{b}], \quad j = \overline{1,t}, \tag{5.9}$$

with:

$$\underline{\hat{b}} = \underline{\hat{b}}^{(q,1)} = (x'\Phi^{-1}x)x'\Phi^{-1}\underline{X} \leftarrow \text{ is the best linear } \theta - \text{unbiased estimator of } \underline{\beta}(\theta),$$

$$Z = Z^{(q \times q)} = ax'\Phi^{-1}x(I + ax'\Phi^{-1}x)^{-1} \leftarrow \text{ the resulting credibility factor.}$$

Remark 5.1. $Z\underline{\hat{b}} + (I - Z)\underline{b}$ is the credibility estimator of $\underline{\beta}(\theta)$.

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